equation is obtained, it is seen that the curve is a hypocycloid of three cusps. This is deduced in the following way:

The radius of curvature is found from (2) to be

$$R = \frac{2p(3-p^2)}{(1+p^2)^{3/2}}. \tag{3}$$

The length of arc is

$$S = \int_0^p \frac{2p(3-p^2)}{(1+p^2)^{5/2}}\,dp = -\frac{8}{3}\,\frac{1}{(1+p^2)^{3/2}} + \frac{2}{(1+p^2)^{1/2}} + \frac{2}{3},$$

from which

$$\frac{3}{2}\left(S - \frac{2}{3}\right) = \frac{3p^2-1}{(1+p^2)^{3/2}}. \tag{4}$$

In order to eliminate p from equations (3) and (4), form the expressions $R^2/4$ and $\frac{9}{4}(S - \frac{2}{3})^2$. It is seen at once that

$$\frac{R^2}{4} + \frac{9}{4}\left(S - \frac{2}{3}\right)^2 = 1, \quad \text{or} \quad R^2 + 9\left(S - \frac{2}{3}\right)^2 = 4, \tag{5}$$

which is the intrinsic equation of a hypocycloid. The curvature of (2) is

$$k = \frac{(1+p^2)^{3/2}}{2p(3-p^2)}, \tag{6}$$

also

$$\frac{dx}{dp} = \frac{2p(3-p^2)}{(1+p^2)^3}, \quad \frac{dy}{dp} = \frac{2p^2(3-p^2)}{(1+p^2)^3}. \tag{7}$$

From (6) and (7) it is seen that when $p = 0$ or $\pm\sqrt{3}$, dx/dp and dy/dp become zero and $k = \infty$. Hence (5) is the equation of a hypocycloid of three cusps.

UNIVERSITY OF COLORADO.

HERMITE'S WORKS.

Œuvres de Charles Hermite. Publiées sous les auspices de l'Académie des Sciences par EMILE PICARD. Vol. IV. Paris, Gauthier-Villars, 1917. 8vo. 593 pp.

THE present volume brings to a close the Œuvres of Hermite. It contains about ninety papers arranged chronologically, dating from 1879 and continuing to the year of his death, 1901.

By a strange coincidence, his first and last papers appeared in the initial volumes of new journals. The *Nouvelles Annales de Mathématiques* founded in 1842 contained the first fruits of his budding genius. Nearly sixty years later his last paper was written to lend the support of his great name to a new enterprise, *Le Matematiche pure ed applicate.* It was only a short note, a sort of open letter to the editor, which terminates with these words: "Prochainement je vous enverrai quelques remarques complémentaires, et une petite Note sur le produit d'une série quelconque par l'exponentielle e^{-x}." He did not live to carry out his benevolent intentions, but it is pleasant to recall that his last efforts were made in behalf of the rising generation and that he wished them to enjoy the encouragement that early publication and recognition alone affords.

The papers of this last volume are for the most part short and treat isolated topics in the elliptic functions, the Γ function, the functions of Legendre, continued fractions, and the theory of numbers. They represent the labors of an aging hero whose exalted official position makes exacting demands on his time and strength, but whose active brain still takes delight in research.

As one looks over the volume, there is much that catches the eye, but the fragmentary nature of most of the papers makes it impossible to give any adequate account of them without undue expenditure of time and space. Perhaps the most noteworthy are two papers on the application of the elliptic functions to the theory of numbers, viz., those beginning page 138, and page 223. They are a continuation of the line of thought which Hermite employed in his celebrated letter to Liouville, "Sur la théorie des fonctions elliptiques et ses applications à l'arithmétique" (1861). Instead of using the factors of the modular equation in the case of complex multiplication to get relations between the class numbers of binary quadratic forms of negative determinant as Kronecker had done, Hermite showed how the comparison of different developments of Θ quotients led to similar results. This method also gave him the number of decompositions of an integer into a sum of three and five squares, a problem rendered celebrated by the researches of Henry Smith and Minkowski.

A paper of general interest is entitled "Sur les racines de la fonction sphérique de seconde espèce," i. e., the roots of

$Q_n(x) = 0$. It was written in 1890, the same year as Klein's paper on the roots of the hypergeometric function $F(\alpha, \beta, \gamma, x)$. Neither author knew of the other's work at the time of publication, moreover the methods employed are totally different. Hermite's work on $Q_n(x) = 0$ led Stieltjes* to attack the same problem from another and more direct point of view. Neither paper seems to have received the attention it deserves.

The many and interesting properties of the Γ function came in for a large share of Hermite's attention. We note as especially important two papers on this subject: the one entitled "Sur une extension de la formule de Stirling," page 378, composed in 1893, the other "Sur la fonction log $\Gamma(a)$," page 412, written in 1895, and dealing with the same problem, viz., the asymptotic expansion of log Γ.

But of all the subjects dear to Hermite in his later years, the elliptic functions without doubt held first place. He knew Jacobi's Fundamenta Nova by heart and he reveled in the endless maze of formulas which to others seem almost an intolerable burden. Here are a few titles dealing with these functions: "Sur la différentiation des fonctions elliptiques par rapport au module," "Sur l'intégration de l'équation différentielle de Lamé," "Sur une proposition de la théorie des fonctions elliptiques," "Sur une formule d'Euler," "Sur une représentation analytique des fonctions au moyen des transcendantes elliptiques." These will suffice to give the reader an idea of the rest.

A feature of this volume will appeal to all mathematicians, as it reveals in no uncertain way Hermite's generous and affectionate nature; we mean the short sketches of the careers of his departed friends and colleagues, Bouquet, Halphen, Kronecker, Kummer, Cayley, Weierstrass and Brioschi. Speaking of Kronecker he says: "La louage se tait devant le deuil de la Science et l'émotion causée dans tout le monde mathématique, par la perte cruelle du grand Géomètre; à ces regrets douloureux, à ces souvenirs d'une vie remplie par tant de travaux et de découvertes, je joins ceux d'une amitié qui a été pendant trente années, l'honneur de ma vie scientifique et que je ne retrouverai plus."

Of Weierstrass he writes: "La vie de notre illustre Confrère a été en entier consacrée à la Science qu'il a servie avec un absolu dévoument. Elle a été longue et comblée d'honneurs;

* *Annales de Toulouse*, vol. 4 (1890).

mais devant une tombe qui vient de se fermer, nous ne rappelons que son génie et cette universelle sympathie qui s'accorde à la noblesse du caractère. Weierstrass a été droit et bon; qu'il reçoive le suprême hommage plein de regrets et de respect que nous addressons à sa mémoire! Elle vivra aussi longtemps que des esprits avides de vérités consacreront leur efforts aux recherches de l'Analyse, au progrès de la science du Calcul."

Let us quote from one more éloge, that of Brioschi: "J'ai été associé aux travaux de Brioschi; nous avons souvent mis en commun nos efforts; j'ai suivi sa carrière qui a été si belle, remplie par l'étude, et de grands services rendus à son pays. Nul ne ressent plus que moi la perte du grand géomètre, et de l'homme d'honneur, le souvenir de son amitié, d'une étroite liaison remontant à notre jeunesse me restera à jamais l'un des meilleurs et des plus chers de toute ma vie."

Other interesting biographical matter is contained in an address by Hermite at the inauguration of the new Sorbonne in 1900. It gives a brief but illuminating analysis of the work of the chief French mathematicians in the first half of the nineteenth century.

Finally we note the touching discours pronounced by Hermite at the celebration of his seventieth anniversary in 1892. A photograph of the medallion by Chaplain presented to him on this occasion by his friends and admirers from all over the world forms the frontispiece of the volume and a facsimile reproduction of an extract of a letter to Tannery relative to Hermite's classic researches on the elliptic modular function in 1858 brings the volume to a fitting close.

JAMES PIERPONT.

BLICHFELDT'S FINITE COLLINEATION GROUPS.

THE author of "Finite Collineation Groups" is certainly under great obligations to Professor H. H. Mitchell for his very generous and thorough review in this BULLETIN of February, 1918; particularly so since Professor Mitchell had evidently read the book with great care. The author is in full accord with the reviewer on a number of the defects pointed out by the latter; and he offers herewith a few remarks in the hope of clearing up those statements in the book that