

THE MAY MEETING IN NEW YORK

The two hundred thirty-sixth regular meeting of the Society was held at Columbia University, on Saturday, May 3, 1924, extending through the usual morning and afternoon sessions. The attendance included the following forty-nine members of the Society.

Archibald, Babb, C. R. Ballantine, J. P. Ballantine, Beal, Bernstein, W. J. Berry, Herman Betz, Borden, Bowden, Daniel Buchanan, Abraham Cohen, Philip Franklin, Gafafer, Gehman, Gill, Glenn, Hazeltine, Hazlett, E. R. Hedrick, Himwich, Huntington, Joffe, Kasner, Kline, Lamson, Langman, Harry Levy, Locke, Longley, MacDuffee, Meder, C. L. E. Moore, Northcott, Pfeiffer, Rainich, Raynor, R. G. D. Richardson, Ritt, L. B. Robinson, Rowe, Seely, Sosnow, Vandiver, Veblen, H. S. White, Whittemore, Norbert Wiener, M. M. Young.

No meeting of the Council was held. At a meeting of the Board of Trustees, the finances of the Society were reviewed.

The three amendments to the By-Laws recommended by the Council at the April meeting were unanimously approved by the Society. The first concerns the elimination of ephemeral regulations governing the business of the Society during the period of incorporation and the first year thereafter. The second permits the Council to transact business by correspondence provided there is a favorable vote of at least fifteen, the favorable vote to be at least three-quarters of the votes transmitted to the Secretary. The third introduces a new class of membership to be known as Sustaining Membership, for which the annual dues shall be not less than one hundred dollars. The revised By-Laws will be printed in full in the List of Officers and Members to be issued late in 1924.

President Veblen presided at the sessions of the Society, relieved by Professors White, Whittemore, and C. L. E. Moore. At the beginning of the afternoon session, a paper was read by Professor J. F. Ritt, at the request of the Program Committee, on *Rational substitutions*.

Titles and abstracts of the papers presented at this meeting follow below. Mr. Kormes was introduced by Dr. Pfeiffer, Mr. Widder by Professor Birkhoff, and Mr. Sheffer and Mr. Roos by Professor Evans. The papers of Bray, Foster, Maria, Michal, Moore, Roos, Sheffer, Taylor, Wedderburn, Widder, and Wilder were read by title.

1. Dr. Norbert Wiener: *The Dirichlet problem.*

The author develops a necessary and sufficient condition for the solvability of the Dirichlet problem in any number of dimensions, and applies this criterion to the deduction and extension of a large number of existing criteria for the solvability of the Dirichlet problem.

2. Dr. Philip Franklin: *Analytic transformations of everywhere dense point sets.*

It is well known that any two enumerable, everywhere dense point sets on a straight line may be mapped on one another by a one to one transformation which preserves order. We show here that we may set up an analytic function, analytic in the interval covered by the first point set, its inverse being analytic in the interval covered by the second point set, which effects the mapping. Some extensions to point sets everywhere dense in a two-dimensional region are given.

3. Dr. Philip Franklin: *Functions with an isolated essential singularity.*

In this note we prove some properties of functions with an isolated essential singularity, derived from Picard's theorem, of which the following are typical: If an entire function has no zeros, any function obtained from it by changing a finite number of coefficients in its Taylor's expansion about any point has an infinite number of zeros. The function formed by adding a polynomial, not a constant, to a periodic function, not a constant, possessing no poles, always has an infinite number of zeros.

4. Professor J. R. Kline: *Concerning the sum of two bounded continua irreducible between the same pair of points.*

In the present paper, the author obtains necessary and sufficient conditions that two distinct bounded continua,

each irreducible between the same pair of points, may have a sum irreducible between two of its points.

5. Mr. H. M. Gehman: *On extending a continuous (1—1) correspondence of two plane continuous curves to a correspondence of their planes.*

In this paper, it is proved that if two plane continuous curves that contain no simple closed curve are in continuous (1—1) correspondence in such a way that sides of arcs are preserved under the correspondence, a continuous (1—1) correspondence of their planes can be defined so that the correspondence of the two continuous curves is preserved.

6. Mr. Mark Kormes: *Treatise on basis sets.*

Algebraic (rational, linear) fields are defined, and their properties discussed. By the use of these fields, a definition of algebraic (linear) basis sets is given. This definition includes the definitions of Hamel, Zermelo, and E. Noether as special cases. The following theorems are proved: There exist algebraic (linear) basis sets for each set M . The basis set of a non-denumerable set M has the same potency as the set M , and there exist 2^{\aleph_μ} different basis sets of M , if \aleph_μ is the potency of M . There exist basis sets that are measurable in Lebesgue's sense, but they have the measure zero. There exist also basis sets that are not measurable (L). A theorem of Sierpinski is generalized: Every algebraic basis set of the continuum is non-measurable in Borel's sense. The basis sets of a linear space are considered. It is shown, in consequence of a general theorem, that the linear basis set of all continuous functions has the potency of the continuum. New sets S^M are defined and considered as to their potency and measurability properties. An example of a perfect set which is a basis set is given.

7. Professor O. E. Glenn: *Differential combinants.* Preliminary report.

The subject of this paper is the algorithms and fundamental theorems of a complete theory of differential combinants and associated forms. Combinants of n forms $\alpha_{ax}^m, \beta_{ax}^m, \dots$ are found to be functions of determinants of order n of a matrix whose elements are derivatives of the coefficients of the forms. A translation principle leading to ternary parameters is explained.

8. Professor H. S. White: *A set of invariants connected with seven points on a gauche cubic.*

While 7 points on one gauche cubic curve are known to determine 7 osculating planes of a second, the algebraic nature of the discovery of the parameters of those 7 planes has not been known in detail. In this paper it is shown that this work leads inevitably to a system of 28 relations, of similar form in the two sets of parameters, *invariant under the well known group of order 168*. These 28 relations are equivalent to 4, and are *solvable rationally* for either set of parameters, except for the 3 residual arbitraries inherent in a collineation.

9. Professor R. L. Moore: *A characterization of a continuous curve.*

If A and B are points belonging to a connected point set M , a point set K is said to separate A from B in M if K is a proper subset of M and $M - K$ is the sum of two mutually separated point sets which contain A and B respectively. It is shown that in order that a plane continuum M shall be a continuous curve it is necessary and sufficient that, for every two points A and B that belong to M , there exist a finite number of continua whose sum separates A from B in M . In order that a bounded plane continuum M shall be a continuous curve which neither contains a domain nor separates the plane, it is necessary and sufficient that, for every two distinct points A and B which belong to M , there exist a point which separates A from B in M .

10. Dr. R. L. Wilder: *A theorem on continua.*

In the above paper by Professor Moore, the following property of continuous curves is established: Of two concentric circles, C_1 and C_2 , let C_1 be the smaller. Denote by H the point set which is the sum of C_1 and C_2 , and the annular domain bounded by C_1 and C_2 . Let M be a continuous curve which contains a point A interior to C_1 and a point B exterior to C_2 . If N is any connected subset of M containing A and B , N will contain at least one point of some continuum which is a subset of M and H , and which has at least one point in common with each of the circles C_1 and C_2 . It is shown in the present paper that this is a property of *any* closed and connected point set M .

11. Mr. D. V. Widder: *A general mean-value theorem.*

This paper deals with the following problem. Given a linear differential expression, $L[u(x)]$, of order n , the coefficients being continuous in an interval (a, b) . Under what conditions will $n + 1$ conditions on the zeros of a function $f(x)$ and of its derivatives, $f^{(h_i)}(x_i) = 0$ ($i = 0, 1, 2, \dots, n$), insure the change of sign of $L[u(x)]$ in (a, b) ? A necessary and sufficient condition is obtained. Special cases of the problem have been treated by G. D. Birkhoff and by G. Pólya. The present paper establishes connections with their results.

12. Mr. A. D. Michal: *Integro-differential invariants of one-parameter groups of projective transformations of function space.* Preliminary report.

In this paper the author studies functionals of a function y and its derivatives admitting a given arbitrary one-parameter group of projective functional transformations defined by $\delta y(x) = [\alpha(x)y(x) + \int_0^1 H(x, s)y(s) ds] \delta a$. In particular, it is shown that a necessary and sufficient condition that an "analytic" functional $F\left[\begin{smallmatrix} y \\ y' \end{smallmatrix}\right]$ admit this group of functional transformations is that it satisfy a certain completely integrable equation in partial functional derivatives (cf. an article by the author in a forthcoming issue of the ANNALS OF MATHEMATICS). The expansion of the solution of this equation is obtained. The above discussion assumes $\alpha(x)$ and $H(x, s)$ to be such as to insure the non-vanishing of a certain Fredholm determinant. The author also considers the invariants when the above general method breaks down.

13. Mr. J. M. Sheffer: *Systems of equations in an infinite number of unknowns whose solutions involve an arbitrary parameter.*

The author considers equations of the form

$$x_i + \sum_{j=i+1}^{\infty} (\lambda_j + b_{ij})x_j = c_i, \quad (i = 1, 2, \dots),$$

which are not of a canonical type, and imposes sufficient conditions that the equations have a solution. He finds a set of solutions depending on an arbitrary parameter appearing in the general solution of incidental difference equations. The system is characterized by the hypothesis that the limit of λ_j with j infinite is not zero.

14. Mr. C. F. Roos: *Mathematical theory of competition.*

The author considers a question in economic dynamics by applying the postulates of competition to the formula for total profit over an interval of time. The question, unlike the case of monopoly, does not reduce to a straight problem in the calculus of variations. Moreover it is related to the theory of integral equations and their application to phenomena of hysteresis.

15. Dr. J. S. Taylor: *The theory of testimony. Second paper: n witnesses.*

In another paper (JOURNAL OF MATHEMATICS AND PHYSICS, vol. 3, No. 3) the author presented certain theorems concerning the probability of the occurrence of an event when it is attested to by a single witness. In the present paper the investigation is extended to the case of n witnesses, with special attention to the case of two witnesses, the traditional treatment of which has been severely criticised by J. M. Keynes. Methods are discussed of determining approximately both the "credibility" and the "personal accuracy" of an observer by means of the testimonies of n observers.

16. Professor J. F. Ritt: *Rational substitutions.*

This address dealt with the work performed during the last few years by Julia, Fatou, and the author on the iteration of rational functions and on related problems.

17. Professor J. F. Ritt: *Analytic functions and periodicity.*

This paper appeared in full in the October number of this BULLETIN.

18. Mr. G. Y. Rainich: *The curvature of space-time, the electromagnetic tensor, and radiation.*

In an earlier paper (PROCEEDINGS OF THE NATIONAL ACADEMY, April, 1924) the author found that the presence of the electromagnetic field imposes on the contracted curvature tensor F_{ij} certain conditions which he gave in geometric form; in the present paper these conditions are given analytically as follows: $F_{\sigma\sigma} = 0$, $F_{\sigma i} F_{\sigma j} = \delta_j^i \omega^2$ and $q_{ij\sigma, \sigma} = 0$ with $q_{ijk} \cdot \omega^2 = F_{\sigma i, j} F_{\sigma k} + F_{\sigma j, k} F_{\sigma i} + F_{\sigma k, i} F_{\sigma j}$. These conditions being satisfied, the curvature tensor determines the electromagnetic tensor as soon as the value of a certain scalar quantity is given at one point. This quantity plays in the simplest case the role of the phase of radiation.

19. Dr. Malcolm Foster: *Note on a special congruence.*

This paper appears in full in the present number of this BULLETIN.

20. Mr. Harry Levy: *Ricci's coefficients of rotation.*

This paper will appear in full in an early number of this BULLETIN.

21. Professor Edward Kasner: *The addition of quadratic differential forms.*

In this paper the author considers quadratic differential forms in n variables which can be regarded as the sum of two or more forms in separated variables. For example, if $n = 4$, the possible types of separation are (1, 1, 1, 1), (1, 1, 2), (1, 3), and (2, 2). In special cases, the given form may belong to more than one type, so that the problem of overlapping types must be studied. The type (1, 1, 1, 1) of course defines flat space. It is proved that if the Einstein equations are to be satisfied the only separated type possible is (2, 2), and then the result is the sum of two binary forms representing equal spheres. The immersing of separated spaces in higher flat spaces is also studied.

22. Professor Edward Kasner: *Einstein solutions in space of seven dimensions.*

In his second paper, the author considers the summation of forms involving common independent variables. This is applied to the decompositions of solutions of the cosmological field equations whose coefficients are functions of a single variable, say x_1 . These may be broken up into the sum of three binary forms belonging to three surfaces of revolution. The final four-dimensional manifold may then be constructed synthetically and immersed in a seven-flat. The rotating curves involve abelian integrals.

23. Professor J. H. M. Wedderburn: *Algebras which do not possess a finite basis.*

In the classification of algebras which do not necessarily possess a finite basis difficulties arise which are not found in the corresponding theory when a finite basis is present. In this paper no basis is assumed to exist, but it is found that the principal results of the ordinary theory can be deduced with the aid of some simple assumptions regarding idempotent elements. Several examples are given.

24. Professor J. H. M. Wedderburn: *A theorem on simple algebras.*

This paper will appear in full in an early number of this BULLETIN.

25. Dr. H. E. Bray: *Green's formula.*

In this paper a new proof of Green's formula, $\iint_S (\partial Q / \partial x) dx dy = \int_C Q dy$, is given, where the boundary C is a rectifiable closed curve without multiple point and the quantities in the two members are respectively Lebesgue and Stieltjes integrals. The theorem is briefly proved in the case where Q is continuous in (x, y) and absolutely continuous in x for every y . Next the condition that Q be continuous is removed; Q is assumed to be bounded and equal to the symmetric superficial derivative of its own Lebesgue integral, and the line integral is treated as a Lebesgue-Stieltjes integral. Finally the Stieltjes potential function $\iint_E \log(1/r) df(e)$ is discussed, and the equation

$$\iint_S dx dy \int \int_E \frac{\cos xr}{r} df(e) = \int_C dy \int \int \log(1/r) df(e)$$

is shown to be valid for nearly every position into which the curve C may be brought by translation.

26. Mr. A. J. Maria: *Functions of plurisegments.*

In the present paper the analysis of Vitali (RENDICONTI DI PALERMO, vol. 46 (1922), p. 388) is applied to the two-dimensional case of functions of plurisegments. To determine the structure of functions of limited variation of this type we associate with every such function an additive function, called a discard. This function measures the quantity by which the function falls short of being absolutely continuous when there are no point values. It is shown that the property which characterizes a discard is that the discard coincides with its own discard. The theorem is proved that every such function can be decomposed into the sum of a function of point values, an absolutely continuous function, and a finite sum or infinite series, each term of which is an elementary discard multiplied by a constant.

R. G. D. RICHARDSON,
Secretary.