

THE TWENTY-SEVENTH REGULAR MEETING OF
THE SAN FRANCISCO SECTION.

THE twenty-seventh regular meeting of the San Francisco Section of the Society was held at Stanford University on November 20, 1915. Sixteen persons were present, including the following members of the Society:

Professor R. E. Allardice, Dr. B. A. Bernstein, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor G. C. Edwards, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor E. W. Ponzer, Professor T. M. Putnam.

The chairman of the Section, Professor Haskell, presided at the opening of the meeting; the chairman-elect, Professor Allardice, then took the chair. The following officers were elected for the ensuing year: chairman, Professor Allardice; secretary, Dr. Buck; programme committee, Dr. Buck and Professors Blichfeldt and Manning.

It was voted that the time and place of the spring meeting be determined by the executive committee. The usual luncheon was served.

The following papers were presented at this meeting:

(1) Dr. B. A. BERNSTEIN: "A simplification of the Whitehead-Huntington set of postulates for Boolean algebras."

(2) Professor H. F. BLICHFELDT: "Some diophantine approximations."

(3) Dr. FRANK IRWIN: "A convergent series derived from the harmonic series."

(4) Dr. H. N. WRIGHT and Dr. FRANK IRWIN: "Some properties of the curves, $y = a$ polynomial in x ."

(5) Mr. T. A. PIERCE: "The numerical factors of the arithmetic forms $\prod_{i=1}^n (1 \pm \alpha_i^m)$."

(6) Professor D. N. LEHMER: "Concerning certain binary forms of higher degrees than the second whose prime divisors are of the form $nx \pm 1$."

(7) Professor M. W. HASKELL: "A note on integration."

Dr. Wright was introduced by Dr. Irwin and Mr. Pierce by Professor Lehmer.

Abstracts of the papers follow.

1. The best set of postulates for Boolean logic, considered from the point of view of elegance and pedagogy, is that given by Whitehead and made rigorous by Huntington. The number of propositions in this set is ten and the number of special elements postulated three. Dr. Bernstein, by replacing three postulates of this set by a new proposition, while retaining elegance and naturalness, reduces the number of postulates to eight and the number of postulated special elements to one.

2. Let X, Y, Z, \dots be n linear homogeneous functions of n variables x, y, z, \dots , having a determinant $\Delta \neq 0$. Minkowski has proved that such integers (not all zero) can be substituted for x, y, z, \dots that the sum $|X| + |Y| + |Z| + \dots \leq k\Delta^{1/n}$, where k is a function of n , independent of the coefficients involved in X, Y, Z, \dots (*Geometrie der Zahlen*, Leipzig, 1910). By applying the processes explained in "A new principle in the geometry of numbers," *Transactions*, 1914, Professor Blichfeldt obtains a much lower value for k , except for small values of n , than those given by Minkowski and by the author in the above article in the *Transactions*.

3. In extension of a result of Dr. Kempner in the *American Mathematical Monthly* for February, 1914, Dr. Irwin shows that we shall obtain a convergent series if we omit from the harmonic series all terms whose denominators contain the digits 9, 8, 7, \dots each more than a given number a, b, c, \dots of times respectively.

4. It is shown by Dr. Wright and Dr. Irwin that if to the curve $y = f(x)$, where $f(x)$ is a polynomial, all the tangents be drawn from a given point, then (1) the sum of their slopes and (2) the sum of the abscissas of their points of contact are independent of the ordinate of the given point. A remarkable line associated with the curve is noticed. Finally a graphical construction is given for the imaginary roots of a cubic equation and of certain biquadratics.

5. In volume 1 of the *American Journal of Mathematics* Lucas published a paper dealing with the properties of numbers given by $\frac{\alpha^n - \beta^n}{\alpha - \beta}$ and $\alpha^n + \beta^n$, where α and β are the roots of

a quadratic equation having integral coefficients. Carmichael has given a different and more exhaustive treatment of these numbers in the *Annals of Mathematics*, 1913. In the present paper Mr. Pierce obtains somewhat similar results for numbers given by the forms $\prod_{i=1}^n (1 \pm \alpha_i^m)$, where the α_i denote algebraic integers defined as the roots of an n th degree equation. The forms of the factors of $\prod_{i=1}^n (1 - \alpha_i^m)$ are determined by use of algebraic number theory, and this perhaps constitutes the most novel result of the work.

6. Lucas has developed the theory of the prime divisors of the functions $U_n = (a^n - b^n)/(a - b)$ and $V_n = a^n + b^n$, where a and b are the roots of a quadratic equation (*American Journal of Mathematics*, volume 1, page 184). Connected with these functions are certain binary forms of degree equal to one half the totient of n , the divisors of which Professor Lehmer has shown to be of the form $2nx \pm 1$. Combining this result with certain results of Mr. Pierce, Professor Lehmer has also obtained a series of numbers the prime factors of which must belong to two such forms, thus restricting notably the character of their divisors.

7. Professor Haskell shows that the condition that a rational fraction whose denominator is the n th power of a quadratic should be rationally integrable, is that the numerator shall be of degree $2(n - 1)$ and that it shall be apolar to the $(n - 1)$ st power of the quadratic factor of the denominator.

THOMAS BUCK,
Secretary of the Section.

TRANSFORMATION THEOREMS IN THE THEORY OF THE LINEAR VECTOR FUNCTION.

BY DR. VINCENT C. POOR.

(Read before the American Mathematical Society, December 31, 1915.)

SINCE the memorable work of Grassmann (1844), the study of the linear transformation has taken various forms, among which are the quaternions of Hamilton, the matrices of Cayley,