# TWO BOOKS ON ANALYSIS

*Lezioni di Analisi Infinitesimale.* By Giulio Vivanti. 2d edition. Torino, S. Lattes and Co., 1920. vii + 693 pages.

*Lezioni di Calcolo Infinitesimale.* By Ernesto Pascal. Milano, Ulrico Hoepli. Part I, 4th edition, 1919, viii + 330 pages. Part II, 4th edition, 1918, xii + 313 pages. Part III, 2d edition, 1918, xi + 325 pages.

Permitting oneself a variation on a well known theme, one could perhaps say to a nation: "Show me your fundamental course in analysis and I will tell you who you are mathematically." For doubtless, a valuable estimate of a nation's state of development in mathematics can be obtained by considering the form in which the fundamentals of the infinitesimal calculus are presented to the students of that nation.

If the two works under review may be taken as representative Italian courses, one must form a high estimate of the esteem in which mathematics is held in that country. In scope, point of view and method of approach, they are broad and scholarly. They cover practically the same ground except for Pascal's third volume which is devoted entirely to the calculus of variations and to the calculus of finite differences. They are not encyclopedic like the French cours d'analyse; neither are they written on the superficial plan of so many of our American college texts. From them a student can learn enough to be well prepared for special study in analysis, as well as for work in the applied sciences—he will at least have acquired that which he is to apply. Both intended for technical students, the authors do not hesitate to include the elements of the theory of point sets and other topics usually regarded in our classes as material unfit for the training of "practical men." In his preface, written May 1917, Pascal says "It is certain that through the profound changes which the critical spirit has made in the foundations of the calculus, even a course intended for those for whom mathematics is a means rather than an aim, cannot but use the new results which have been reached . . . it would therefore exhibit a shortsighted view and little esteem for the ability of the future engineer, to believe that it would be sufficient for them, at least if they can, to learn to operate the calculus in about the way in which a workman knows how to operate a machine made by others, and of which he does not know the inner connections."

Is not this a point of view worth the consideration of our teachers of engineering students? Not merely for the mathematical specialist, but for the person concerned with the applied sciences, a fundamental theoretical course, not merely a working course, is requisite if these applications are to be more than mere mechanical adaptations of the thoughts of others.

The two books differ in their method of treatment, inasmuch as Vivanti is in favor of and Pascal opposed to the fusion of differential and integral calculus. The *Lezioni* of Vivanti consists of the following six parts: I. Analytical introduction (90 pp.), II. Derivatives and integrals of functions of one variable (150 pp.), III. Derivatives and integrals of functions of several variables (84 pp.), IV. Geometrical applications (183 pp.), V. Differential equations (138 pp.), VI. Calculus of variations (20 pp.).

In the analytical introduction are assembled those notions from the theory of functions of a real variable which form the irreducible minimum for a sound development of the calculus: real numbers, sequences and series, greatest lower and least upper bounds, continuity, uniform convergence, etc. This part reveals what an excellent expositor the author is. The reviewer noticed with pleasure several neat proofs, such as the one on page 39, in which it is shown that $\lim_{x \to c} f(x) = a$ implies that $\lim_{x \to c} b^{f(x)} = b^a$. In the arrangement of the material, in the working out of details, in the style and the form, a masterly hand shows itself. This section provides for the reader an enjoyment which I would only spoil by dwelling on the details.

With such preparation a thorough treatment of the calculus becomes possible. A curious slip occurs on page 140, where from the uniform convergence of a series of functions $f_i(x)$ on an interval $(ab)$, the conclusion appears to be drawn that a series of functional values $f_i(x_i)$ must converge for arbitrary choice of the $x_i$. It is surprising to find in this book the notation $\partial f(\alpha, \beta)/\partial \alpha$ where $[\partial f(x, y)/\partial x]_{\alpha, \beta}$ is meant, a notation no longer used in our better texts and, in my judgment, very undesirable. Space is lacking for detailed mention of the many instances of elegance in treatment found throughout these parts of the book.

It seems curious that with such free fusion of the two divisions of the calculus, there should be such a sharp separation between the calculus and its geometric applications, to which the largest single part of the book is devoted. In the first 20 pages of this section we find an exposition of the elements of vector analysis which follows the methods of Burali-Forti and Marcolongo. Throughout the rest of the section these methods, as well as the ordinary methods, are used in the applications to differential geometry, which in many instances are improved and simplified in this way. The incorporation of the vector treatments, assembled in an appendix in the first edition of the *Lezioni*, as an organic part of the book in this new edition, bears witness to the vogue which the work of Burali-Forti and Marcolongo has given these methods.

In Part V, particular mention should be made of the simple and straightforward discussion of the singular integral and its geometrical interpretation, as well as of the treatment of the linear homogeneous equation with constant coefficients by the use of simple relations holding between the properties of the linear differential operator and the associated algebraic functions. A brief section devoted to the calculus of variations treats a few of the classical problems and merely mentions some of the others. Numerous examples are worked out in the text. For further exercises the reader should consult the companion volume, the author's *Esercizi*.* The misprints are very few and do not seriously mar the typographical excellence of this book.

* See the review of the book by R. C. Archibald in this BULLETIN, vol. 20 (1914), p. 482.

In the three volumes of Pascal's *Lezioni* on the other hand, misprints are quite numerous, particularly in volume 3, where some fifty were noticed on slightly over three hundred duodecimo pages. Enough has been said above in comparing the two works to make further discussion of the first two volumes of Pascal's *Lezioni* superfluous. The third volume is little different from the first edition of 1897 of which the first part appeared in a German translation by Schepp in 1899.* It appears to be a reprint of this earlier volume rather than a new edition. It is this which accounts for the apparent disregard of the important developments in the calculus of variations during the last twenty years. It still frankly represents the old school, so that a detailed criticism from the modern point of view, for which the book furnishes ample opportunity, would clearly be out of place. The extensive bibliographies inserted at various points form a valuable feature. It is surprising, however, that in the list of treatises on page 19, apparently revised since the earlier edition, Hadamard's *Leçons* is not mentioned, while there is mention made of a *Lehrbuch der Variationsrechnung* by Carathéodory and Zermelo, which, although announced repeatedly, has not yet appeared and does not occur on Teubner's later lists of future publications.

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# SHORTER NOTICES

*Allgemeine Theorie der Raumkurven und Flächen.* By V. Kommerell and K. Kommerell. Vols. I and II, third edition. (Sammlung Schubert, XXIX and XLIV.) Berlin and Leipzig, Vereinigung wissenschaftlicher Verleger, 1921. 184 + 196 pp. 28 + 13 figs.

The third edition of these two volumes is so nearly a reprint of the second edition, which has already been reviewed in this BULLETIN,† that an account of its contents is quite unnecessary. The arrangement of material is precisely the same in the two editions, but many of the discussions have been slightly amplified in the later one and the few errors in printing have been corrected. The only new material appears in the derivation of curves from given properties and in the definition of the Christoffel symbols.

These two volumes should be intelligible to the student who has little training beyond the calculus, yet they present an excellent treatment of the essentials of differential geometry. The student who has read them should have no difficulty with the more extensive treatments of Bianchi, Eisenhart, Forsyth and others. Such a presentation of differential geometry as this by Kommerell and Kommerell, if available in English, would increase the teaching of that important subject to the advanced students in our American universities.

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* See the review by E. R. Hedrick in this BULLETIN, vol. 12 (1906), p. 172.

† Vol. 21 (1915), pp. 99–100.