THE SECOND ITHACA COLLOQUIUM*

The tenth colloquium of the American Mathematical Society and the second Ithaca Colloquium was held at Ithaca, N.Y., in conjunction with the thirty-first summer meeting of the Society on September 8-12, 1925.

Tied as we are to the decimal system of notation, the attainment of a tenth in any sequence is in the nature of the passing of a milestone. So it might be interesting to give in this connection a brief summary of the colloquia which have been held by the Society up to the present time.†

The colloquium idea was an outgrowth of the desire on the part of members of the Society to hear more extensive treatments of some of the recent developments of the science than can be given in the brief papers which are read at regular meetings of the Society. It was originally planned to hold these colloquia annually, but experience seems to have shown that a larger interval was better. They have been held at intervals of from two to five years, but the plan is to hold them at intervals of not more than two years in the future.

The main facts concerning the first ten colloquia are as follows.‡

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* A report prepared by Professor T. H. Hildebrandt at the request of the Secretary of the Society and the editors of this BULLETIN.

† A résumé of the first five colloquia is to be found in the report of V. Snyder on *The Fifth Colloquium*, this BULLETIN, vol. 13 (1906-7) pp. 72-73. Material covering the first seven is contained in the preface to the volume of *Madison Colloquium Lectures*. See also the list of colloquium lectures published by this Society, on the inside of the front cover of this number.

‡ Perhaps the Evanston Colloquium, held on August 28 to September 9, 1893, should count as the *zeroth* or as the *preliminary* colloquium. Although it was not held under the auspices of this Society (which was not yet existent as such), the Society did acknowledge its interest and indebtedness by republishing the lectures in 1911 (reported by A. Ziwet and first published in 1893).

THE FIRST COLLOQUIUM. Buffalo, N.Y., September 2-5, 1896. Attendance, 13.

M. BÔCHER: *Linear Differential Equations and their Applications.* Not published in full, but parts are contained in a pamphlet on *Linear Differential Equations* (Harvard University, 1898), and a paper in the ANNALS OF MATHEMATICS, (1), vol. 12.

JAMES PIERPONT: *Galois Theory of Equations.* Published in the ANNALS OF MATHEMATICS, (2), vols. 1 and 2.

THE SECOND COLLOQUIUM. Cambridge, Mass., August 22-27, 1898. Attendance, 26.

W. F. OSGOOD: *Some Methods and Problems of the General Theory of Functions.* Published in this BULLETIN, vol. 5, pp. 59-87.

A. G. WEBSTER: *The Partial Differential Equations connected with Wave Propagation.* Not published.

THE THIRD COLLOQUIUM. Ithaca, N.Y., August 21-24, 1901. Attendance, 25.

O. BOLZA: *The Simplest Type of Problems in the Calculus of Variations.* Expanded to his book on *Calculus of Variations* in the DECENNIAL PUBLICATIONS OF THE UNIVERSITY OF CHICAGO.

E. W. BROWN: *Modern Methods of Treating Dynamical Problems and in Particular the Problem of Three Bodies.* An abstract was published in this BULLETIN, vol. 8 (1901-2), pp. 103-113.

THE FOURTH COLLOQUIUM. Boston, Mass., September 2-5, 1903. Attendance, 31.

E. B. VAN VLECK: *Selected Topics in the Theory of Divergent Series and Continued Fractions.*

H. S. WHITE: *Linear Systems of Curves on Algebraic Surfaces.*

F. S. WOODS: *Connectivity of Euclidean Spaces.*

Published for the Society under the title, *The Boston Colloquium,* by the Macmillan Company, in 1905.

THE FIFTH COLLOQUIUM. New Haven, Conn., September 5-8, 1906. Attendance, 43.

E. H. MOORE: *Theory of Bilinear Functional Operations.*

E. J. WILCZYINSKI: *Projective Differential Geometry.*

MAX MASON: *Selected Topics in the Theory of Boundary-Value Problems of Differential Equations.*

Published for the Society under the title, *The New Haven Colloquium*, by the Yale University Press, in 1910.

THE SIXTH COLLOQUIUM. Princeton, N. J., September 15-17, 1909. Attendance, 28.

G. A. BLISS: *Fundamental Existence Theorems.*

E. KASNER: *Geometric Aspects of Dynamics.*

Published by the Society under the title, *Princeton Colloquium Lectures*, in 1913.

THE SEVENTH COLLOQUIUM. Madison, Wis., September 10-13, 1913. Attendance, 51.

L. E. DICKSON: *Certain Aspects of a General Theory of Invariants with special Consideration of Modular Invariants and Modular Geometry.*

W. F. OSGOOD: *Selected Topics in the Theory of Analytic Functions of several Complex Variables.*

Published by the Society under the title, *The Madison Colloquium Lectures*, in 1914.

THE EIGHTH COLLOQUIUM. Cambridge, Mass., September 6-8, 1916. Attendance, 69.

G. C. EVANS: *Theory of Functionals.*

O. VEBLEN: *Analysis Situs.*

Published by the Society under the title, *The Cambridge Colloquium Lectures*, in 1918 and 1922.

THE NINTH COLLOQUIUM. Chicago, Ill., September 8-11, 1920. Attendance, 90.

G. D. BIRKHOFF: *Dynamical Systems.*

F. R. MOULTON: *Theory of Functions of Infinitely Many Variables.*

To be published by the Society.

THE TENTH COLLOQUIUM. Ithaca, N.Y., September 8-12, 1925. Attendance, 122.

L. P. EISENHART: *The New Differential Geometry.*

D. JACKSON: *The Theory of Approximation.*

The committee in charge of the present colloquium consisted of L. E. Dickson, H. H. Mitchell, J. H. Tanner, and H. S. White. Professor Dickson resigned before the choice of lecturers, and H. H. Mitchell was appointed chairman of the committee in his stead. Two groups of five lectures each were given in the Lecture Room of Baker Laboratory, one by each lecturer on Tuesday afternoon, Wednesday afternoon, Thursday evening, Friday afternoon, and Saturday morning. The following one hundred twenty-two persons, the largest number in attendance at any colloquium held up to the present time by the Society, were present at the lectures.

C. R. Adams, R. B. Adams, W. E. Anderson, Archibald, Bacon, Bareis, Bell, A. A. Bennett, T. Bennett, H. Betz, Birkhoff, Bliss, H. S. Brown, M. Buchanan, W. G. Bullard, R. W. Burgess, Bussey, E. B. Callahan, B. H. Camp, A. D. Campbell, C. W. Carter, Carver, J. T. Colpitts, Coolidge, L. P. Copeland, Court, E. F. Cox, C. F. Craig, Denton, Dresden, Dunkel, Eiesland, Eisenhart, G. C. Evans, Everett, F. Farnum, W. B. Ford, Fort, Fry, Gaba, Gale, D. C. Gillespie, Gilman, M. C. Graustein, W. C. Graustein, L. M. Graves, E. R. Hedrick, Herr, Hildebrandt, Hille, Hollcroft, A. M. Howe, Hurwitz, Ingraham, D. Jackson, O. D. Kellogg, E. H. Kennard, B. F. Kimball, Kuhn, W. D. Lambert, Lefschetz, C. A. Lindeman, Long, Lubben, J. V. McKelvey, M. M. McKelvey, MacColl, MacCreadie, MacDuffee, Maria, H. A. Merrill, Michal, G. A. Miller, N. Miller, H. H. Mitchell, Molina, C. L. E. Moore, C. N. Moore, D. S. Morse, H. M. Morse, F. H. Murray, Nassau, Olds, Olson, F. W. Owens, L. R. Perkins, Poritsky, Ranum, Rasor, R. G. D. Richardson, D. E. Richmond, H. Rickard, H. L. Rietz, Schelkunoff, Seely, Sellew, H. C. Shaub, Shewhart, Shohat, Slaught, Smail, C. E. Smith, H. F. Smith, Snedecor, Sullivan, J. Tamarkin, Tappan, J. H. Taylor, E. M. Thomas, J. M. Thomas, T. Y. Thomas, B. M. Turner, Vivian, G. W. Walker, W. J. Wallip, Watkeys, H. E. Webb, Wedderburn, Weisner, W. L. G. Williams, Williamson, Yeaton.

Below are the synopses of the lectures as prepared by the speakers, one lecture being devoted to each section, with the exception of the material in Section III of the series by Professor Eisenhart, which was covered in his third and fourth lectures.

THE NEW DIFFERENTIAL GEOMETRY

BY PROFESSOR L. P. EISENHART

I. *Riemannian Geometry.* The fundamental quadratic form and the measurement of lengths and angles. Geodesics. Geodesic coordinates and Riemannian coordinates. Parallelism of Levi-Civita; its intrinsic character. Curvature of a curve. Associate directions of Bianchi. Parallelism and associate directions in a sub-space.

II. *Linear Connection of a Space.* Infinitesimal displacement of a contravariant vector. Coefficients of the linear connection. Covariant differentiation. The generalized identities of Ricci and the generalized curvature tensor. Fundamental tensors of the second order. Finite displacement of a contravariant vector along a curve. Displacement of a contravariant vector around an infinitesimal closed circuit. Infinitesimal displacement of a covariant vector. Schouten's generalization. The paths of the space. Curvature of a curve and the associate direction of a vector displaced along a curve. Linear connections determined by n independent contravariant vectors (Weitzenboeck). Different linear connections for which the directions of a displaced vector are the same. Schouten's semi-symmetric linear connections.

III. *Geometry of Paths.* Affine connection of a space. Normal coordinates (Veblen). Path coordinates. Affine connections with the same paths. Projective geometry of paths. The Weyl tensor and projective flat-spaces. Normal affine connection (Cartan). Projective geometry of a path (T. Y. Thomas). Projective normal coordinates (Veblen and J. M. Thomas). Projective invariants. First integrals of equations of paths. Weyl's affine connection.

IV. *Geometry of a Sub-Space of a Linearly Connected Space.* Components in the sub-space of a tensor in the enveloping space. Induced linear connection of the sub-space. Relative curvature and associate direction of a vector displaced along a curve. Generalized equations of Gauss and Codazzi. Generalizations of lines of curvature and asymptotic lines.

THE THEORY OF APPROXIMATION, BY PROFESSOR DUNHAM JACKSON

I. *The Approximate Representation of Continuous Functions.* Weierstrass's theorem. Approximate representation by trigonometric sums in the case of a function satisfying the Lipschitz condition. Approximate representation of functions satisfying other conditions of continuity. Corresponding theorems on polynomial approximation. Convergence and degree of convergence of the Fourier series for a continuous function. Convergence and degree of convergence of the Legendre series for a continuous function. Other forms of approximating function.

II. *Discontinuous Functions and Functions of Limited Variation.* Riemann's theorem on the Fourier coefficients of an arbitrary function. Convergence of the Fourier series in an interval of continuity. Convergence at a point of discontinuity. Uniformity of convergence in an interval of continuity. Convergence in the case of a function of limited variation. Order of magnitude of the coefficients under hypotheses of limited variation; order of magnitude of the mean square error. Degree of convergence in an interval of continuity. Partial analogies in the case of Legendre series.

III. *The Principle of Least Squares and its Generalizations.* The least square property of the Fourier series. Bernstein's theorem on the derivative of a trigonometric sum. Proof of convergence on the basis of the least-square property. Convergence under relaxation of the requirement of a minimum. Convergence with a weight function. Convergence with an arbitrary positive power of the error. Bernstein's theorem for polynomials. Polynomial approximation: Legendre series, series of Tchebycheff polynomials, and approximation with an arbitrary power of the error; convergence throughout the interval; convergence in the interior of the interval. Polynomial approximation over an infinite interval. Other generalizations.

IV. *Interpolation.* Formal solution of the problem of trigonometric interpolation with an odd or even number of points equally spaced over a period. Formal solution of the problem with unequally spaced points. Convergence and degree of convergence of the interpolation formulas for a continuous function, in the case of equally spaced points. An interpolation formula analogous to Fejér's integral in the theory of Fourier series. Convergence and degree of convergence of the interpolation formulas for a discontinuous function. A special problem of polynomial interpolation.

V. *The Geometry of Function Space.* Geometrical significance of the principle of least squares; distance and angle in function space. Application to the theory of statistics. The geometry of frequency functions. Theorems of orthogonality in space of many dimensions. Beginnings of vector analysis.

As announced elsewhere in the present issue of this BULLETIN, the publication of these colloquium lectures has been referred by the Council of the Society to the Committee on Printing. It is anticipated that they will be published in full at an early date.

T. H. HILDEBRANDT