

which mathematics gives rise, such as infinity, countability; (c) the introduction of a mathematical point of view, and as far as possible, of mathematical methods into the discussion of problems belonging to the domains of epistemology, of ethics, and to other philosophical fields. For progress in the treatment of these questions the closest cooperation between philosophers and mathematicians is essential. Such cooperation will certainly be stimulated by the reading of Professor Shaw's book.

ARNOLD DRESDEN.

*Functions of a Complex Variable.* By THOMAS M. MACROBERT.

London and New York, Macmillan, 1917. xiv + 298 pp.

THIS book is designed for students who have acquired a good working knowledge of the calculus and desire to become acquainted with the theory of functions of a complex variable and its principal applications. The material has been well chosen for accomplishing this purpose. The first two chapters are intended to familiarize the student with the geometrical representation of complex numbers in a plane and with simple rational and irrational functions of a complex variable. Some fundamental properties of holomorphic functions are established in Chapter III and these are used in defining certain elementary transcendental functions. Chapters IV to VII deal in order with the theory and practice of integration, convergence of series and the Taylor and Laurent expansions, uniformly convergent series and infinite products, and various summations and expansions. Up to this point (page 131) the book gives to the student an introduction to the elementary parts of the classic theory of functions of a complex variable with some of the simpler applications.

The theory of the gamma functions is presented in Chapter VIII (pages 132-159) in a way which is not very satisfying. The development of the initial elementary properties of elliptic functions is to be found in Chapters IX, X, XI (pages 160-207) in an exposition which is pleasing and particularly well suited to the needs of the young learner. The remaining chapters (IX to XV, pages 208-276) are devoted to differential equations, in part to general theorems and in other part to special cases, such as the equations of Legendre and Bessel. This division of the book also is well suited to the needs of the beginner.

Besides numerous worked examples the book contains many

problems. In addition to those interspersed throughout the text, there are to be found at the ends of the chapters and in a set of miscellaneous examples at the end of the book no fewer than 550 problems. Many of these are quite elementary in character and serve merely to familiarize the reader with the notions involved in the text. Many others contain significant theorems which would be demonstrated in a more extensive treatise.

Concerning the character of his exposition the author says in his preface: "In order to avoid making the subject too difficult for beginners, I have abstained from the use of strictly arithmetical methods, and have, while endeavoring to make the proofs sufficiently rigorous, based them mainly on geometrical conceptions." It is evident that the desirable golden mean here is a matter on which there is likely to be difference of opinion. The reviewer believes that the statements are sometimes too loose. Thus "closed region" is first defined on page 92. Yet several times on the earlier pages statements have been made involving the word "region" which are correct only when one understands "closed region." For cases in point see page 24, line 6 up; page 26, line 7; page 39, line 7 up. In theorem 2 of page 93 it should be specified that the path of integration is finite in length or the proof should be modified. For theorem 2 on page 39 one should read "No [closed] region can contain an infinite number of isolated singularities [and no other singularities]," the words in brackets being those which one must insert into the theorem to make it accurate. These may be taken as examples of loose statements which are all too frequent, especially in the first 131 pages, in which the more general matters are treated.

A student who has been forewarned against these somewhat loose statements will find the book one by the reading of which he will be much profited. Many persons will probably find it also a suitable source of elegant elementary problems for enforcing a clear understanding of numerous fundamental notions and theorems.

R. D. CARMICHAEL.

*Leçons sur les Fonctions Elliptiques en Vue de leurs Applications.* Par R. DE MONTESSUS DE BALLORE. Paris, Gauthier-Villars, 1917.

THE object of these lectures, which M. de Montessus de Ballore delivered at the Faculté des Sciences of Paris in 1915-