As for more important criticism, one may quarrel with the author for his abstract style, for his euclidean manner of grading the proofs, so that no difficulties remain and none but mild climaxes are reached, for his finish that may excite admiration but hardly activity on the reader's part. One may crave for a book that is built like a drama around a single idea —a more sketchy book, leaving more to the reader's imagination, a book with a less diversified and more emphatic message. But such remonstrance would be like quarrelling with Beethoven for having written symphonies instead of operas. There is no such thing as *the* book. Hausdorff's Grundzüge is a treatise, and as a treatise it necessarily falls short of the summum bonum. But as a treatise it is of the first rank.

HENRY BLUMBERG.

SHORTER NOTICES.

The Casting-Counter and the Counting Board. By FRANCIS PIERREPONT BARNARD. Oxford, Clarendon Press, 1916. 358 pp. + 63 plates.

When we consider that Gerbert, the greatest mathematician living in Europe at the close of the tenth century, wrote upon the use of counters as an aid to computation; that Robert Recorde, who is often called the founder of the English school of mathematicians of the sixteenth century, did the same; and that nearly all computation in Europe before the year 1500 (in Italy before c. 1200) was performed by the aid of some type of abacus, we may well infer that the "casting-counter," as Professor Barnard calls it, has played an important rôle in the history of calculation. Indeed, our very word "calculate" is, it need hardly be said, due to this very fact, the word "calculus" meaning a pebble, calculi being used in numerical work in the classical period of the Greek and Roman civilizations.

When we also consider the fact that it was the bamboo rods, used by the early Chinese algebraists to express coefficients, that suggested to the Japanese the *sangi* which were used for the same purpose, and also suggested the idea of determinants which their scholars developed in the seventeenth

century, anticipating the expansion of these forms by Leibniz, the counter takes on a dignity that might hardly have been anticipated. And finally when we consider that it was the humble abacus that led to such devices as the suan pan, the soroban, the tschotü, and the choreb now used in, numerically speaking, the greater part of the civilized world; that led Pascal to invent and Leibniz to work upon the modern calculating machine; and that finds frequent place in the literature of Chaucer, Shakespeare, and other makers of our language, it may well occur to the mathematician to look with interest upon the treatise under review, which Professor Barnard has published in such sumptuous form.

The counters that have come down to us date from the thirteenth to the eighteenth century, the earlier pieces having been melted up for the metal or lost because of the very fact that they were too common to be held in much esteem. There are, of course, many disks of bone, baked earth, or metal that have come down to us from ancient times and which may have been used for purposes of calculation by the calculones, the calculatores, or the numerarii; but since they bear no inscriptions, we are uncertain of their use and they may have been merely pieces employed in playing such ancient games as backgammon or checkers. From the thirteenth century on for five hundred years, however, specimens were preserved, and these still remain to tell the story, decade by decade, of the use of these devices, of their change in form, of the Rechenmeisters for whom they were struck, and of the slow decay of the abacus as the power of the eastern numerals came to be the better understood. To-day, about all that we have in common use to remind us of the ancient and medieval counter (projectilis, jetton, augrim stone, calculus, abaculus, Worpghelt, Legpennig, Rechenpfennig, méreau à compte, tessera ad computandum, and the like) of our ancestors—is the poker chip, the wire with its disks above the billiard table, and the string of beads used in various religious ceremonials throughout the Buddhist, Mohammedan, and Christian world.

In the work under review Professor Barnard has given us the benefit not only of knowing about his collection of some 7,000 jettons and about some 40,000 other specimens which he has examined, but of benefitting by his rare scholarship in all that relates to medieval history. As to the former, he

has reproduced by photographic process a hundred and twenty-six of the rarest counters in his collection, generally both in obverse and reverse, and has given scientific descriptions of a large number of others. As to the opportunity that he has given the reader of having some share in his scholarship—he has shown the painstaking care in research into historical details, and the interesting style of presenting facts that characterize his other writings upon the period to which he has devoted a life of study. The fact that the bibliography of works consulted contains more than six hundred items is proof of the care with which he has considered the literature relating to the subject.

The work consists of three parts. Part I relates to the history of the casting-counter and to descriptions of the specimens found in England, Italy, France, the Low Countries, Germany, and Portugal. Part II contains the only worthy description that we have of the counting boards and counting cloths used in the medieval and Renaissance periods, and is admirably illustrated by numerous plates. Part III sets forth the methods of casting with jettons, a subject upon which we have plenty of information from such sources as Robert Recorde (c. 1542), John Awdeley (1574), Nicholas Cusa (1514), Martinus Siliceus (1526), Köbel (1514), and various other writers of the sixteenth century, upon whose works the author has freely drawn.

There are two elaborate indexes, one of legends and inscriptions on the counters and the other of a general nature—and nothing is more conducive to the comfort of a student who has occasion to consult a work of this nature than a good index.

It may be said of the work as a whole that it represents the most elaborate study that we have upon any of the minor features of this kind in the history of mathematics, and that it deserves a place in every college and university library and on the shelves of every one who is working in the special field of the history of computation. DAVID EUGENE SMITH.

Tables des Nombres Premiers, et de la Décomposition des Nombres de 1 *à* 100,000. By G. INGHIRAMI, reviewed and corrected by Dr. PROMPT. Gauthier-Villars et Cie., 1919. xi + 35 pp.

THIS little factor table gives the smallest divisor of all