BULLETIN

OF THE

AMERICAN MATHEMATICAL SOCIETY

THE THIRTY-FIRST SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY

A notable occasion in the annals of the Society was the holding of the thirty-first summer meeting and tenth colloquium of the Society at Cornell University, Ithaca, N.Y., from Tuesday to Saturday, September 8-12, 1925. For such a gathering the location was ideal; the local arrangements left nothing to be desired; there was a record attendance, and enthusiasm ran high.

The colloquium lectures, by Professors Eisenhart and Jackson, were delivered on Tuesday, Wednesday, and Friday afternoons, Thursday evening, and Saturday morning. On Thursday and Friday mornings the Society met in sections for the reading of papers, the sections of Geometry and of Point Sets and Foundations meeting on Thursday, and those of Analysis and of Algebra and Theory of Numbers on Friday. Sessions of the Mathematical Association of America were held on Tuesday and Wednesday mornings. The joint dinner of the Society and the Association, held at the Bank Restaurant in Ithaca on Wednesday evening, with Professor Slaught as toastmaster, was attended by one hundred eighty-five persons.

Many attending members recalled with pleasure the two earlier summer meetings of the Society held at Cornell, in 1901 and 1907. The Society held its Third Colloquium in connection with the 1901 meeting; on this occasion Professor

Oskar Bolza lectured on *The simplest type of problems in the calculus of variations*, and Professor E. W. Brown on *Modern methods of treating dynamical problems and in particular the problem of three bodies*. The 1925 colloquium should therefore be known as the Second Ithaca Colloquium. A special report of this colloquium by Professor Hildebrandt is given in the present issue of this BULLETIN on page 24.

The attending mathematicians and their friends were entertained at the Sage College dormitory of the University, and the meetings were held in the Baker Chemical Laboratory. On Tuesday evening President Farrand and the Mathematics Department welcomed the visitors at a reception in Sage College. Through the courtesy of the Automobile Club of Ithaca an excursion was made on Thursday afternoon to Enfield Glen, a very beautiful gorge near Ithaca. The Ithaca Country Club extended an invitation to visitors to make use of its facilities. Other enjoyable occasions were a luncheon at the home of Mrs. Owens for attending ladies, a reception at the Ithaca Country Club given by Mrs. Tanner, a visit to the Fuertes Observatory, and an organ recital by Professor Ballard of the department of Electrical Engineering. A hearty vote of thanks was passed to the various hosts and to the local members of the Committee on Arrangements, Professors Tanner, Gillespie, and Hurwitz.

One hundred twenty-two members of the Society heard the Colloquium lectures. Attendance at the sessions of the Society included the following one hundred fifty members:

C. R. Adams, R. B. Adams, Akers, W. E. Anderson, Archibald, Bacon, Bareis, E. R. Beckwith, Beisel, Bell, Bennett, Herman Betz, Birkhoff, Bliss, Boothroyd, H. S. Brown, Margaret Buchanan, W. G. Bullard, R. W. Burgess, Bussey, Callahan, B. H. Camp, A. D. Campbell, Carr, Carus, Carver, Coble, Teresa Cohen, J. T. Colpitts, Coolidge, L. P. Copeland, Court, Craig, Dale, Decker, Denton, Dowling, Dresden, Dunkel, Eiesland, Eisenhart, W. W. Elliott, G. C. Evans, G. W. Evans, Everett, Peter Field, W. B. Ford, Fort, Fry, Gaba, Gale, Garretson, D. C. Gillespie, Gilman, Glenn, J. W. Glover, M. C. Graustein, W. C. Graustein, Gravatt, L. M. Graves, E. R. Hedrick, Herr, Hildebrandt, Hille, Hollcroft, A. M. Howe, Hurwitz, Hutchinson, Ingraham, Dunham Jackson, B. W. Jones,

O. D. Kellogg, B. F. Kimball, Kuhn, Lambert, Lefschetz, Long, Lubben, J. V. McKelvey, M. M. McKelvey, MacColl, MacCreadie, MacDuffee, Maria, H. A. Merrill, Michal, E. A. Miller, G. A. Miller, Norman Miller, H. H. Mitchell, Molina, C. L. E. Moore, C. N. Moore, Morenus, Richard Morris, D. S. Morse, Marston Morse, F. H. Murray, Nassau, Olds, Olson, F. W. Owens, H. B. Owens, Packer, L. R. Perkins, Poritsky, Ranum, Rasor, R. G. D. Richardson, H. L. Rietz, Robinson, E. D. Roe, J. R. Roe, Rosenbaum, Schelkunoff, Seely, Sellew, Sharpe, Shewhart, Shohat, Slaught, Smail, C. E. Smith, H. F. Smith, Snedecor, Virgil Snyder, Stocker, Sullivan, Tamarkin, Tappan, J. H. Taylor, E. M. Thomas, J. M. Thomas, T. Y. Thomas, Trevor, B. M. Turner, Vandiver, Vivian, Waddell, G. W. Walker, Watkeys, H. E. Webb, Wedderburn, Weisner, Westfall, W. L. G. Williams, Williamson, E. W. Wilson, Yeaton.

The Assistant Secretary announced that the following persons and institutions had been elected to membership since the April meeting of the Council:

To sustaining membership:

International Life Insurance Company, St. Louis, Mo.;
Massachusetts Institute of Technology, Cambridge, Mass.;
Northwestern University, Evanston, Ill.;
Philadelphia Electric Company, Philadelphia, Pa.;
Western and Southern Life Insurance Company, Cincinnati, Ohio.

To ordinary membership:

Mr. Glenn Potter Aldrich, State University of Iowa;
Professor Mildred Allen, Mt. Holyoke College;
Mr. Ben Raymond Beisel, Cornell University;
Mr. Charles Franklin Bowles, South Dakota State School of Mines;
Dr. Gregory Breit, Department of Terrestial Magnetism, Carnegie Institution of Washington;
Mr. William Marshall Bullitt, Louisville, Ky.;
Mr. Robert Hamilton Coats, Dominion Bureau of Statistics, Ottawa, Canada;
Mr. Irving Smith Cranford, Columbia University;
Mr. George Adams Ellis, New York, N.Y.;
Mr. Hallett Barker Hammatt, Harvard University;
Mr. George Hartwell, Cheltenham Magnetic Observatory;
Dr. William Jackson Humphreys, United States Weather Bureau;
Mr. Wilmer Atkinson Jenkins, University of Michigan;
Professor Louis Vessot King, McGill University;
Mr. Alfred Korzybski, New York, N.Y.;
Mr. Silvio G. Lanzon, Toronto, Canada;
Mr. Lincoln LaPaz, Dartmouth College;
Mr. Morris Marden, Harvard University;
Mr. Tsao Matsumura, Miyazaki Normal School;

Vice-President Paul Vaughan Montgomery, Southland Life Insurance Company, Dallas, Tex.;
Mr. Lucius Terrell Moore, Johns Hopkins University;
Professor Harold R. Phalen, Armour Institute of Technology;
Mr. Boris Podolsky, Los Angeles, Calif.;
Mr. Elder Alexander Porter, Indianapolis Life Insurance Company;
Professor Adrien Pouliot, Laval University;
Mr. Henry William Raudenbush, Columbia University;
Mr. Pierre Gain Robinson, Iowa State College;
Professor Herbert E. Russell, University of Denver;
Miss Faith Saunders, Logan College;
Professor Albert Norman Shaw, McGill University;
Miss Mary Frazer Smith, Wellesley College;
Mr. Martin Joseph Spinks, Champion Bridge Company, Wilmington, Ohio;
Mr. Arthur Francis Chesterfield Stevenson, University of Toronto;
Professor Jacques Tamarkin, Dartmouth College;
Miss Frances Edget Thomas, Wells College;
Dr. Manuel Sandoval Vallarta, Massachusetts Institute of Technology;
Mr. Hubert Stanley Wall, University of Wisconsin;
Mr. Morgan Ward, California Institute of Technology;
Professor Walter F. Willcox, Cornell University;
Mr. Don M. Yost, California Institute of Technology;
Dr. Otto Julius Zobel, American Telephone and Telegraph Company, New York, N.Y.;

Nominees of the Aetna Life Insurance Company, Hartford, Conn.:
Paul Dorweiler, Morley I. Doxsie, Roland S. Haradon, Elton B. Hill, Walter S. Paine;

Nominees of the American Life Insurance Company, Detroit, Mich.:
William H. Brown, C. F. Cross, Walter H. Ekberg;

Nominees of the Detroit Life Insurance Company, Detroit, Mich.:
Henry R. Carstens, Clements H. Kettenhofen, Eva M. Roe, Margaret Toft, E. C. Wightman;

Nominees of Harvard University, Cambridge, Mass.:
Professors Harry E. Clifford, Hector J. Hughes, Theodore Lyman, Har low Shapley, George F. Swain;

Nominee of the International Life Insurance Company, St. Louis, Mo.:
T. C. Rafferty;

Nominees of the Maccabees, Detroit, Mich.:
W. P. Coler, Roy Deng, Maurice Hartwell, F. H. Lee, J. E. Little;

Nominees of the Massachusetts Institute of Technology:
Professors Charles Edward Fuller, Harry Manley Goodwin, William Hovgaard, Frederick George Keyes, Edward Pearson Warner;

Nominees of the Missouri State Life Insurance Company, St. Louis, Mo.:
Lucy Andrews, O. J. Burian, Alfred Jekel, B. E. Shepherd, Douglas Wood;

Nominees of the National Life Insurance Company of the United States of America, Chicago, Ill.:

Miguel Antonio Basoco, University of Chicago; Esther Comegys, Wellesley College, Chester George Jaeger, University of Missouri; Herbert S. Thurston, Brown University;

Nominees of the Pacific Mutual Life Insurance Company, Los Angeles, Calif.:

Waldo D. Boss, Leslie J. Cooper, Alfred G. Hann, Arthur N. Havens, William H. Otis, Oscar Swenson, Ellis E. Thomas, Cecil V. VanWyck, Roy Wheeler;

Nominees of the Philadelphia Electric Company, Philadelphia, Pa.:

Raymond Bailey, C. W. Bates, P. H. Chase, John L. Conner, N. E. Funk, Howard S. Phelps, Farley Ralston, George Schleicher, W. G. Wagner, Robert A. Walton;

Nominees of the Prudential Insurance Company, Newark, N. J.:

R. S. Brush, A. H. Fitzgerald, F. D. Kineke, E. L. Lundgren, G. S. Mower;

Nominees of the Travelers Insurance Company, Hartford, Conn.:

Charles H. Davis, James Strode Elston, B. D. Flynn, Joseph D. Flynn, Edward Bontecou Morris.

The Council announced the appointment of the following committees: Professor W. B. Ford, Mr. Robert Henderson, Professors D. N. Lehmer, E. J. Townsend, and Oswald Veblen (Chairman) to nominate officers and members of the Council for 1926; Professors R. C. Archibald, W. D. Cairns, Arnold Dresden, W. A. Luby, and E. B. Stouffer (Chairman) to arrange for the Western Christmas meeting in Kansas City, December, 1925. Professor Harris Hancock will represent the Society at the semicentennial anniversary of Vanderbilt University, October 15-19, 1925.

The invitations of Hunter College, New York City, for the next Annual Meeting; of the Ohio State University for the Summer Meeting in 1926; and of the University of Wisconsin for the Summer Meeting and a Colloquium in 1927, were accepted with the thanks of the Society.

It was announced that the Bulletin for 1926 is to be printed by the George Banta Publishing Company, Menasha, Wisconsin, and that it has been decided to continue the issue of six numbers annually. The problem connected with the printing and distribution of the Colloquium Lectures was referred to the Committee on Printing.

Professor J. L. Coolidge presented his resignation as chairman of the Committee on Endowment to take effect at the end of the present year. Professor Arnold Dresden also resigned. The goal set by the Committee has been reached, provided the present number of Sustaining Memberships can be maintained. Professor Tomlinson Fort (Chairman) and Dr. R. W. Burgess were added to the Committee.

A report of the Congress on the use of Esperanto in Science was received from Professor Fréchet, the delegate of the Society; an abstract appears in the present issue of this BULLETIN. President Birkhoff announced that he had sent a congratulatory cable to the London Mathematical Society on the occasion of a dinner to celebrate its sixtieth anniversary.

The Council adopted resolutions of thanks to the Assistant Secretary for his able and devoted service in carrying the additional duties of the Secretary during the six months absence of the latter in Europe, and to Mrs. Anna J. Pell for a contribution to the Endowment Fund in honor of her late husband, Professor Alexander Pell.

Titles and abstracts of the papers read before the sectional sessions of the Society follow below. The papers numbered 1 to 22 were read before the section of Geometry on Thursday morning, Vice-President Evans presiding; numbers 23 to 31 before the section of Point Sets and Foundations, Thursday morning, Vice-President Hildebrandt presiding; numbers 32 to 46 before the section of Analysis, Friday morning, President Birkhoff presiding; and numbers 47 to 60 before the section of Algebra and the Theory of Numbers, Friday morning, Professor Bell presiding. Mr. Agnew was introduced by Professor McKelvey, and Professor Schouten by Professor Kasner. Professor R. L. Moore's papers were read by Professor Bennett, and the papers of the following authors were read by title: Brinkmann, Dines, Eiesland (second paper), Ettlinger, Ford, Garabedian, Gronwall, Murnaghan, Nelson, Rainich, Schouten, Tamarkin and Wilder, Vandiver, Weisner (second paper), Wilson.

1. Professor F. R. Sharpe: *Space involutions having a web of invariant rational surfaces.*

The author shows that if the web has at least one simple basis point the involution can be mapped on a rational cubic variety. In all the cases found the cubic variety has at least one double point and can therefore be mapped on a second space. The involution is therefore reducible to a perspective monoidal involution. If there is no simple basis point there is a finite number of independent types of involutions.

2. Professor W. B. Carver: *Note on six points in a plane and the six conics determined by them.*

The author notes certain properties of a set of six points in a real projective plane which are invariant under the group of real linear transformations, the six points not lying on a conic and no three of them being collinear. Two characteristics of each point with respect to the set may be noted: (1) Each point P_i is either inside or outside of the conic C_i determined by the remaining five points; or, briefly, each point is an *inside* or *outside* point of the set. (2) Through each point P_i there pass five conics C_j, $j \neq i$, with tangents t_j at P_i; and also there are five lines l_j joining P_i to each of the other five points P_j; the five tangents t_j are projective with the five lines l_j; and hence, going round the point P_i, the two sets of lines t_j and l_j are met either in the same or in opposite order; and one may say that P_i is respectively a *consistent* or a *contradictory* point of the set. Every set of six points is of one of three types: (A) Each of the six points is inside and consistent. (B) Two points are inside and consistent, and four are outside and contradictory. (C) Three points are inside and contradictory, and three are outside and consistent. If the six points are "nearly" on a conic, the set is of type (C).

3. Professor T. R. Hollcroft: *On the reality of singularities of plane curves.*

By the use of Plücker's equations, Klein's theorem, and the Lefschetz postulate, the maximum number of real cusps of a curve of given order and genus is found. Limits for other real and imaginary singularities are also derived. The maximum number of cusps of a curve of given order and genus has been found by Lefschetz. It is known that, in general, not all of these cusps can be real, and limits for the number of real cusps are found in each case.

4. Dr. Louis Weisner: *Self-projective plane 5-points.*

The problem considered in this paper may be stated as follows: Determine the largest collineation group under which an arbitrary set of five coplanar points, no three of which are collinear, is invariant, and determine all special 5-points which are invariant under a larger group. It is shown that in general a 5-point is invariant only under the identical transformation. There are four special 5-points which are invariant under a larger group. Two of them are imaginary. The two real special 5-points are (1) a set of five points, four of which are the vertices of an isosceles trapezoid whose axis of symmetry passes through the fifth; (2) the vertices of a regular pentagon.

5. Professor A. D. Campbell: *Plane cubic curves in the Galois fields of order* 2^n.

In this paper are completely tabulated, for the first time, the classes of non-degenerate and degenerate plane cubic curves in the finite analytical geometries whose algebras are those of the Galois fields of order 2^n. A typical cubic curve is given for each class. A complete new machinery is set up to test these cubic curves for nodes or cusps and points of inflection, and to discover the cubic curves that have no real points on them. The Hessian of one of these cubics is the curve itself. There occur cubics with 0, 1, 3, 7, or 9 real points of inflection on them. The coefficient k of the xyz term in the equation of a cubic is a remarkable relative invariant. A corresponding net of conics is given for the general cubic (for which net this cubic is the discriminant of a general conic). The polar conics of these cubics have strange properties.

6. Dr. F. H. Murray: *Generalization of certain theorems of Bohl.* Second paper.

As in the first paper, the motion of a dynamical system is assumed to be defined by a system of differential equations in the canonical form, which are satisfied by a set of constant values of the variables; the behavior of the trajectories near this point depends on the nature of the characteristic exponents for the equations of variation. In this paper it is shown that with fewer restrictions on the characteristic exponents, existence theorems can be proved which are similar to those obtained in the first paper; in addition the asymptotic properties of certain trajectories remaining near the fixed point are studied by methods similar to those of Bohl.

7. Professor C. L. E. Moore: *Ricci notation for geometrical products.*

The author shows how to express the usual geometrical products by the use of Ricci's "system ϵ."

8. Mr. G. Y. Rainich: *Projection of a fixed vector on a surface.*

On a surface in ordinary space a vector field is formed by projecting a fixed vector on the tangent plane at each point; this field satisfies a second order differential equation. It can also be given by means of a scalar field whose values are lengths of projections of the same fixed vector on the normals. The scalar field also satisfies a second order equation. In the case of a minimal surface this differential equation expresses an internal property. On a one-sided minimal surface the solutions are two-valued, a solution changing its sign when we pass from one side to the other. Some of the results are extended to hypersurfaces and curved spaces.

9. Mr. G. Y. Rainich: *Mass in curved space time.*

Contrary to the opinion which seems to be generally accepted, the mass is not connected with the *contracted* but is implicitly contained in the complete (not contracted) Riemann tensor. In the case of the centro-symmetric solution, mass cannot be separated from the electric charge, i. e., it is impossible to separate linearly from the Riemann tensor a part which contains the constant corresponding to the mass and does not contain the electric charge constant, unless we separate time and space. In the spaces obtained as a result of such a separation, mass appears as a residue of a field determined by the Riemann tensor of space-time.

10. Professor B. H. Camp: *Mutually consistent regression surfaces for three-dimensional frequency solids.*

In the normal solid of frequency, all regressions are linear in form, but for solids that admit of non-linear regression there are interrelations between regression surfaces and total regression curves which severely restrict one's freedom in choosing forms to fit observed surfaces. Some mutually consistent sets are described. It becomes apparent that simple polynomial regression surfaces may imply very complicated forms for the total regressions, save in the all linear case only. This fact has applications in various fields. When complicated two-variate regression is encountered, this does not necessarily mean that the functional relationship would not be simple after a third variate was added. It is suggested that the form "y (polynomial in x) equals polynomial in x" is of frequent occurrence among two-way regressions and should be tried instead of "y equals polynomial in x" when the regression fails to be linear.

11. Professor A. A. Bennett: *New properties of an orthocentric system of triangles.*

Let T_1, T_2, T_3 be any triangle, and let T_0 be the orthocenter. Let T_0', T_1', T_2', T_3' be the reflexions of T_0, T_1, T_2, T_3, respectively, in N, the center of the nine-point circle. The author states the following as the result of computation with concomitants of simultaneous quadric and cubic binary forms. There is a pencil of cubic curves through the nine points N, T_0, T_1, T_2, T_3, T_0', T_1', T_2', T_3'. One of these, K_M, passes also through the six midpoints M_1, M_2, M_3, M_1', M_2', M_3', each of which is the midpoint of two mutually perpendicular sides. Another, K_P, passes through the six pedal points. A third, K_C, is a circular cubic. The real asymptote of K_C is the inflectional tangent of K_M at N and is the second polar with respect to the two circular points of the medial line triple NM_1, NM_2, NM_3. The entire system is determined completely by any one of the eight triangles $T_0T_1T_2$, $T_1T_2T_3$, $T_2T_3T_0$, $T_3T_0T_1$, $T_0'T_1'T_2'$, $T_1'T_2'T_3'$, $T_2'T_3'T_0'$, $T_3'T_0'T_1'$, in contrast to the more familiar figures associated with a triangle.

12. Dr. H. W. Brinkmann: *Solutions of the Einstein equations for empty space.*

Various solutions of the Einstein equations for empty space, $R_{ij}=\frac{1}{4}Rg_{ij}$, are obtained when certain simplifying assumptions are made concerning the coefficients of the fundamental quadratic differential form. In particular, it is shown that the Schwarzschild solar field line element furnishes essentially the only solution for which said quadratic form looks like this: $ds^2 = Edx_1^2+2Fdx_1dx_2+Gdx_2^2+A(edx_3^2+2fdx_3dx_4+gdx_4^2)$, where E, F, G, A are functions of x_1, x_2 alone and e, f, g are functions of x_3, x_4 alone.

13. Dr. H. W. Brinkmann: *Einsteinian 4-spaces imbedded in euclidean 5-space.*

In a recent paper (MATHEMATISCHE ANNALEN, vol. 94 (1925), Theorem IX, on p. 140) the author has given an example of a 4-space satisfying the Einstein equations $R_{ij}=0$ (R_{ij} being the contracted Riemann-Christoffel tensor) which may be imbedded in euclidean 5-space. In the present paper it is shown that any 4-space having this property can be put in the form there given.

14. Dr. H. W. Brinkmann: *Riemann spaces of class one.*

A Riemann n-space is of class one if it can be imbedded in euclidean $(n+1)$-space. The present note gives a simple condition which is sufficient to ensure that a given n-space ($n>3$) be of class one. This condition is not necessary, but is satisfied by such spaces "in general."

15. Dr. H. W. Brinkmann: *The torsion of a Riemann n-space imbedded in a euclidean m-space* ($m\geqq n+2$).

The torsion of a Riemann space imbedded in a euclidean space is defined and its properties are investigated. It is found necessary to introduce "torsion vectors" and antisymmetric "torsion tensors" of rank two. The most pleasing results are obtained for the case $m=n+2$, where the study of the torsion depends upon that of a single Pfaffian expression.

16. Professor C. A. Garabedian: *Solution of the problem of the thick rectangular plate with two opposite edges supported and two edges free, and under uniform or central load.*

In a recent note on beams (COMPTES RENDUS, vol. 179 (1924), p. 381) space prevented the author from giving more than the central deflection of the plate here considered (the load being uniform). Although the complete solution had been obtained at that time, it was in rather lengthy form. It is interesting to try to express this solution in terms of the thin plate solution and its derivatives, as has been shown possible, for example, in the case of the clamped or supported plate (COMPTES RENDUS, vol. 180 (1925), p. 257). The author has just succeeded in writing the thick plate problem in the concise form desired. And, as in the example cited, the formulas for central load are obtained at once from those for uniform load by merely using the appropriate thin plate solution and suppressing the terms in which the letter p (load) appears explicitly. An abstract of these results will be found in the COMPTES RENDUS. The author plans to give the details in a subsequent paper on beams.

17. Professor C. A. Garabedian: *Rectangular plates of constant or variable thickness.*

This paper is a sequel to the author's *Circular plates of constant or variable thickness* (TRANSACTIONS OF THIS SOCIETY, vol. 25 (1923), pp. 343-398). It develops in detail the method of series in rectangular coördinates, and obtains *usable* solutions of a number of outstanding problems in thick rectangular plates and in beams of rectangular section. The differential equations that now present themselves for solution are *partial* rather than *total*, but happily this increase in complexity is offset by a widening of the field of applications. In circular plates not all the solutions in constant thickness were new, and the differential equations in variable thickness were integrable in certain cases; in the present paper the partial differential equations in variable thickness are somewhat complicated, but the applications to problems in constant thickness are all believed to be new, a situation apparently due to the lack hitherto of a suitable method of attack.

18. Professor J. A. Eiesland: *A generalization of the tetrahedral complex in odd* S_{n-1}. Preliminary report.

If we consider two pencils of flats (P, E), (P', E') in $(n-1)$-space and let a projective relation be established between them, all the flats that intersect corresponding flats in the two pencils will define a quadratic flat-complex analogous to the tetrahedral complex in 3-space. The singular surface of the complex is then considered. It is generated by ∞^2 flats which belong to a linear flat-complex of rank $n-2$. The surface is of $(n+2)/2$ dimensions and has two conics for double edges. This class of spreads has been studied by the author in an earlier paper (TÔHOKU MATHEMATICAL

JOURNAL, vol. 16). Its transform in sphere-space is a generalized Dupin cyclide of type $n-2$. The result translated into sphere-space is as follows: The singular surface of a quadratic sphere-complex, consisting of all the spheres which touch corresponding spheres of two projective pencils of spheres, is a Dupin cyclide of the type $n-2$.

19. Professor J. A. Eiesland: *The loci of point singularities on a generalized Kummer surface in odd S_{n-1}.* Preliminary report.

The author studies the loci of double, triple, and quadruple points on the generalized Kummer surface in flat-geometry; their respective orders are obtained, and an important contact-locus on the surface is also examined.

20. Dr. C. A. Nelson: *Note on rational plane cubics.*

This paper appears in full in the present issue of this BULLETIN.

21. Dr. T. Y. Thomas: *A projective theory of affinely connected manifolds.*

This paper gives a reduction of the projective theory of a given n-dimensional manifold of affine connection to the affine theory of an associated manifold involving one additional dimension. Thus the ordinary methods of the Ricci calculus may be applied immediately to the development of the projective geometry. The associated manifold is subject to a particular group of transformations, but this causes no difficulty and one easily arrives at a method of projective extension, a system of projective normal coördinates, the replacement (reduction) theorem, a complete set of projective invariants, etc., all of which have their counterpart in the ordinary affine theory of the manifold. The paper will be published in the MATHEMATISCHE ZEITSCHRIFT.

22. Professor F. D. Murnaghan: *A study of the conformal mapping $w=az+b/z+c/z^2$, and its application to aerodynamics.*

The mapping $w=az+b/z+c/z^2$ sends the unit circle in the z plane into a bicircular quartic and the constants may be chosen so that the curve has a cusp and has the general shape of the well known Joukovsky profiles. The left and turning moment are calculated in the usual way.

23. Professor A. D. Campbell: *On the use of fractions in the algebra of logic.*

In this paper a proposition such as "All a is b," when used as a premise or conclusion of a syllogism, is written algebraically as a fraction a/b.

Four rules are given for the use of these fractions in the algebraic representation of syllogisms. For example, the syllogism "All b is c, all a is b, therefore all a is c" is expressed algebraically by writing $(b/c)(a/b) < a/c$. All the types of syllogisms are then given algebraically, using these fractions.

24. Professor R. L. Moore: *Concerning the relation between separability and the proposition that every uncountable point set has a limit point.*

In this paper it is shown that in order that every subset of a given class D of Fréchet should be separable it is necessary and sufficient that every uncountable subclass of that class should *have* a limit point, and in order that this condition should be fulfilled it is necessary and sufficient that every uncountable subclass of D should *contain* a point of condensation of itself. It follows that a class D is separable if, and only if, every subset of it is separable.

25. Professor R. L. Moore: *Concerning the separation of point sets by curves.*

The following proposition is proved and, with its help, the author establishes other results concerning accessibility and the separation of point sets by simple closed and open curves. Theorem: If T, the common part of two bounded continua M and K, is totally disconnected and neither M nor K separates the plane, and $K-T$ is connected, then there exists a simple closed curve which encloses $K-T$ but no point of M and which contains T but no point of $M+K-T$.

26. Dr. R. G. Lubben: *The double elliptic case of the Lie-Riemann-Helmholtz-Hilbert problem of the foundations of geometry.*

Hilbert has formulated a set of axioms concerning a group of motions which is sufficient to necessitate that this group be simply isomorphic with either the euclidean or the Bolyai-Lobatschefskian group of rigid motions in the plane. He assumes, however, that the set of points which undergoes the transformation is a number manifold. R. L. Moore has given a treatment in which this assumption is not made in advance, but in which there is a simultaneous analysis of the group of transformations and of the space which undergoes this transformation. In the present paper we shall give a similar analysis for the double-elliptic case. The problem (solved by Hilbert for the euclidean case by methods which do not apply here) of proving the congruence of the base angles of an isosceles triangle takes up a large part of the paper and presents particular difficulties. The problem is solved with the use of trigonometry built up by methods similar, in part, to those used by W. H. Young for the euclidean and Bolyai-Lobatschefskian spaces.

27. Dr. R. G. Lubben: *Concerning limiting sets.*

Let G be a collection of point sets and let K be the accumulation set of G. The following results are established in this paper. (1) If K is bounded and P is a point of K, there exists a sub-collection of G which has a sequential limiting set containing P. (2) There exist at most a countable number of elements of G which are not subsets of K. (3) If the elements of G are connected point sets, and M is a maximal connected subset of K, then there exists a sub-collection of G whose accumulation set is M. (4) If G is a collection of connected point sets, then a necessary and sufficient condition that K be connected is that there exist a sub-continuum M of K such that the accumulation set of every infinite sub-collection of G contains points of M. (5) If G is a bounded collection of closed point sets, then, if for each positive number e and each point P belonging to an element of G there exist an element g_{eP} of G whose upper distance from P is less than e, G is a separable set of sets.

28. Dr. R. G. Lubben: *Surrounding theorems with applications to questions of accessibility.*

The author gives the following generalization of surrounding theorems established by Zoretti and by R. L. Moore. If K is a bounded point set and if L is either K or a closed point set consisting of the points belonging to a collection of maximal connected subsets of K, and e is a positive number, then there exist a finite number of domains whose sum covers L such that no one of these domains plus its boundary contains any points or boundary points of any of the remaining domains; no point or boundary point of one of these domains is at a distance from L greater than e; the boundary of each domain consists of a finite number of simple closed curves none of which contain a point of K. The author also shows that if M and N are bounded point sets, M' and N' have no points in common, and there exists an arc AXB which contains no point of M and an arc AYB which contains no point of N, then there exists an arc AZB which contains no point of $M+N$. The author also states a necessary and sufficient condition that a boundary point of domains be accessible from the domain.

29. Dr. L. M. Graves: *Taylor's theorem in general analysis.*

Consider an abstract set X of elements x, which together with certain operations of addition, multiplication by real or complex numbers a, and taking the modulus, constitute a space called by Fréchet "*espace* (*D*) *vectoriel*" (Fréchet, COMPTES RENDUS, vol. 180 (1925), p. 419; Banach, FUNDAMENTA MATHEMATICAE, vol. 3 (1922), p. 134). Let Y be a second such space, which is also complete, in the sense that every regular sequence has a limit in the space. Let F be a function defined and of the class $C^{(n)}$ on a convez region X_0 of X, and with functional values in the space Y. The definition of the class $C^{(n)}$ generalizes that given by Bolza in his *Variationsrechnung*, page 14. Then the function F may be expanded about an arbitrary element x of the region X_0 in a Taylor's series with remainder. The formula for the remainder generalizes one given by Jordan in his *Cours d'Analyse* (vol. 1, p. 247 (2d edition), formula (2)). A fuller account of the results obtained is found in a note by the author in the COMPTES RENDUS, vol. 180 (1925), p. 1719.

30. Professor W. A. Wilson: *On the oscillation of a continuum.*

If c is a point of a continuum A and C_δ is a sub-continuum of A which contains all points of A whose distance from c is less than δ, then the lower bound of the diameters of the sub-continua C_δ for all positive values of δ is a function $t(c)$. This function has properties similar to those of the oscillatory function $\sigma(c)$ of Mazurkiewicz and might well be used as a definition of oscillation. Closely associated with this function are certain sub-continua of A called *oscillatory sets.* These concepts have proved to be useful tools in the study of continua, particularly irreducible continua. In this paper some of their properties for continua in general are given, and they are used to investigate the properties of the oscillation of a continuum in the vicinity of points where it differs from zero.

31. Professor W. A. Wilson: *Some properties of a continuum limited and irreducible between two points.*

For a limited continuum ab irreducible between two points the oscillatory set about each point is unique and has a diameter equal to the oscillation of the continuum at that point. Oscillatory sets of a certain type are called complete; each of these is a continuum of condensation and defines a partition of ab into two continua having this set as a common part. When the points of the *first genre* are everywhere dense, it is found that each point of ab lies on one and only one complete oscillatory set and that the aggregate of these sets has the order type of the linear continuum. From this it follows that there is a correspondence $x=f(t)$ between the points $\{x\}$ of ab and $\{t\}$ of a linear segment cd such that $f(t)$ is one-valued and continuous at an everywhere dense set of points and, at the remaining points $\{t'\}$ of cd, $f(t)$ is many-valued and has as its values the aggregate of limiting values of $f(t)$ as t approaches t' over all possible sequences.

32. Professors E. R. Hedrick and M. B. Porter: *A proof of Weierstrass's theorem with application to Dirichlet's principle.*

In this paper, a new method of demonstrating Weierstrass's theorem is given, which is capable of immediate generalization to functions of several variables, and which is fundamentally simpler in nature than former proofs, particularly for functions of several variables. Applications to the proof of Dirichlet's principle and to the proof of the existence of Green's functions are indicated.

33. Professors E. R. Hedrick and M. B. Porter: *On a generalization of Gibbs' phenomenon.*

The phenomenon usually called *Gibbs' phenomenon*, in the case of Fourier series, is here expressed by means of the point oscillation ω of the limiting function $f(x)$, and the two-dimensional point oscillation Ω of the approximating trigonometric polynomials $\phi_n(x)$, thought of as functions of x and n. The phenomenon as described by Bôcher leads to a ratio Ω/ω, which may be called the *Bôcher ratio.* The concepts involved are generalized in this paper to approximating functions $\phi(x, y)$ to any function $f(x)$; and it is shown that if $f(x)$ is continuous and if $\Omega=\omega$, the approach is uniform, and conversely, so that for a continuous limit, the equation $\Omega=\omega$ is both necessary and sufficient to insure uniform approach. Generalizations to the case when $f(x)$ is discontinuous are immediate, and are stated. The applications of these theorems to the discussion of uniform approach are discussed. In particular, it is pointed out that any Dirichlet function can be approached by trigonometric polynomials either without a Gibbs' effect at any point, or else with preassigned Gibbs' effects at each of a preassigned finite number of points.

34. Professor R. G. D. Richardson: *A problem in the calculus of variations with an infinite number of auxiliary conditions.*

In ordinary problems of the calculus of variations involving one or more auxiliary conditions, an integral possesses either a maximum or a minimum, the other extremum being infinite. By imposing an infinity of auxiliary conditions, we may discuss problems where both maximum and minimum exist. In the simplest problem treated, $\int_0^1 (pu'^2 - qu^2)dx =$ extremum, $\int_0^1 ku^2dx = 1$, $\int_0^1 kU_iudx = 0$, $(i=1, \cdots, m-1;\ s+1, \cdots)$, where U_i are solutions of the differential equation $(pu')'+qu+\lambda ku=0$, $u(0)=u(1)=0$, $p>0$, $k>0$, corresponding to the characteristic numbers λ_i, the Euler and Jacobi conditions, together with the Hamiltonian function and the Hilbert integral, appear as limiting cases of the corresponding formulas for the finite problem. But the corresponding analogues of the Legendre and Weierstrass conditions suggest $p>0$ for a minimum and $p<0$ for a maximum, and this contradiction obviously demands reexamination of the bases of validity of these criteria. A discussion of the Jacobi condition involving an infinite determinant of integrals is vital for the treatment of the problem, and it is found that the Legendre and Weierstrass conditions, when of any significance whatever, must play a role subsidiary to it.

35. Professor C. N. Moore: *On convergence factors in multiple series.*

Convergence factors may be conveniently divided into two classes. The first class includes those which merely have the property of reducing a divergent series to convergence or increasing the rapidity of convergence of a convergent series throughout a certain range of values of the parameters of which the convergence factors are functions. The second class includes those which have the additional property of assigning a value to the original series by the process of allowing the parameters to approach certain limit-values, this value being consistent with the definition of convergence or one of the standard definitions of summability. Necessary and sufficient conditions for convergence factors of both types in the case of double series summable by Cesàro's method have been obtained in a paper previously presented to the Society (this BULLETIN, vol. 30 (1924), p. 220). In the present paper, these results are generalized to the case of multiple series of any order.

36. Professor Marston Morse: *Relations between the singular points of n ordinary differential equations of the first order.*

Poincaré, in a series of four papers *Sur les courbes définies par les équations différentielles*, has included among his results certain relations between the numbers of the different types of singular points in regions bounded by manifolds "without contact." His relations are limited to a number of special cases in which $n=1$, 2, or 3, and in which the number of bounding manifolds is 1 or 2. In the present paper the relations recently obtained by the author between the different types of critical points of a function of n variables serve as a means of arriving at a general formulation of the relations of which Poincaré's results are special cases.

37. Professor Einar Hille: *A class of reciprocal functions.*

This paper contains a study of integrals of the form

$$h(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-(z-t)^2} g(t)dt$$

with varying assumptions concerning $g(t)$. Reciprocal relations between $h(z)$ and $g(t)$ are noted. Incidentally the paper shows many connections with the theory of Hermite's polynomials and with Fourier's integral theorem, and gives a contribution to the summation theory for Hermite's series. The paper marks a first step in the author's study of integral equations with the kernel $\exp[-(x-y)^2]$ and the range $(-\infty, +\infty)$.

38. Professor J. Tamarkin: *Boundary problems and expansion theorems in the theory of integro-differential equations.*

In this paper the integro-differential equation

$$L(y) = p(x) \int^{b} q(t) M(y) dt + f(x)$$

with boundary conditions of the form

$$A_i(y)\,\big|_{x=a}+B_i(y)\,\big|_{x=b}+\int_a^b M_i(y)dt=0, \qquad (i=1,2,\cdots),$$

is discussed, where $L(y)$ denotes a linear differential operator of order n, and $A_i(y)$, $B_i(y)$, $M_i(y)$, $M(y)$ are analogous operators of orders not greater than $(n-1)$. The coefficients of all these operators are polynomials in a certain complex parameter ρ, whose coefficients are functions of the independent variable x. Various problems concerning the existence and asymptotic expressions of characteristic numbers and fundamental functions of the Green function, as well as the general expansion problem of an arbitrary function in series of fundamental functions are solved. The methods used in the paper are essentially those previously used by Birkhoff and the present author.

39. Professors J. Tamarkin and C. E. Wilder: *Second law of the mean in the theory of definite integrals.*

This paper will appear in full in an early issue of this BULLETIN.

40. Professor J. A. Shohat: *On a general formula in the theory of Tchebycheff's polynomials and its applications.*

In this paper the author derives a general formula which gives the solution of the following problem: For all polynomials $A_n(x) = \Sigma_{i=0}^n g_i x^i$ satisfying the condition $|q(x)A_n(x)| \leqq M$ on (a, b), find the upper limit of $|\Sigma_{i=0}^n \alpha_i g_i|$. Here $M>0$ and α_i are arbitrarily given real constants, and $q(x)$ is defined on a certain interval (a, b), finite or infinite. The solution is given by means of Tchebycheff's polynomials. Specifying the α_i and $q(x)$, the author derives certain general minimum properties of Tchebycheff's polynomials, and also some inequalities concerning polynomials in general, on a finite or infinite interval. Some of the results thus obtained generalize those given by Tchebycheff, V. Markoff, and Szegö.

41. Professor H. J. Ettlinger: *Note on a fundamental theorem concerning the limit of a sum.*

This paper appears in full in the present issue of this BULLETIN.

42. Professor J. A. Schouten: *On the conditions of integrability of covariant differential equations.*

The problem of finding the conditions of integrability for a system of differential equations has been solved in the general case before. But the direct application of the known results to *covariant* equations does not lead to convenient expressions. In the present paper the author treats with the aid of intrinsic operations the case where the first members of the equations are linear in the covariant derivatives and have constant coefficients, and the general solution depends on a *finite* number of arbitrary constants.

43. Professor L. R. Ford: *The fundamental region for a Fuchsian group.*

This paper appeared in full in the November-December (1925) issue of this BULLETIN.

44. Mr. R. P. Agnew: *On the form of the solid of revolution of minimum resistance when the normal resistance varies as the nth power* $(n>0)$ *of the normal velocity.*

In this paper the extremals are investigated by means of the Euler, Legendre, and Weierstrass conditions, and solutions are obtained which satisfy the Weierstrass-Erdmann corner condition. The Euler differential equation is integrated when $n=1, 3/2, 2$, and the loci plotted. Numerical tables are given.

45. Professor O. D. Kellogg: *Some properties of bounded polynomials in several variables.*

Let $P_n(x, y)$ denote a polynomial, of degree not greater than n, with real or imaginary coefficients, the real variables x and y being restricted to the region $x^2+y^2 \leqq 1$. Let $|P_n(x, y)| \leqq 1$ in this region. Then if $P_n(x, y)$ is homogeneous, its coefficients have moduli which do not exceed the corresponding binomial coefficients; these bounds are attained (except for the cases of the coefficients of x^n and y^n) only when $P_n(x, y)$ is a number of modulus 1 times the real or imaginary part of $(x+iy)^n$. The magnitude of the gradient of $P_n(x, y)$ does not exceed n for $x^2+y^2 \leqq 1$. Whether $P_n(x, y)$ is homogeneous or not its gradient cannot exceed n^2 for $x^2+y^2 \leqq 1$. These results are proved, and generalized to polynomials in m variables, by elementary considerations based on Descartes' rule of signs. Connection is made with the theorems of Bernstein and Markhoff.

46. Professor L. L. Smail: *Simplification of a general method of summability.*

This note gives a simplification of the conditions and proofs of a method of summability of divergent series discussed by the author in the ANNALS OF MATHEMATICS ((2), vol. 20, pp. 149-154).

47. Professor G. A. Miller: *Multiply transitive substitution groups.*

A substitution group G of degree n is said to be r-fold transitive if each of the $n(n-1)\cdots(n-r+1)$ permutations of its n letters taken r at a time is represented by at least one substitution of G. It is not sufficient to say that each of its possible sets of r letters is transformed into every other one of these sets by substitutions of G. Another well known definition of an r-fold transitive group of degree n is that this group contains a transitive subgroup of each of the degrees $n, n-1, \cdots, n-r+1$. These two definitions are equivalent and if a group is r-fold transitive it is also $(r-\alpha)$-fold transitive, where α is a positive integer $\leqq r-1$. Although a study of multiply transitive groups of degree n which do not involve the alternating group of this degree was successfully undertaken by E. Mathieu, who found one such 5-fold transitive group of each of the degrees 12 and 24, little progress along this line has been made since then. The main theorem of the present article may be stated as follows: *If a substitution group is of degree $lp+k$, p being a prime number, $p>l<k$, then this group cannot be more than k-fold transitive, $k>2$, unless it is the alternating or the symmetric group.*

48. Professor O. E. Glenn: *A program on ordinary differential parameters.* Preliminary report.

The method of typical representation which the author has developed in a previous paper furnishes, with great generality, explicit complete systems of differential parameters of binary forms. The present paper extends this subject and, when completed, is to give also certain lists of differential combinants and parameters derived from them by translation, as explained by the author in a recent article in the PROCEEDINGS OF THE OF THE NATIONAL ACADEMY (vol. 11, No. 6).

49. Dr. Louis Weisner: *Groups in which the normaliser of every element except identity is abelian.*

This paper has appeared in full in the October (1925) number of this BULLETIN.

50. Professor W. L. G. Williams: *On the formal modular invariants of binary forms.*

This paper owes its origin to an attempt to determine a fundamental system of formal modular invariants, mod p, a prime, of the binary cubic form. Several general methods of finding formal modular invariants are developed, and the discussion of two special cases makes clear a fundamental difference between the cases where p is of the form $3m+1$ and of the form $3m+2$.

51. Mr. H. S. Vandiver: *Application of the theory of relative cyclic fields to both cases of Fermat's last theorem.*

In this paper the author attacks the Last Theorem by a new method based on the theory of power characters in the field $\Omega(\theta)$, where θ is a primitive (pk)th root of unity, k prime to the odd prime p. The following result, among others, is obtained: If $x^p+y^p+z^p=0$ is satisfied in integers none zero and all prime to the odd prime p, v is any number in the set t, $1-t$, $1/t$, $1/(1-t)$, $(t-1)/t$, $t/(t-1)$, and $-x/y=t$, then if $\alpha=e^{2i\pi/p}$, $\beta=e^{2i\pi/(n-1)}$,

$$q(n)\prod_{a=1}^{n-2}\left((1-v)\operatorname{ind}(\alpha\beta^a-1)-q(n)\right)\equiv 0 \pmod p,$$

where $\mathfrak{q}=(\beta-r,\, n)$, r is a primitive root of n, $(\alpha\beta^a-1)^{(N(\mathfrak{q})-1)/p}\equiv \alpha^i(\bmod \mathfrak{q})$, $i=\operatorname{ind}(\alpha\beta^a-1)$, $q(n)=(n^{p-1}-1)/p$, n is any prime $\not\equiv 0$ or $1 \pmod p$, and $N(\mathfrak{q})$ is the norm of $\mathfrak{q}$.

52. Mr. H. S. Vandiver: *On algorisms for the solution of the quadratic congruence.*

The problem considered here is the setting up of an algorism for the determination of the integer x in $x^2\equiv q \pmod p$ where p and q are odd primes, without the use of methods of trial. A scheme is described which depends on the expansion of $\sqrt{pq}$ as a continued fraction. For the case where p and q are each of the form $4n+3$ the method is devoid of trial and includes a proof of the law of quadratic reciprocity. For other forms of p and q the element of trial is introduced.

53. Mr. H. S. Vandiver: *Laws of reciprocity and the first case of Fermat's last theorem.*

This paper contains the proof of the following theorem: Suppose $x^p+y^p+z^p=0$ is satisfied in integers none zero and all prime to the odd prime p. Also, let the principal ideal $(\omega(\alpha))$ be the pth power of any ideal in the field defined by $\alpha=e^{2i\pi/p}$ which is prime to (z) and (p); then

$$f_{p-n}(t)\quad\left[\frac{d^n\log\omega(e^v)}{dv^n}\right]_{v=0}\equiv 0 \qquad (\bmod p);$$

$f_k(t)=\sum_{s=1}^{p-1} s^{k-1}t^s$; $t\equiv -x/y \pmod p$; $n=1, 2, \cdots, p-2$, and e is the Napierian base. A number of corollaries are obtained to this theorem, including the criteria $f_{p-n}(t)f_n(1-t)\equiv 0 \pmod p$, $n=1, 2, \cdots, p-1$.

54. Mr. H. S. Vandiver: *A new theory of the representation of integers as definite quadratic forms.*

This paper presents first a proof of the following theorem: If m is any positive integer, and a is a positive integer prime to m, then there is at least one and not more than two sets (x, y) such that $ay \equiv \pm x \pmod{m}$, where x and y are integers $0 < x < \sqrt{m}$, $0 < y < \sqrt{m}$, and two sets (x_1, y_1) and (x_2, y_2) are regarded as the same if and only if $x_1y_2 = x_2y_1$. It is also possible, by the use of a certain continued fraction algorism, to determine each set directly. This proof and algorism were given by the author in an earlier paper for the case m a prime. The theorem is shown to furnish a method for finding all the representations of a number N in the form $ax^2 + bxy + cy^2$, where $b^2 - 4ac < 0$.

55. Mr. H. S. Vandiver: *Note on the condition that a cubic equation have an integral root.*

This note is devoted to the proof of the following theorem: If $x^3 + px + q = 0$ and p and q are rational integers, then a necessary and sufficient condition that x be a rational integer is that $(-27q + 3\sqrt{-3\Delta})/2$ be the cube of an integer in the algebraic field defined by $\sqrt{-3\Delta}$, where Δ is the discriminant of the given equation.

56. Professor L. L. Dines: *Definite linear dependence.*

In this paper, which will appear in the ANNALS OF MATHEMATICS, the author defines definite linear dependence as follows. The m sets of n real constants (1) $a_{i1}, a_{i2}, \cdots, a_{in}$, $(i = 1, 2, \cdots, m)$ are said to be *definitely linearly dependent* if there exist m real constants $c_1, c_2, \cdots, c_m$, of which none is negative and at least one is positive, such that $c_1a_{1j} + c_2a_{2j} + \cdots + c_ma_{mj} = 0$, $(j = 1, 2, \cdots, n)$. In a recent paper (ANNALS OF MATHEMATICS, (2), vol. 23 (1922)) Carver found this particular type of linear dependence to be a necessary and sufficient condition for the inconsistency of the system of linear inequalities (2) $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n > 0$, $(i = 1, 2, \cdots, m)$. In an earlier paper (ANNALS OF MATHEMATICS, (2), vol. 20 (1919)) the present writer studied this system of inequalities, characterizing the solutions by means of a concept which he called the I-rank of the matrix of elements a_{ij}. In the present paper he establishes the equivalence of the following three conditions: (a) definite linear dependence of the system (1); (b) inconsistency of the system (2); (c) zero I-rank for the matrix of elements a_{ij}. The possibility of extension to the field of continuous functions is noted, and the author hopes to present results along these lines at a later date.

57. Professor L. L. Dines: *On certain symmetric sums of determinants.*

In his second paper, the author presents formal identities of two types. The determinants occurring in the first type are those obtainable from any given determinant by a certain well defined transformation. The determinants occurring in the second type have as elements the elements of a given persymmetric array, affected by binomial coefficients according to a definite law. The paper will appear in the AMERICAN JOURNAL.

58. Professor A. A. Bennett: *Proof that large primes have four consecutive quadratic residues.*

The author proves by elementary methods that for e ch prime $p>53$, there are at least four consecutive positive (non-zero) integers which are reduced quadratic residues, modulo p. A complete set of mutually exclusive possibilities are examined for each of which an actual numerical example of four consecutive quadratic residues is furnished. Certain systems related respectively to the primes 11, 13, 53 require special consideration. One is inclined to venture the conjecture that for each natural number n there exists an N such that for each prime $p>N$ there are at least n consecutive quadratic residues modulo p.

59. Dr. T. H. Gronwall: *The algebraic structure of the formulas in plane trigonometry.* Second paper.

In continuation of his first paper on the subject, read May 2, 1925, the author investigates the formulas connecting the angles of a plane triangle from the point of view of Hilbert's theory of algebraic forms (MATHEMATISCHE ANNALEN, vol. 36 and 42).

60. Professor E. T. Bell: *An algebra of sequences of functions.*

The algebra is devised to facilitate the derivation of relations between the elements of any sequence of functions and those of any given sequences. It is of use in the applications to arithmetic of several types of polynomials, including those given as coefficients (functions of the moduli) in the power series expansions of multiply periodic functions. This paper will appear in the TRANSACTIONS OF THIS SOCIETY.

ARNOLD DRESDEN,
Assistant Secretary.
R. G. D. RICHARDSON,
Secretary.