of the first chapter (pages 8–11); it is too long for reproduction in the review. The whole treatment proceeds in intimate dependence upon this logical basis. Certain central results are first obtained in association with any group for which hypotheses $H$ are satisfied.

Chapter II is devoted to a class of linear groups: for the case of one variable these became the fuchsian groups whose properties have been treated by Poincaré; for the case of two variables they become the hyperfuchsian groups of Picard; for the case of more than two variables they are groups which have been studied by Fubini. All the groups of the class are shown to satisfy hypotheses $H$ of the first chapter. Further properties of this special class are developed. In Chapters III and IV there is a similar treatment of certain quadratic groups, and of certain groups formed from a set of several given groups. The treatment in these four chapters (pages 8–90) is general and abstract in character and is intimately dependent upon the basic postulates $H$. The final Chapter V (pages 91–123) is devoted primarily to the functions of Poincaré. On account of the special features of the more restricted theory, certain results become more precise than in the more general theory; and this fact is brought out by a derivation of the detailed results.

R. D. CARMICHAEL.

*Archimedes.* By SIR THOMAS LITTLE HEATH. Society for the Promotion of Christian Knowledge, London, 1920. vi + 60 pp.

AFTER becoming familiar with the larger works which have made Sir Thomas Heath so widely known, the reader who takes up the little work under review will do so with a feeling of surprise. The academic world has come to expect from his pen only such extended treatises as he has written upon Apollonius of Perga, Diophantus, Aristarchus, Euclid, and Archimedes,—treatises filled with erudition and written in that classical style of which he is a master. If the reader is a man of the cloister, the surprise will be unpleasant; if he is a man among men, it will be the opposite. Since the spirit of the time makes scholars more and more men of the world, the balance of judgment is certain to be in favor—shall we say of the appellant or the defendant?

What Sir Thomas Heath has done is to give a brief and popular summary of the results which he has set forth in his well known treatise on the great Syracusan, and in his pamphlet on the recently discovered manuscript (1906) on the method of treating mechanical problems. He has done this by means of seven brief and easily read chapters as follows: I. Archimedes; II. Greek Geometry to Archimedes (substantially the subject of the first volume of his forthcoming history of Greek mathematics); III. The Works of Archimedes; IV. Geometry in Archimedes; V. The Sandreckoner; VI. Mechanics; and VII. Hydrostatics. To this summary he has added a brief bibliography and a chronological table.

The story is told in a style that will easily appeal to the popular taste,—that is, to a taste that has not been trained to enjoy the more severe intellectual food served at the tables of erudition. The opening chapter, on the life of Archimedes, will fascinate any youthful reader and will be read with pleasure by those of more mature years. The chapter on Greek geometry requires a little knowledge of algebra and geometry, but can easily be read by any one with a high school education. The third chapter sets forth briefly the nature of the extant works of Archimedes and gives a list of those which are now known only by title. The chapter on geometry in Archimedes reaches a little further into the domain of mathematics, but will give the college freshman, in brief space, the essential information which he needs with respect to the contributions of the Greeks to the calculus. The sandreckoner is within the easy reach of the high school student, but the chapters on mechanics and hydrostatics will be found, in view of the denatured courses in physics in our American high schools, beyond the average pupil.

The frontispiece is from David Gregory's edition of Euclid (Oxford, 1703), already used in the author's work on Diophantus, but for some reason (perhaps the size of the page) there is no indication of the interesting source.

As a piece of technical bookmaking the work is, of course, not in the same class with the other publications of the author. The products of but few publishing houses can compare with those of the Cambridge and Oxford presses. Moreover, the book is published for popular use, and expense is a weighty consideration. The Society for the Promotion of Christian Knowledge has done and is doing a praise-worthy work in placing such works within the reach of every purse. It is a matter of great regret that we have not, in this country, a similar foundation which enables us to publish a series of works of this nature at a nominal price, as is done in several European countries. No better field than this is today open in this country for establishing a relatively small foundation which should seek to satisfy a hunger for good reading. With the work of this British society in mind, one can readily excuse the lack of an index, and the poor paper which war conditions have imposed.

DAVID EUGENE SMITH.

*The Theory of the Imaginary in Geometry together with the Trigonometry of the Imaginary.* By J. L. S. HATTON, principal and professor of mathematics, East London College. Cambridge, England, University Press, 1920. 216 pp. and 96 figures.

THE word theory in the above title is to be understood in a very non-technical sense. Indeed, apart from the idea of the invariant elements of an elliptic involution on a straight line, no theory is found at all. The purpose of the book is rather to furnish a certain graphical representation of imaginaries under a number of conventions more or less well known. Three concepts run through the work: first, an incompletely defined idea of the nature of an imaginary; second, the analogy with the geometry of reals; third, the use of coordinate methods, assuming the algebra of imaginaries.

Given a real point $O$ and a real constant $k$, an imaginary point $P$ is defined by the equation $OP^2 = -k^2$. The two imaginary points $P$ and $P'$ are the double points of an involution having $O$ for center, and $ik$ for parameter. The algebra of imaginaries is now assumed, and a geometry of imaginary distances on a straight line is built upon it. The reader is repeatedly reminded that in themselves there is no difference between real and imaginary points; that differences exist solely in their relations to other points. In the extension to two dimensions both $x$ and $ix$ are plotted on a horizontal line, while $y$ and $iy$ are plotted on a vertical line. Imaginary lines are dotted, and points having one or both coordinates imaginary are enclosed by parentheses, but otherwise the same figures are used for proofs, either by the methods of elementary geometry, or by coordinate methods.

In the algebra of segments it is shown that an imaginary distance $O'D'$ can be expressed in the form $iOD$, wherein $OD$ is a real segment, or at most by $OD$ times some number. Now