

Perhaps the most illuminating chapter in the book is the one dealing with systems of curves on a surface. Here we find a fundamental theorem that two curves are algebraically equivalent if, and only if, they are homologous in the ordinary sense of analysis situs. Another interesting fact, as pointed out by Lefschetz, is that the so-called "virtual" curves, which have a purely symbolic existence from the algebraic point of view, may be interpreted in topological terms as non-algebraic cycles of the surface. The book also contains other interesting chapters on algebraic manifolds of higher dimensions and on abelian functions, not to mention two notes in the form of postscripts.

J. W. ALEXANDER

Sophus Lie's Gesammelte Abhandlungen (Samlede Avhandlingar).

Edited by Friedrich Engel and Poul Heegaard. Volume V: *Abhandlungen über die Theorie der Transformationsgruppen, erste Abteilung (Avhandlingar om Transformationsgruppernes Teori, første Avdeling)*, edited by Friedrich Engel. Leipzig, B. G. Teubner, and Kristiania, H. Aschehoug and Co., 1924. xii + 776 pages.

The fifth volume of Lie's collected memoirs is the second of the series to be published; it is preceded by the third volume which was published in 1922 (and was reviewed in this BULLETIN, vol. 29 (1923), pp. 367-369). The published memoirs of Lie are to be gathered together in six volumes while a seventh is to be devoted to the principal works found among his literary remains. The memoirs on geometry are to go into volumes I and II; those on differential equations into volumes III and IV; and those on transformation groups into volumes V and VI. Naturally these divisions are not rigorously separated from each other; and each volume will contain memoirs belonging in part to all three divisions. In each of the three divisions the arrangement is chronological; and, generally, the material in the first of any two related volumes is that which was first published at Christiania, while the second of these volumes is devoted principally to Memoirs from the MATHEMATISCHE ANNALEN and the publications of the Leipzig Akademie. In this way it is brought about that no volume will contain two memoirs one of which is mainly a reworking of the other.

As the subtitle indicates, the fifth volume is given to memoirs on transformation groups. It contains the earlier of them, up to February 1889. Of the 560 pages required to print the included memoirs, about 320 pages are given to those in which "Transformationsgruppen" is the main word in the title; a large part of the remaining space is given to memoirs dealing with the applications of the theory of transformation groups to differential equations. The notes from the Norwegian (none of them long) are translated into German, so that the whole body of this volume appears in German. On pages 669-673 of the notes an

address of Lie's is published in French, the language in which it was spoken. (In volume III the French articles were published in French, all others appearing in German.) Some verbal changes have again been made in the articles on reprinting; and a list of these is given on pages 561-582.

The notes and supplementary matter in volume III made up more than a third of the whole. The editor stated in that volume that volume V would require much fewer notes; but the event has shown that he was mistaken, the space given to supplementary matter in volume V being only a little less than in volume III. On pages 583-614 we have an account of the beginnings of the theory of transformation groups as set forth by Lie in his letters to Adolph Mayer in 1873 and 1874. Pages 615-755 are given to the editor's notes on the separate memoirs (and include the address already mentioned). These notes contain numerous matters of detail, explanatory remarks, suggestions and cross references, together with a few discussions of considerable length (such as that on pages 643-668). An extensive index to the volume is given on pages 756-774.

The editorial work (here as in volume III) is marked throughout with evidence of that care and patience which belong only to a labor of love. It is done in such way as to render great service to all those who will have occasion to use the memoirs of Lie which are here reproduced. The next volume of the series which is to be printed is the sixth, and in the preface it is indicated that work upon it will begin immediately. We can not hope more for it than that it will be edited and printed with the same care as volumes III and V.

R. D. CARMICHAEL

Johannes Kepler. Mysterium Cosmographicum. Das Weltgeheimnis.
Uebersetzt und eingeleitet von Max Caspar. Augsburg, Dr. Benno
Filser Verlag, 1923. xxxi+150 pp.

The large increase in the number of students of mathematics and natural science who are unfamiliar with the Latin language makes it more and more useful and desirable to have the greatest works of the pioneers of modern science translated into modern tongues. Among these pioneers Kepler deserves a distinguished place, first because of his work in geometry, in which he prepared the way, through his "infinitesimal method," for the invention of integral calculus; and secondly because in astronomy he laid the foundation for the modern view of the solar system, through his famous three "laws."

Thus there is a real place in the literature of the history of science for such works as the one under review. The *Mysterium Cosmographicum* was Kepler's first work, and it attracted sufficient notice