L'Analysis Situs et la Géométrie Algébrique. By S. Lefschetz. Paris, Gauthier-Villars, 1924.

As a condensed account of the theory of algebraic manifolds, this little volume is so successful that it would seem beside the point to criticise the somewhat hasty manner in which the author disposes of several rather delicate details. Professor Lefschetz has not only given the theory a new unity, but has clarified and extended it at more than one point by the systematic use of topological methods.

First, we are given a brief résumé dealing with the theory of cycles on an algebraic surface, the connectivity numbers and coefficients of torsion, the intersection numbers associated with cycles intersecting in points. Skilful use is made throughout the book of the simple theorem that if one of two cycles intersecting in points is a bounding cycle, the algebraic sum of the points of intersection is zero. It may be mentioned here, in passing, that there are a number of topological invariants associated with the theory of cycles that meet, not in points, but in curves, surfaces, etc. So far as the reviewer knows, the significance of these invariants in the theory of algebraic manifolds has never been studied.

There follows a discussion of the topological characteristics of an algebraic surface, during the course of which a number of the fundamental constants of algebraic geometry reappear as topological invariants. The surface is thought of as swept out by its intersection with a plane varying in a pencil of planes. A general plane section of the surface is a curve C of genus p on which $2p$ 1-cycles, $\gamma_1, \gamma_2, \cdots, \gamma_{2p}$, independent with respect to C, may be traced. If the plane is made to vary in its pencil so as, ultimately, to return to its original position, the cycles γ_i undergo a linear transformation. It would be interesting to see how far the theory of algebraic manifolds could be carried without making use of anything more than the group of linear transformations engendered on the cycles γ_i. Along these lines, it might be possible to develop a perfectly general theory of surfaces without going into the question of the resolution of singular points and curves.

There is an important theorem due to Lefschetz himself to the effect that a cycle is algebraic if and only if no double integrals of the first kind have periods with respect to it. This and other considerations suggest that, in the theory of algebraic manifolds, it might be advisable to modify the definition of an homology so as to make it read as follows: A cycle C is homologous to zero, $C \sim 0$, if, and only if, it is bounding *or algebraic*. The invariants (Betti numbers, coefficients of torsion, etc.) that would follow from this definition of an homology would have the advantage of being unaltered under birational transformations, whereas the strictly topological invariants that follow from the ordinary definition do not have this property.

Perhaps the most illuminating chapter in the book is the one dealing with systems of curves on a surface. Here we find a fundamental theorem that two curves are algebraically equivalent if, and only if, they are homologous in the ordinary sense of analysis situs. Another interesting fact, as pointed out by Lefschetz, is that the so-called "virtual" curves, which have a purely symbolic existence from the algebraic point of view, may be interpreted in topological terms as non-algebraic cycles of the surface. The book also contains other interesting chapters on algebraic manifolds of higher dimensions and on abelian functions, not to mention two notes in the form of postscripts.

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Sophus Lie's Gesammelte Abhandlungen (Samlede Avhandlinger). Edited by Friedrich Engel and Poul Heegaard. Volume V: *Abhandlungen über die Theorie der Transformationsgruppen,* erste Abteilung (*Avhandlinger om Transformationsgruppernes Teori,* første Avdeling), edited by Friedrich Engel. Leipzig, B. G. Teubner, and Kristiania, H. Aschehoug and Co., 1924. xii + 776 pages.

The fifth volume of Lie's collected memoirs is the second of the series to be published; it is preceded by the third volume which was published in 1922 (and was reviewed in this BULLETIN, vol. 29 (1923), pp. 367–369). The published memoirs of Lie are to be gathered together in six volumes while a seventh is to be devoted to the principal works found among his literary remains. The memoirs on geometry are to go into volumes I and II; those on differential equations into volumes III and IV; and those on transformation groups into volumes V and VI. Naturally these divisions are not rigorously separated from each other; and each volume will contain memoirs belonging in part to all three divisions. In each of the three divisions the arrangement is chronological; and, generally, the material in the first of any two related volumes is that which was first published at Christiania, while the second of these volumes is devoted principally to Memoirs from the MATHEMATISCHE ANNALEN and the publications of the Leipzig Akademie. In this way it is brought about that no volume will contain two memoirs one of which is mainly a reworking of the other.

As the subtitle indicates, the fifth volume is given to memoirs on transformation groups. It contains the earlier of them, up to February 1889. Of the 560 pages required to print the included memoirs, about 320 pages are given to those in which "Transformationsgruppen" is the main word in the title; a large part of the remaining space is given to memoirs dealing with the applications of the theory of transformation groups to differential equations. The notes from the Norwegian (none of them long) are translated into German, so that the whole body of this volume appears in German. On pages 669–673 of the notes an