THE APRIL MEETING IN CHICAGO

The twenty-third Western meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 10 and 11, 1925.

Ninety-six persons registered at this meeting, among whom were the following eighty-one members of the Society:

F. E. Allen, R. W. Babcock, Barnard, W. S. Beckwith, H. A. Bender, Birkhoff, Bubb, C. C. Camp, Carmichael, Carr, Chittenden, Crathorne, Curtiss, Dickson, Dostal, Dresden, Emch, Feltges, C. A. Garabedian, Garver, Gorrell, V. G. Grove, W. L. Hart, Hildebrandt, Holl, L. A. Hopkins, Mildred Hunt, Huntington, Louis Ingold, Ingraham, Dunham Jackson, Kempner, Kerékjártó, Kinney, Kouperman, Krathwohl, E. P. Lane, Laves, Libman, Logsdon, MacDuffee, W. D. MacMillan, March, William Marshall, T. E. Mason, Mathews, B. I. Miller, G. A. Miller, Miser, E. H. Moore, E. J. Moulton, F. S. Nowlan, Parkinson, Reinsch, H. L. Rietz, Roever, Roth, Rowland, Runge, Schottenfels, Showman, Simmons, W. G. Simon, E. B. Skinner, Slaught, E. R. Smith, H. L. Smith, Steimley, Stouffer, Teach, E. L. Thompson, Townsend, J. S. Turner, Van Vleck, Wahlin, E. M. Weaver, K. P. Williams, Frederick Wood, F. E. Wood, J. M. Young, J. W. A. Young.

At the Council meeting the following elections to membership took place:

To sustaining membership:

Amherst College, Amherst, Mass.;
Harvard University, Cambridge, Mass.;
The Maccabees, Detroit, Mich.;
The Missouri State Life Insurance Company, St. Louis, Mo.;
The Pacific Mutual Life Insurance Company, Los Angeles, Cal.;
The Prudential Insurance Company, Newark, N. J.;
The Travelers Insurance Company, Hartford, Conn.

To ordinary membership:

Professor Ethel Beatrice Callahan, Cedar Crest College;
Mr. Claude Bernhart Dansby, Morehouse College;
Mr. Harold John Hornberger, Great Northern Life Insurance Company;
Professor Béla de Kerékjártó, University of Szeged;
Mr. Donald D. Laun, University of Chicago;
Rev. Paul Muehlman, Loyola University;
President Ellen Fitz Pendleton, Wellesley College;
Mr. Clarence Edward Rose, Arkansas Cold Storage Company;
Dr. Charles Edward St. John, Mount Wilson Observatory;
Professor Jennie Leatitia Tate, McMurry College;
Rev. George Warren Walker, Margaretville, N. Y.

Nominees of the John Hancock Life Insurance Company, Boston, Mass.:

Eleanor Alice Abbott; Harold A. Garabedian; Harold Alden Grant; L. H. Howe; Earl M. Thomas.

Nominees of the Connecticut Mutual Life Insurance Company, Hartford, Conn.:

William Pond Barber, Jr.; Thomas Kilburn Dodd; Gladstone Marshall; Leslie R. Martin; H. I. B. Rice.

Nominees of the National Life Insurance Company, Montpelier, Vt.:

Andrew John Blackmore; Austin Harris Hobson; Henry Holt; Henry Hollister Jackson; Clarence Edgeston Moulton.

Nominees of the Edison Electric Illuminating Company, Boston, Mass.:

Leonard L. Elden; Robert S. Hale; Harold Cyrle Hamilton; Sidney Hosmer; Irving E. Moultrop.

The Assistant Secretary announced the following elections by mail vote of the Council:

To sustaining membership:

The Aetna Life Insurance Company, Hartford, Conn.;
The American Life Insurance Company, Detroit, Mich.;
The Connecticut Mutual Life Insurance Company, Hartford, Conn.;
The Detroit Life Insurance Company, Detroit, Mich.;
The Eastman Kodak Company, Rochester, N. Y.;
The Edison Electric Illuminating Company, Boston, Mass.;
The Equitable Life Insurance Company of Iowa, Des Moines, Ia.;
The John Hancock Mutual Life Insurance Company, Boston, Mass.;
The Metropolitan Life Insurance Company, New York, N. Y.;
The National Life Insurance Company, Montpelier, Vt.;
The National Life Insurance Company of the United States of America, Chicago, Ill.;
The New England Life Insurance Company, Boston, Mass.

To ordinary membership:

Dr. Carl Louis Alsberg, Food Research Institute, Stanford University;
Miss Rose Lucile Anderson, Bryn Mawr College;
Mr. Frank Ayres, Agricultural and Mechanical College of Texas;
Mr. Lancelot Minor Berkeley, New York City;
Professor Durga Prasanna Bhattacharyya, Bareilly College;
Mr. Perry Aquila Caris, Philadelphia;
Mr. Edwin R. Carter, National Life Insurance Company, Chicago;
Professor William Frederick Durand, Stanford University;
Mr. James Drouie Flynn, Travelers Insurance Company, Hartford;
Mr. Thomas Henry Galbraith, Central Life Insurance Company of Illinois, Chicago;
Miss Marion Cameron Gray, Bryn Mawr College;
Mr. Leroy Francis Harza, Chicago;
Mr. Ray Nelson Haskell, Michigan Agricultural College;
Professor Mildred Hunt, Illinois Wesleyan University;
Dr. Carey M. Jensen, University of Wisconsin;
Professor Reginald Stevens Kimball, State Normal School, Worcester, Mass.;
Miss Helen Kunte, Hunter College;
Professor Anna Delia Lewis, Lake Erie College;
Professor John Hector McDonald, University of California;
Miss Laura Frances McDonough, Moravian College for Women;
Dr. Lee Horace McFarlan, University of Missouri;
Professor E. Loula McNeer, Morris Harvey College;

Professor Braxton Davis Mayo, Virginia Military Institute;
Mr. Earl Lewis Mickelson, Hamline University;
Professor Rayburn Z. Newson, Rusk College;
Mr. Frank Ordway, Phillips High School, Birmingham, Ala.;
Professor Delfin de la Paz, University of the Philippines;
Professor Carl John Rees, University of Delaware;
Mr. Charles Frederick Roos, Rice Institute;
Miss Elizabeth Thatcher Stafford, University of Texas;
Mrs. Ormelle Haines Stecker, Pennsylvania State College;
Mr. Henry J. Sternberg, Columbia University;
Professor Telesforo Tienzo, University of the Philippines;
Mr. William Arthur Watt, Pilot Life Insurance Company, Greensboro, N. C.;
Miss E. Kathryn Wyant, University of Missouri.

Nominees of Allyn and Bacon, Boston, Mass.:

Max G. Carman, University of Illinois; Alonzo Church, Princeton University; Orrin Frink, Columbia University; Raymond Garver, University of Chicago; John Haven Neelley, Yale University.

Nominees of the Connecticut General Life Insurance Company, Hartford, Conn.:

Walter Bjorn; Ward Van Buren Hart; Earl C. Henderson; Edward H. Hezlett; John Melvin Laird.

Nominees of the Eastman Kodak Company, Rochester, N. Y.:

Frederick E. Altman; F. M. Bishop; Charles Warnock Frederick; L. A. Jones; Ludwik Silberstein.

Phoenix Mutual Life Insurance Company, Hartford, Conn.:

Alden Thomson Bunyan; Henry W. Dewey; Henry N. Kaufman; Alson C. Patton; Harold Merle Springer.

Twenty-six applications for membership and thirty-two nominations by sustaining members were received.

The Council voted a resolution of thanks to Professor H. A. Perkins of Trinity College, Hartford, Conn., for the very valuable assistance he has rendered in connection with the endowment campaign, and to the Carnegie Corporation for a second contribution to the endowment fund.

The following appointments by President Birkhoff were announced:

To represent the Society at the Inauguration of Chancellor Benner at the University of Porto Rico, Professor C. G. P. Kuschke; to represent the Society at the Fiftieth Anniversary of the Founding of the George Peabody College for Teachers at Nashville, Tenn., Mr. Lewis C. Cox; to arrange for the next Josiah Willard Gibbs lecture, Professors E. R. Hedrick, Luby, E. H. Moore, Roever, and Stouffer

(Chairman); to consider the fees for colloquium lectures, Professors Archibald, H. H. Mitchell (Chairman), and Van Vleck; to consider the printing problems, Professors Cohen, Eisenhart (Chairman), Graustein, Hedrick, Slaught.

It was announced that the summer meeting and colloquium had been set for September 8–12, 1925, in Ithaca, N. Y.; that the Gibbs lecture in December, 1925, would be given by Professor J. Pierpont.

The Society decided to hold the Western Christmas meeting in Kansas City in conjunction with the meetings of the American Association for the Advancement of Science.

At the dinner held at the Del Prado Hotel on Friday evening, 67 persons were present. Professor Bliss presided and toasts were responded to by Professors E. H. Moore, Huntington, Kerékjártó, Stouffer, Birkhoff and Dresden. Professor Kerékjártó called attention to a new journal established in Hungary, to which further reference is made in the Notes in this issue.

On Saturday a resolution was passed thanking the Department of Mathematics of the University of Chicago for their hospitable reception.

The papers read at the meeting are listed below. Not mentioned in this list is the symposium address delivered on Friday afternoon by Professor W. D. MacMillan on *Some mathematical aspects of cosmology.* On Friday forenoon the Society met in two sections; the one for geometry, before which the papers numbered 1, 2, 3, 4, 5, 26, 27, and 30 were presented, was presided over by President Birkhoff; the other for algebra and theory of numbers, which heard the papers numbered 6, 7, 8, 9, 10, 11, 12, 13, 28, and 29, was presided over by Vice-President Hildebrandt. The latter also presided on Friday afternoon, while President Birkhoff took the chair during the closing session on Saturday forenoon.

The papers by Bennett, Caris, Reilly, Shohat, Wilson, and Ingraham's second paper were read by title; Mr. Jenkins was introduced to the Society by Professor J. W. Glover, and Mr. Robinson by Professor E. P. Lane.

1. Professor R. M. Mathews: *Cubic curves and desmic surfaces.*

The twelve points of contact of the four tangents from each of three collinear points on a cubic of the sixth class form a Hessian $(12_4, 16_3)$ configuration on the curve. The sides of the three quadrangles of the points meet by threes in a similar configuration on a conjugate cubic which meets the given one in the nine vertices of the diagonal triangles of the quadrangles. A desmic surface is a quartic surface with twelve nodes which group into three tetrahedra each pair of which is perspective from each vertex of the third. These twelve nodes determine a counter-set of twelve nodes and for each point P of space there are two desmic surfaces, one on each set of nodes. The projection from P of the space configuration upon an arbitrary plane gives the configuration of the cubics. Various properties of the surfaces and curves are derived, among them relations with a 16_6 configuration and with a Poncelet hexagon and associated Steiner lines.

2. Professor Rufus Crane: *Another poristic system of triangles.*

This paper discusses the loci of points of a variable triangle having incircle and nine-point circle fixed. The loci of the centroid, circumcenter, orthocenter, and Nagel point are circles. The locus of the excenters is a limaçon; that of vertices of the triangle is a bicircular quartic. The relations of certain fixed points to these loci are also discussed.

3. Professor V. G. Grove: *A general theory of nets on a surface.*

In this paper, we first refer the sustaining surface to the asymptotic net, and then to an arbitrary non-conjugate net. We thus find that all of the projective properties of the net are expressible in terms of quantities characterizing the surface, and two arbitrary functions determining the most general non-conjugate net on the surface. From the forms of the formulas involved, one can tell at a glance those properties of the net which are really properties of the net, those which are properties of either of the two oneparameter families of curves forming the net, and those which are properties entirely of the surface.

The R-reciprocal congruences associated with a net are Green reciprocals if and only if the tangents to the curves of the net form with the asymptotic tangents a constant cross ratio. There exists one and only one pair of Green reciprocal congruences that are in relation R with respect to a non-conjugate, non-asymptotic net.

4. Professor Arnold Emch: *On surfaces and curves which are invariant under involutory Cremona transformations.*

The present paper extends to projective spaces of n dimensions the investigations of the curves which are invariant under involutary Cremona transformations of two superposed planes begun in some papers in the TOHÔKU MATHEMATICAL JOURNAL.

If the transformation is given the form $\varrho x_i' = \varphi_i$, $i = 1, 2, \cdots, n+1$; and $\sigma x_i = \varphi_i(x')$, we form the Plückerian coordinates $\varrho p_{ik} = x_i\varphi_k - x_k\varphi_i$ of the line joining two corresponding points P and P'. These, as well as $\sigma q_{ik} = x_i\varphi_k + x_k\varphi_i$ are invariant under the transformation. By means of the identities $p_{ik}p_{jl} = q_{il}q_{jk} - q_{ij}q_{kl}$, it is found that in every involutory transformation in S_n there are two types of invariant surfaces (curves in S_2) representable by the equations $K\ (p_{12}, \cdots p_{ik} \cdots) = 0$, $L\ (q_{11}, q_{12}, \cdots, q_{ik}, \cdots, q_{ii}, \cdots) = 0$. When K is of even degree in the p_{ik}'s, then it may be classed with the form L. When K is of odd degree, then its square only belongs to the class L. Thus every invariant hypersurface, or its square, belongs to class L. Accordingly, invariant hypersurfaces belong to two classes: Those which belong to L but are not squares of K; those whose squares only belong to L. The second class is connected with the contact problem. As applications are given the quadratic transformation and its connection with the Cayley cubic surface, the Geiser transformation, transformations in S_3, the Schur sextic curve, the Enriques sextic surface, etc.

5. Dr. E. E. Libman: *Mean curvature curves on quadric surfaces.*

The name "mean curvature curve" is given to a curve (on a surface) at every point of which the mean curvature of the surface is the same. This paper is an investigation into the properties of such curves on quadric surfaces. Some of the more interesting results follow. The mean curvature curves on a quadric are algebraic curves of order twelve. They touch both cyclic sections along a conic, the central section by a plane normal to the cyclic sections. Along a mean curvature curve the tangent of the angle at which two generators intersect upon the curve is proportional to the square of the distance of the tangent plane from the center. The mean curvature curve on a central quadric along which the mean curvature of the quadric is zero is the locus of the intersections of perpendicular pairs of generators. It is a sphero-conic upon the director sphere. The zero mean curvature curve on a paraboloid is a conic section, the intersection of a plane normal to the direction of the infinite center.

6. Professor J. F. Reilly: *Interpolation formulas containing parameters.*

An interpolation formula which gives satisfactory results throughout one portion of a table may yield rather inaccurate values throughout other portions of the same table. In this paper formulas are developed containing one or two parameters, which may be adjusted to fit any portion of the table. Sprague's third order differences osculatory interpolation formula is obtained by specializing the parameters. The use of such formulas is illustrated by showing how to adjust the parameters in order to obtain the best interpolated values in the least squares sense.

7. Professor A. A. Bennett: *On sets of three consecutive integers which are quadratic residues of primes.*

This paper appears in full in the present number of this BULLETIN.

8. Mr. P. A. Caris: *A solution of the quadratic congruence, modulo p, $p = 8n + 1$, n odd.*

Explicit solutions of the quadratic congruence $x^2 \equiv a (\bmod p)$, where p is prime and of the form $4q + 3$ or of the form $8n + 5$ have been obtained by Legendre and others. The present paper contains explicit solutions for $a = -1, 2$ and n when $p = 8n + 1$ and for a in general when n is odd.

9. Professor C. C. MacDuffee: *The nullity of a matrix relative to a field.*

The row-nullity ϱ of a matrix x relative to a field F is defined as the number of linearly independent linear relations with coefficients in F among the rows of x; the column nullity $\varkappa$ is similarly defined. If a and b are two non-singular matrices with elements in F, then ϱ and $\varkappa$ are invariants of x under the transformation $x' = axb$.

Certain relative projective invariants of algebraic forms written as determinants give rise to invariant matrices, e. g., the hessian matrix $H \equiv (f_{x_ix_j})$ of f_n is transformed according to the equation $H' = \breve{a}Ha$ by the projective transformation of matrix a. Thus the row-nullity of H relative to the field of the coefficients is an arithmetic invariant h of the form f_n itself. The complete solution of Hesse's problem is stated very simply in terms of h. Moreover, if the rank of H is 1, $h = n-1$, and f_n is a constant times a perfect nth power of a linear function.

10. Dr. H. A. Bender: *On groups of order p^m which contain an abelian subgroup of order p^{m-1}.*

In every group of order p^m, p being an odd prime number, which contains an abelian subgroup of order p^{m-1} the abelian subgroup must be transformed in the same manner as it is transformed by an isomorphism of order p. In this paper we establish the necessary conditions that an isomorphism be of order p, and with this we proceed to show the possible ways a central and a commutator subgroup may be selected so as to give groups of order p^m and to determine the number of distinct groups there are for a given central and a given commutator subgroup.

11. Dr. H. A. Bender: *On orders of operators in the group of isomorphisms of prime power abelian groups.*

If $Q_n(x) = 0$ be the algebraic equation whose roots are the primitive nth roots of unity without repetition, then the group of isomorphisms of the abelian group of order p^m, p being a prime number, and type $(1, 1, 1, \cdots)$ contains operators of orders $p^\alpha \Pi_d Q_d(p)$ where d varies through all the distinct divisors of $a_1, a_2, \cdots, a_n$. The a's may be distinct and are to assume all possible values which satisfy the relation $\alpha_1+\alpha_2+\alpha_3+\cdots+\alpha_n=\beta\leqq m\ (\beta=0,1,2,\cdots,m)$ $(p^{\alpha-1}<m-\beta\leqq p^{\alpha})$.

12. Professor G. A. Miller: *Imprimitive substitution groups.*

Various possible definitions of imprimitive substitution groups are observed, including the following: A necessary and sufficient condition that a transitive group G is imprimitive is that some co-set of G with respect to a subgroup G_1 composed of all the substitutions of G which omit a fixed letter, generates an intransitive group, or that such a co-set generates a proper subgroup of G. It is also necessary and sufficient that G contains a set of letters which does not include all its letters and is transformed into itself by all the substitutions of G which transform a fixed letter of G into some other fixed letter. To every proper subgroup generated by a co-set of G with respect to G_1, or by more than one such co-set, there corresponds a system of imprimitivity of G, and vice versa.

The number of possible sets of systems of imprimitivity of a transitive group G each of which involves two letters is always odd whenever there is a least set of such systems, and there is an infinite number of imprimitive groups which have the common property that each of them involves any arbitrary given odd number of different such systems of imprimitivity. In a transitive group whose order is of the form p^m, p being an odd prime, the number of the different systems of imprimitivity each of which involves exactly p letters is always of the form $kp+1$. This is not necessarily the case when the order of G is not of the form p^m. In fact, for every value of p it is possible to construct imprimitive groups which have exactly two different systems with just p letters in one system.

13. Professor G. E. Wahlin: *On a problem in Diophantine analysis.*

The author develops a method for obtaining the solutions of a Diophantine equation in which one member is a quadratic form in n variables and rational integral coefficients; and the other member a product of two or more unknown factors. It is based on the construction of an algebra over the field of rational numbers and consideration of the ideals of certain quadratic fields which are contained in the algebra.

14. Dr. C. C. Camp: *Expansions in terms of solutions of partial differential equations.* Third paper, involving partial differential equations possessing a fundamental set of solutions.

There is a striking analogy between the equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{2\partial^2 u}{\partial x \partial y}+\frac{\partial^2 u}{\partial y^2}+a_1(x,y)\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)+a_0(x,y)u=0,$$

where the a's are analytic near $(0,0)$, and an ordinary linear differential equation of the second order. By employing the operator $D=\frac{\partial}{\partial x}+\frac{\partial}{\partial y}$ as the analog of ordinary differentiation, one obtains a fundamental set of independent solutions such that the general solution may be written $u=\varphi_1 u_1+\varphi_2 u_2$. Here the φ's are functions of $x-y$. The initial conditions $u(x,0)=p(x)$, and $Du(x,0)=q(x)$, and analytic, determine a unique analytic solution. The theory extends to equations of higher order as well as to those of the first order. For the latter by Lagrange's method of solution one obtains a relation $y=f(x)-h$. With suitable boundary conditions the problem of expansion of an arbitrary function $f(x,y)$ involves a Birkhoff expansion along this curve and the coefficients are obtained by line integrals. The series converges to the mean value as illustrated by the expansion of xy in the case of the equation $Du+\lambda u=0$.

15. Professor K. P. Williams: *Non-synchronized relative invariant integrals.*

The paper deals with relative linear invariant integrals of the system $dz_i/dt=Z_i(z_1,\cdots,z_m)$. Such integrals are taken along closed paths drawn on a tube of trajectories. If the path passes through points that correspond to the same values of t we have a synchronized invariant $J=\int\sum L_i\delta z_i$. If it passes through points corresponding to different values of t we have a non-synchronized invariant $I=\int\sum L_i\delta z_i+K\delta t$. The paper develops a criterion on K so as to render I invariant if J is invariant.

16. Mr. J. A. Shohat: *On some properties of polynomials.*

In this paper the author considers integrals of the type $\int_a^b(x-a)^{\alpha-1}\,(b-x)^{\beta-1}q(x)\;M(x)\,dx\,(\alpha,\beta>0,\;q(x)\geqq q$ in

(a, b), $M(x)$ a polynomial; (a, b) finite). Using formulas previously established by the author in the theory of orthogonal Tchebycheff polynomials, he obtains some inequalities involving the maximum and minimum of $M(x)$, $M'(x)$ in (a, b); for example, a precise form of the law of the mean for polynomials. These results are applied, in particular, to polynomials which are monotonic in (a, b). A formula similar to one developed in this paper was given by Tchebycheff, using the theory of polynomials deviating the least from zero.

17. Professor H. L. Smith: *On functions of closest approximation.*

In the first part of this paper the notion of linear independence of n functions $p_1(x), \cdots, p_n(x)$, bounded in (ab), relative to a function $u(x)$ of limited variation on (ab) is defined. The existence of a linear combination $(c)(p) = c_1p_1 + \cdots + c_np_n$ for which the Stieltjes integral $\int_a^b |f-(c)(p)|^m du$ has its minimum value is then proved where f is a fixed function integrable (u) and m a fixed number $\geqq 1$. The minimizing function is proved to be unique when $m > 1$. A theorem is developed concerning the sequence of approximating functions corresponding to a convergent sequence of functions u and the theorem is applied to the study of the minimizing of finite sums.

In the second part the functions $p_1, \cdots, p_n$ are replaced by $(2n+1)$ trigonometric functions, the convergence of the approximating function to f as n approaches infinity is studied and the results are applied to the case of finite sums.

18. Professor Dunham Jackson: *On the convergence of certain processes of closest approximation over an infinite interval.*

This paper is concerned with polynomials affording the closest approximation to a given function over an infinite interval, according to the criterion of least squares (or least mth powers), with a weight function of appropriate character. Mr. Shohat (BULLETIN, Nov.–Dec., 1924) has discussed some of the problems relating to such polynomials of approximation. It is shown in the present paper that under specified hypotheses the approximating polynomial will converge to the value of the given function, as the degree of the polynomial is indefinitely increased. The analysis applies in particular to the series of Hermite's polynomials used in the theory of statistics under the name of the Gram-Charlier series.

19. Professor Dunham Jackson: *On vector analysis in function space.* Preliminary communication.

This paper contains a further development of certain ideas which were presented for consideration at an earlier meeting of the Society (October, 1924). In particular, there is a discussion of a functional operator analogous to the divergence of a vector point function in ordinary space.

20. Professor W. H. Wilson: *Two related functional equations.*

The equation $f(x+y)=f(x)g(y)+g(x)f(y)+2\lambda f(x)f(y)$, where λ is a constant which may be zero, is discussed in part I of this paper and the functions, $f(x)$ and $g(x)$, satisfying this equation are expressed in terms of the functions $\varphi(x)$ and $\psi(x)$ whose addition theorems are $\varphi(x+y)=\varphi(x)+\varphi(y)$ and $\psi(x+y)=\psi(x)\,\psi(y)$, respectively. In part II the equation $g(x+y)=g(x)g(y)+\mu^2 f(x)f(y)$, where μ is a constant, is discussed and the functions, $f(x)$ and $g(x)$, satisfying this equation are expressed in terms of $\varphi(x)$ and $\psi(x)$.

21. Dr. R. W. Babcock: *On thermal convection.*

The differential equations which apply to slow steady convective motion of a viscous fluid are the standard equations of hydrodynamical theory, combined with equations expressing the law of thermal interchanges in a moving fluid. In the present paper approximate solutions are found following the method outlined by Oberbeck (ANNALEN DER PHYSIK, 1879, p. 271). A comparative study of the exact solution of a special system of non-homogeneous ordinary equations and its Oberbeck approximate solution shows that the magnitude of the constants in the system controls the rate of convergence of the approximation, and often produces divergence. Solutions with graphs of temperature and velocity fields have been computed for special cases of plane and cylindrical boundaries with sinusoidal impressed boundary temperatures. If this temperature has a vertical gradient only, the Oberbeck method produces a null solution, corresponding to equilibrium of the fluid in stratified layers. Arithmetical solutions for common liquids at ordinary temperatures show that the Oberbeck solutions diverge unless the gradient is much too small to be easily applied in laboratory practice.

22. Professor H. W. March: *The deflection of a rectangular plate with two opposite edges supported and two edges free.*

Solutions of this problem for the case of a uniform load and of other problems relating to the rectangular plate with two opposite edges supported were given by Estanave (ANNALES DE L'ECOLE NORMALE, 1900) using the method of Lévy. This paper gives a simpler solution of the problem for the case of a uniform load and also a solution for the case of a load which is a function of x alone, x being measured in the direction perpendicular to the supported edges. The simplification effected results from first choosing a solution of the non-homogeneous differential equation which satisfies part of the boundary conditions and then adding to this a suitably chosen solution of the corresponding homogeneous equation.

23. Professor M. H. Ingraham: *Solution of certain functional equations relative to a general number system.*

Relative to any normally-orderable associative division number system $\mathfrak{A}$, there exists a range P and a field F, either the field of rationals or a field of integers modulo a prime, such that $\mathfrak{A}$ is isomorphic with the set V of all finitely non-zero functions of P to F. Relative to functions on such a system V to V it is shown that any solution of the functional equation

$$\sum_{i}^{0,n+1} (-1)^{i}_{n+i} c_i f(v_0 + iv_1) = 0 \qquad (v_0, v_1)$$

is, if F is rational or has a modulus $n+1$, a polynomial of degree equal to or less than n in the coordinates of the elements of V with coefficients in V and conversely. The trivial case in which F has a modulus $m \leqq n+1$ is dealt with. The paper is an extension of the results of Hamel for the equation $f(x+y) = f(x) + f(y)$ and the real number system. Hamel made use of the Zermelo "axiom of selection".

24. Professor E. V. Huntington: *Postulates for order on a closed line*: I. *Reversible order* (*separation of point-pairs*).

The theory of reversible order on a closed line (also known as the theory of separation of point-pairs), which is of fundamental importance in the development of projective geometry, was first systematically studied by Pasch in 1882. The first set of postulates for this theory was given by Vailati in 1895, and has been substantially followed by later writers. The present paper gives a more exhaustive analysis of the properties of this type of order (expressed in terms of a tetradic relation $ABCD$), and leads to the following new set of six postulates (of which the first two serve only to exclude obviously trivial cases):

0. There is at least one true tetrad, say $XYZW$.
00. If $ABCD$ is true, then the elements A, B, C, D are distinct.
I. If $ABCD$ is true, then $BCDA$ is true. (Cyclicality.)
II. If $ABCD$ is true, then $ABDC$ must be false. (Homogeneity.)
III. If $ABCD$ is true, and X is any fifth element, then at least one of the relations $AXCD$ and $ABCX$ is true. (Connexity.)
IV. If $ABCD$ is true, then $DCBA$ is true. (Reversibility.)

This paper (which extends and completes an unpublished paper presented to the Society in December, 1916) will be followed by a second paper on irreversible order, in which Postulate IV is replaced by its opposite.

25. Professor J. B. Shaw: *On the classification of linear algebras.*

The classification of linear algebras, whether associative or not, may be made from two very different standpoints. In one the basis is that of the properties of any number, the general number, considered as held fixed, while all other numbers are multiplied by it. This would correspond geometrically to a collineation. The classification on this basis I have carried out in previous papers. A different basis is that initiated by Cayley in his paper on double algebras, and is based upon the numbers that are invariant in the algebra in the sense of being their own squares, or are zero. That is they are idempotent or nilpotent. The latter basis is geometrically that of quadratic transformations. In both cases there are invariants that must count double, triple, etc., and the number of these furnishes the basis for the divisions. In the paper a study is made of the conditions this multiplicity imposes upon the system. It shows up in the formation of certain sub-algebras. Non-associative algebras may be classified from either standpoint, the property of associativity becoming relatively unimportant.

26. Professor Louis Ingold: *Associated types of linear connection.*

In this paper the transformation $\bar{\Gamma}^s_{i\lambda} = A^s_\alpha\left(\frac{\partial a^\alpha_i}{\partial u^\lambda} + a^r_i \Gamma^\alpha_{r\lambda}\right)$ on the functions Γ^k_{ij}, which are used to define the neighborhood connections in a geometry, is associated with an arbitrary matrix $||a^i_j||$. The elements a^i_j are functions of the coordinates of the space and $A^s_\alpha \cdot |a^i_j|$ is the cofactor of a^s_α. It is shown that the resultant of the transformations associated with two matrices is again a transformation of the same type,—in fact, the one associated with the resultant matrix. The geometries corresponding to different sets of functions Γ related in this way are called associated geometries. The effect of the transformations on certain fundamental tensors is considered and a series of properties is found which hold in all associated geometries.

27. Professor Ingold: *The geometry of a set of n vectors.*

A set of n independent vectors is taken as the basis of a geometry. For convenience they are represented in a function space by the functions $\theta_i(x; u^1, u^2, \cdots, u^n), i = 1, 2, \cdots, n$. Quantities analogous to the fundamental quantities and Christoffel symbols of differential geometry are formed from the functions θ_i and their derivatives with respect to the coordinates u^i. These serve to define the neighborhood connections. If instead of the functions θ_i, a set of n independent functions φ_j are used as basis, where $\varphi_j = a^i_j \theta_i$, the coefficients Γ^k_{ij} of the linear connection undergo the transformation discussed in the previous paper.

28. Mr. W. A. Jenkins: *On a central difference summation formula.*

Three finite difference formulas are discussed each of which interpolates a certain number of values between each of a given series of equidistant ordinates and sums approximately the interpolated along with the given values, in terms of the original ordinates and their differences. Two of these, due to Lubbock and De Morgan, are proved inferior in convergency to the third, a central difference formula derived by Woolhouse in the JOURNAL OF THE INSTITUTE OF ACTUARIES, volume 11, page 307. An approximate test of accuracy is developed.

29. Professor J. S. Turner: *Note on prime factors.*

This note presents a short and direct proof of the theorem "If a prime p is a factor of a product ab, where a, b are positive integers and a is prime to p, then p is a factor of b."

30. Mr. P. G. Robinson: *Surfaces with constant absolute invariants.*

Consider a surface such that the homogeneous coordinates of any point of it are analytic functions of two parameters. Let the asymptotic tangents of any point of the surface be distinct, and let neither have more than second order contact with the surface at any point. Wilczynski has shown that under these conditions and by a proper choice of the tetrahedron of reference, the surface may be represented by a development of the form

$$z = xy + \frac{1}{6}(x^3 + y^3) + \frac{1}{24}(Ix^4 + Jy^4) + \cdots$$

where I and J are absolute invariants of the surface.

This paper shows first that there are no surfaces for which either I or J is equal to zero and the other equal to a constant different from zero, at all points which satisfy the conditions formulated above. Then the case is considered where both I and J are equal to non-vanishing constants. In this case the differential equations of the surfaces are obtained and integrated. From these results are obtained properties of the surfaces.

31. Professor M. H. Ingraham: *A general theory of linear sets.*

In this paper the author, using the postulational method, considers classes of vectors, on a finite range, whose elements belong to a general division algebra, and gives a general basis preliminary to the more intensive study of classes of vectors on a general range and of number systems over a division number system. This paper has appeared in the April, 1925, number of the TRANSACTIONS OF THIS SOCIETY.

ARNOLD DRESDEN,
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