

It must be said, however, that within this space Mr. Ford has succeeded well in the task which he has set for himself.

ARNOLD EMCH.

*A Course in Interpolation and Numerical Integration for the Mathematical Laboratory.* By DAVID GIBB. (Edinburgh Mathematical Tracts, No. 2.) London, G. Bell and Sons, 1915. viii+90 pp.

*A Course in Fourier's Analysis and Periodogram Analysis for the Mathematical Laboratory.* By G. A. CARSE and G. SHEARER. (Edinburgh Mathematical Tracts, No. 4.) London, G. Bell and Sons, 1915. viii+66 pp.

THESE two little volumes of the series edited by Professor Whittaker treat some of the more essential parts of the subjects of interpolation and numerical approximation, the first being devoted chiefly to the non-periodic case of polynomial interpolation, the second mainly to trigonometric interpolation in the representation of periodic functions. In the first volume, after a very brief introductory chapter on finite differences, Chapter II is devoted to the various standard interpolatory formulas of Lagrange, Newton, Stirling, etc., and closes with a brief account of numerical differentiation. Chapter III, on the construction and use of mathematical tables, is in part devoted to explaining in detail the application of the foregoing principles to direct and inverse interpolation, and in part to special methods for computing tables of logarithms. Finally Chapter IV is concerned with numerical integration.

The second volume begins with a chapter which gives in barest outline and quite without proofs the main facts which the practical man must know about Fourier's series. This chapter closes with Bessel's elegant deduction of a finite trigonometric sum which gives the best approximate representation of a function in the sense of the method of least squares when the values of the function at equally spaced points are known. It is the evaluation of the coefficients of these finite sums (not of Fourier's series) which is considered in Chapter II by various methods, chief among which are the systematized methods of computation devised by Runge in the cases of 12 and 24 ordinates. Certain graphical methods are also explained, but the instruments which effect this interpolation mechanically are explicitly excluded. Chapter III, which is entitled Periodogram Analysis, is devoted to a discussion of

the following problem: Some natural phenomenon is represented graphically by an undulating (but not periodic) curve. It is required to represent this curve, if possible, either completely or with a small irregular residuum, by the superposition of a number of simple harmonic curves, whose periods, phases, and amplitudes are all to be determined. Two different methods of treating this problem are here given, one of which goes back, in part, to Lagrange. Chapter IV opens with a very brief description of spherical harmonics and the development of arbitrary functions on a spherical surface which proceed according to them. The rest of it is devoted to a discussion of F. Neumann's method for the practical calculation of the coefficients in such developments.

The exposition of the purely formal sides of the subjects treated is clearly and attractively done. The task of compressing the preliminary theoretical matter into very brief space, without making it wholly unintelligible, is such a difficult one that one is inclined not to criticize, especially as these are not the essential parts of the books. One wishes, however, that a little more stress might have been laid on some aspects of interpolation which the writers would perhaps class as theoretical. The degree of accuracy attained by the various approximative formulas is hardly touched upon, the very serviceable remainder in Simpson's rule, for instance, being not even mentioned. It should surely have been pointed out that Bessel's formulas for trigonometric interpolation given at the close of Chapter II of the second volume under review contain undetermined parameters when the number of ordinates is less than the number,  $2r + 1$ , of coefficients. This is the case which is used in Chapter III, where the number of ordinates is even. Moreover, the reader is left in doubt whether the 12 and 24 ordinate formulas of Runge give *exact* coincidence with the desired values at the points in question; or, perhaps, if the reader has not passed beyond the stage of counting constants, he will not even be in doubt. That we do have exact coincidence might easily have been demonstrated, and should at least have been explicitly stated.

Throughout both volumes great stress is laid on actual numerical computation, substantial numerical examples being almost everywhere worked out and others left for the reader to carry through. This is the strong side of the books and

one of their characteristic features. In other ways also much valuable material has here been brought together for which persons wishing to learn or teach the subject will feel grateful to the authors.

MAXIME BÔCHER.

*Ueber die analytische Fortsetzung des Potentials ins Innere der anziehenden Massen.* Preisschriften gekrönt und herausgegeben von der Fürstlich Jablonowskischen Gesellschaft zu Leipzig. XLIV. By GUSTAV HERGLOTZ. Leipzig, Teubner, 1914. 52 pp.

THE problem for which this memoir is a solution was stated as follows: A given ellipse is transformed by the method of reciprocal radii into a certain oval. Consider a plane surface of homogeneous material (flat plate) bounded by such an oval. In the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften*, 1909, C. Neumann proved on the basis of the theory of the logarithmic potential that, so far as its action on exterior points is concerned, the surface in question can be replaced by a material line bounded by two mass points. Since it might be of interest to extend this result to space of three dimensions, the Society proposed the following question: In the theory of the newtonian potential what is the analogue of Neumann's theorem? That is, what can be said about the newtonian potential of a homogeneous solid bounded by the ovaloid which is obtained from an ellipsoid by the method of reciprocal radii?

The point of view adopted by the author is exhibited first in a generalization of the result obtained by Neumann for the case of the logarithmic potential. If the potential of a body for outside points can be represented as the potential of a suitable mass distribution within the body, then certainly it can be continued analytically inside the body up to the mass distribution. Conversely, if the potential can be continued analytically to points inside the attracting body and if the singular points of the potential which are encountered be enclosed by any surface (curve in the plane problem)  $F$ , then the potential, because it is regular outside  $F$ , can be represented as the potential of a mass distribution on  $F$ . In general the mass consists of a single and a double distribution, the density of which depends on the surface chosen.

The exact meaning of the analytic continuation of the