SHORTER NOTICES

James Stirling: A Sketch of his Life and Works, along with his Scientific Correspondence. By Charles Tweedie. Oxford, Clarendon Press, 1922. x + 213 pp.

This book is an interesting and valuable contribution to the history of mathematics, giving as it does a considerable amount of new information about the work of the early successors of Newton. One of the ablest of these was James Stirling, and thus his hitherto unpublished mathematical correspondence contains much of interest.

The book is divided into three parts, "Life," 22 pages, "Works," 26 pages, and "Correspondence," 160 pages. The first part adds some interesting details to the rather meagre stock of information available concerning Stirling's life. He was one of the non-juror students at Oxford, and as such under suspicion at the time of the accession of George I, when there were riots and a revolution in favor of the restoration of the Stuarts seemed imminent. In fact, because of his Jacobite leanings, Stirling was deprived of a scholarship which he had previously held; but the usual account * that he was expelled from the University and "fled to Italy" is incorrect, as the author brings out the fact that Stirling only left Oxford in 1717, to go to Venice on the offer of a mathematical professorship there. This was the year in which he published (in Oxford) his book on cubic curves. The offer in Venice was eventually declined, on account of the religious conditions attached to it. He remained in Italy, however, till about 1724, when he located in London. He was a friend of Newton and of Maclaurin, and was highly esteemed by other contemporary mathematicians, particularly Machin, DeMoivre, and Cramer. The latter part of his life (from the age of 43 on) he devoted to commercial work, and thus his mathematical productivity came to an end just at the time when he was in a position to have added much to the development of the subject.

Stirling's chief works are *Lineae Tertii Ordinis* (1717) and *Methodus Differentialis* (1730). These are well described in the second part of the book under review. The *Methodus Differentialis* contains most of Stirling's own contributions to mathematics; these relate to the summation of series, to interpolation, and to approximation formulas, the most famous of the last being "Stirling's Formula,"

$$n! = \sqrt{2n\pi}\cdot n^n\cdot e^{-n}.$$

He finds the exterpolated value of the term preceding the first in the series of factorials,

$$1,\ 1,\ 2,\ 6,\ 24,\ 120,\ \text{etc.}$$

to be 1.7724538502, and says that this is equal to $\sqrt{\pi}$. He also introduces the Beta Function as an integral, for the interpolation of

$$a,\quad \frac{r}{p}a,\quad \frac{r(r+1)}{p(p+1)}a,\quad \cdots,$$

* *Dictionary of National Biography*; Cantor, *Geschichte der Mathematik*, vol. 3, p. 372.

and obtains a result equivalent to

$$\frac{B(p + n', q)}{B(p'q)} = \frac{p(p + 1)\cdots(p + n - 1)}{(p + q)(p + q + 1)\cdots(p + q + n - 1)}.$$

The third part of the book is the most important, both in volume and in the additional light which it throws upon Stirling's work in its relation to his contemporaries. Thus there are 11 letters from Maclaurin to Stirling, 10 from Cramer, 3 from Nicholas Bernoulli (1687–1750), the nephew of John Bernoulli, 2 from Machin, and one, perhaps the most valuable in the entire collection, from Euler. There are 9 letters written by Stirling (4 to Maclaurin, 1 to Cramer, 1 to N. Bernoulli, 1 to Castel, 1 to Bradley, and 1 to Euler). All the letters to Stirling testify to the high regard in which the writer held him, and to the influence which his writings exerted upon contemporary mathematics. The lett er from Euler mentioned above, contains a rich fund of information about his work in series, including $\Sigma_{n=0}^{\infty} 1/n^k$, $\Sigma_{n=0}^{\infty} 1/(2n + 1)^k$, and others. A curious infinite product which he gives is perhaps not generally known:

$$\frac{3\cdot5\cdot7\cdot11\cdot13\cdot17\cdot19\cdot23\cdot29\cdots}{4\cdot4\cdot8\cdot12\cdot12\cdot16\cdot20\cdot24\cdot28\cdots},$$

where the factors of the numerator are the successive prime numbers, and each factor of the denominator is the integer of form $4k$ nearest to the corresponding factor in the numerator. Euler says the limit of this fraction is $\pi/4$, but does not give any hint as to his method of proof.

Mr. Tweedie has done a valuable piece of work, which will be found useful not only as a contribution to the history of mathematics, but also as giving in very readable form a source of information as to one of the founders of the modern theory of finite differences.

R. B. McClenon

The Fourth Dimension and the Bible. By W. A. Granville. Boston, Richard G. Badger, 1922. 9 + 119 pp.

The author of this little book points out in his preface that there is little hope of building a system of theology on a foundation either of philosophy or of the physical sciences, owing to the fact that neither of these branches of knowledge present results of a sufficiently authoritative character. He calls attention to the fact that the very foundations of the physical sciences themselves are at the present time open to the most serious challenge. Mathematics is the only existing body of knowledge which may be regarded as entirely authoritative. The author accordingly hopes "to throw some light, even though very dim, on some of the questions connected with our Christian beliefs." His chief aim is "to point out the remarkable agreement which exists between numerous Bible passages and some of the concepts which follow naturally from the mathematical hypothesis of higher spaces." As a popular introduction to some of the fundamental notions of four-dimensional space, the book is well done, although it seems to the reviewer that the author has rather over-emphasized the notion of reasoning by analogy. As to the value of his contribution to theology, let others speak.

J. W. Young