where $\epsilon = e^{2\pi i/n}$. It is shown that on the unit circle $|z| = 1$ we have

$$(1) \qquad |F(z)| < \frac{1}{n}\sum_{\nu=0}^{n-1} \frac{1}{\sin \frac{2\nu+1}{2n}\pi}$$

except when $F(z)$ has the form $e^{\alpha i}f(\epsilon^{-k}z)$, where α is real, k an integer and

$$(2) \qquad f(z) = \sum_{\nu=0}^{n-1} \frac{(\epsilon^{-\frac{1}{2}}z)^\nu}{n \sin \frac{2\nu+1}{2n}\pi},$$

in which case the upper bound of $|F(z)|$ is reached at $z = \epsilon^{\frac{1}{2}+k}$. The polynomial $f(z)$ has all its zeros on the unit circle, one in each of the intervals from ϵ to ϵ^2, ϵ^2 to ϵ^3, $\cdots$, ϵ^{n-1} to 1. The asymptotic value of the upper bound in (1) is

$$\frac{2}{\pi}\left(\log n + C + \log \frac{2}{\pi}\right) + \sigma(1)$$

where C is Euler's constant, and $\sigma(1)$ tends to zero as n increases indefinitely.

34. Dr. Gronwall shows that when $w = z + a_2z^2 + \cdots a_nz^n + \cdots$ maps the circle $|z| < 1$ conformally on a simply connected and nowhere overlapping region in the w-plane, then $|a_n| \leqq n$ for $n = 2, 3, \cdots$. When $|a_n| = n$ for any particular n, then also $|a_n| = n$ for every n, and the function w reduces to the one which gives extreme values to the distortion.

ARNOLD DRESDEN,
Acting Secretary.

THE CHICAGO COLLOQUIUM.

THE ninth colloquium of the American Mathematical Society was held in connection with its twenty-seventh summer meeting at the University of Chicago, September 8–11, 1920. At the annual meeting of 1917, the Council, on the invitation of the Department of Mathematics of the University of Chicago, appointed a committee, consisting of Professors E. H. Moore, E. W. Brown, Max Mason, H. S. White, and the Secretary, to arrange for a summer meeting and colloquium to be held at Chicago in 1919. At the annual meeting of 1918 the Council authorized the postponement of the Chicago meeting until the summer of 1920. The courses of lectures were announced in the preliminary circular of May, 1920, and printed syllabi were distributed at the meeting. The colloquium opened Wednesday afternoon, September 8, and continued until Saturday noon; three lectures were delivered on Thursday, and Friday, and two on each of the other days. At the close of each lecture opportunity for discussion was given, on points of that or previous lectures.

The following ninety persons were in attendance, a number considerably exceeding that of any previous colloquium.

Mr. E. S. Akeley, Professor R. C. Archibald, Professor G. N. Armstrong, Professor I. A. Barnett, Professor S. R. Benedict, Professor A. A. Bennett, Professor G. D. Birkhoff, Professor G. A. Bliss, Professor R. L. Borger, Professor J. W. Bradshaw, Professor W. C. Brenke, Professor W. H. Bussey, Professor W. D. Cairns, Professor J. W. Campbell, Mr. F. E. Carr, Mr. W. E. Cederberg, Professor E. W. Chittenden, Professor A. R. Crathorne, Dr. G. H. Cresse, Professor D. R. Curtiss, Professor E. L. Dodd, Professor L. W. Dowling, Professor Arnold Dresden, Professor Otto Dunkel, Mr. J. D. Eshleman, Professor G. C. Evans, Professor H. S. Everett, Dr. L. R. Ford, Professor W. B. Ford, Professor M. G. Gaba, Professor D. C. Gillespie, Professor R. E. Gilman, Dr. T. H. Gronwall, Professor C. F. Gummer, Professor W. L. Hart, Professor M. W. Haskell, Professor Olive C. Hazlett, Professor E. R. Hedrick, Professor T. H. Hildebrandt, Professor L. A. Hopkins, Professor W. A. Hurwitz, Professor Dunham Jackson, Mr. D. C. Kazarinoff, Miss Claribel Kendall, Professor S. D. Killam, Professor H. W. Kuhn, Professor Gillie A. Larew, Professor Flora E. LeStourgeon, Mrs. Mayme I. Logsdon, Professor A. C. Lunn, Mr. C. C. MacDuffee, Professor Helen A. Merrill, Professor Bessie I. Miller, Professor W. L. Miser, Professor C. N. Moore, Professor E. H. Moore, Professor E. J. Moulton, Professor F. R. Moulton, Professor A. L. Nelson, Mr. H. L. Olson, Miss Eleanor Pairman, Professor Anna H. Palmié, Professor Anna J. Pell, Dr. T. A. Pierce, Professor A. D. Pitcher, Professor S. E. Rasor, Professor R. G. D. Richardson, Professor H. L. Rietz, Professor W. H. Roever, Miss I. M. Schottenfels, Dr. Caroline Seely, Professor E. W. Sheldon, Dr. W. G. Simon, Professor E. B. Skinner, Professor H. E. Slaught, Professor H. L. Smith, Professor C. E. Stromquist, Professor K. D. Swartzel, Dr. Bird M. Turner, Professor A. L. Underhill, Professor E. B. Van Vleck, Mr. J. H. Van Vleck, Professor Oswald Veblen, Professor W. P. Webber, Mr. F. M. Weida, Professor Mary E. Wells, Professor C. E. Wilder, Professor F. B. Wiley, Professor C. H. Yeaton, Professor J. W. A. Young.

Two courses of five lectures each were given, as follows:

I. Professor G. D. BIRKHOFF: "Dynamical systems."

II. Professor F. R. MOULTON: "Topics from the theory of functions of infinitely many variables."

Abstracts of the lectures follow below. The lectures will be published later in full as Volume VI of the Colloquium Series.

I.

LECTURE I. PHYSICAL, FORMAL, AND COMPUTATIONAL ASPECTS OF DYNAMICAL SYSTEMS.

1. The conservation of energy; rates of doing work of external forces:

$$Q_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + R_i \qquad (i = 1, 2, \cdots, n),$$

$$\sum_{i=1}^{n} \dot{q}_i R_i \equiv 0.$$

2. Lagrangian systems; $R_1 = R_2 = \cdots = R_n = 0$. Internal and external characterization.

3. Variational form of equations under no external forces: $\delta \int L dt = 0$. Change of variables.

4. Equations of variation.

5. Integrals linear or quadratic in the velocities.

6. The Hamiltonian equations: $\dfrac{dp_i}{dt} = \dfrac{\partial H}{\partial q_i}$; $\dfrac{dq_i}{dt} = -\dfrac{\partial H}{\partial p_i}$.

Their fundamental properties. Formal series.

7. Methods of computation and their validity.

8. Relativistic dynamics.

9. Dissipative systems.

Lecture II. Types of Motions Such as Periodic and Recurrent Motions, and Motions Asymptotic to Them.

1. Existence of periodic motions; minimum and minimax methods; method of analytic continuation; other methods.
2. Hyperbolic periodic motions and their asymptotic motions.
3. Stable elliptic periodic motions.
4. Unstable elliptic periodic motions and their asymptotic motions.
5. Recurrent motions and their asymptotic motions.
6. Other types of motions.
7. The extension in General Analysis.

Lecture III. Interrelation of Types of Motion with Particular Reference to Integrability and Stability.

1. Transitivity and intransitivity.
2. Distribution of periodic motions.
3. Distribution of recurrent and other special motions.
4. Criteria for various types of integrability.
5. Non-integrability of general case.
6. Criteria for various types of stability.

Lecture IV. The Problem of Three Bodies and Its Extension.

1. The equations of motion and the classical integrals.
2. Regularization at double collision.
3. Impossibility of triple collision if area constants are not all zero.
4. Proof that for given area constants not all zero, and small initial mutual distances, the sum of the mutual distances becomes infinite.
5. Deduction of Sundman's theorem on mutual distances as corollary.
6. Extension to more general law of force.
7. Extension to case of n bodies.

LECTURE V. THE SIGNIFICANCE OF DYNAMICAL SYSTEMS FOR GENERAL SCIENTIFIC THOUGHT.

1. The dynamical model in physics.
2. Modern cosmogony and dynamics.
3. Dynamics and biological thought.
4. Dynamics and philosophical speculation.

II.

LECTURE I. INFINITE SYSTEMS OF LINEAR EQUATIONS.

1. Completely reduced systems. Historical examples.
2. The formal method of reduced systems. Historical examples.
3. Normal infinite determinants. The Hill-Poincaré form; the von Koch form.
4. Infinite systems of linear equations having normal determinants and bounded right members.
5. Absolutely convergent infinite determinants.
6. Infinite systems of linear equations having absolutely converging determinants and bounded right members.
7. Infinite systems of linear equations having absolutely converging determinants and coefficients analytic functions of a parameter.
8. Irregular solutions of infinite systems of linear equations. Examples.
9. The general theory of Schmidt. Solutions for which

$$\sum_{i=1}^{\infty} |x_i|^p \qquad (p > 1)$$

converges, including the limiting cases $p = 1$, $p = \infty$.

10. The method of successive approximations.

LECTURE II. ON PROPERTIES OF FUNCTIONS OF INFINITELY MANY VARIABLES.

1. Hilbert space (H-space) and parallelepipedon space (P-space). The mutual independence of H-space and P-space.
2. Definitions of limit points. Existence of a limit point in an infinite set of points.
3. Types of continuity and relations among them.
4. Convergent functions and uniformly convergent functions.

5. Independence of continuity and convergence.
6. Representation of convergent functions by series.
7. A continuous function of infinitely many variables in a closed P-space has a maximum and a minimum and all intermediate values.
8. Definition of analytic functions of infinitely many variables.
9. Definition of normal functions of infinitely many variables.
10. Finite operations on functions of infinitely many variables.
11. Limiting processes on functions of infinitely many variables.
12. The mean-value theorem for completely continuous functions having first derivatives.
13. Taylor's theorem for functions of infinitely many variables.

Lecture III. Infinite Systems of Implicit Function Equations.

1. Analytic solutions of reduced normal equations.
2. Analytic continuation of the solutions.
3. Solutions of normal equations having normal determinants of the coefficients of the linear terms of the dependent variables.
4. Properties of the solutions of normal equations.
5. Solution of reduced normal equations by the method of successive approximations.
6. Extension of the solution to a boundary of the region of definition of the equations.
7. Solutions of infinite systems of equations having continuous first derivatives.
8. Properties of the solutions.
9. Extension of the solution to a boundary of the region of definition of the equations.

Lecture IV. Infinite Systems of Differential Equations.

1. Analytic solutions of normal equations.
2. Solution of normal equations by the method of successive approximations.

3. Solution of equations satisfying the Lipschitz condition by the method of successive approximations.
4. Properties of the solutions.
5. Extension of the solution to a boundary of the region for which the equations are defined.
6. Solution by the Cauchy-Lipschitz method.
7. Solutions of infinite systems of linear differential equations having constant coefficients.
8. Solutions of infinite systems of linear differential equations having periodic coefficients.

LECTURE V. APPLICATIONS OF FUNCTIONS OF INFINITELY MANY VARIABLES.

1. Hill's problem of the motion of the lunar perigee.
2. Solutions of linear differential equations in the vicinity of singular points.
3. The determination of the moon's variational orbit.
4. Determination of periodic solutions of certain finite systems of differential equations.
5. The dynamics of a certain type of infinite universe.

At the close of the colloquium, Professor E. B. Van Vleck expressed the appreciation of those present for the excellence of the lectures, and tendered the thanks of the American Mathematical Society to the University of Chicago for the generous provision it had made for the colloquium, and for the welfare of the participants. An appropriate reply was made by Professor E. H. Moore.

W. A. HURWITZ.

NOTE ON VELOCITY SYSTEMS IN CURVED SPACE OF N DIMENSIONS.

BY PROFESSOR JOSEPH LIPKA.

(Read before the American Mathematical Society April 24, 1920.)

§ 1. *Introduction.*

IN a previous paper,* the author gave a complete geometric characterization of the families of curves (termed natural

* "Natural families of curves in a general curved space of n dimensions," *Trans. Amer. Math. Society*, vol. 13 (1912), pp. 77–95. We shall hereafter refer to this paper as "Natural families."