This edition of these very valuable lectures will be welcomed.

The whole field of functional calculus is a new territory but recently open for settlement, though an adventurous investigator occasionally explored small parts of it in the past century. The important extensions of mathematics have come from the problems of inversion, such as the Galois theory, theory of ideals, differential equations, integral equations, and now the calcul fonctionnel. These developments of Professor Volterra are of the highest importance mathematically aside from all of their physical interest, for the reason that they furnish a very practical path of entry into this new field and occupy a considerable part of the field itself. Fortunately we can follow them more in detail in the two recent courses of his lectures, Leçons sur les fonctions des lignes, and Lecons sur les équations intégrales.

JAMES BYRNIE SHAW.

SHORTER NOTICES.

Mysticism in Mathematics. By HASTINGS BERKELEY. Henry Frowde, Oxford University Press, 1910. vi + 264 pp.

IT may be well to begin by stating what Mr. Berkeley's thesis is not. The kind of mysticism which he thinks he detects in mathematics is not any of the kinds of mysticism that one encounters in the history and philosophy of religion. It is not contended that devotion to mathematics begets or tends to beget in the devotee a sense of an immediate and ineffable union or identification with deity. It is not argued that there is any essential likeness between Euclid and Timæus or between Gauss and Angelus Silesius. What Mr. Berkeley calls mysticism in mathematics is, he says, so called by him because no other name seems to him so appropriate. It appears to be impossible to state with mathematical precision what the thing is. Of course the author is not to be blamed for that. He succeeds in the difficult enterprise of making the matter as clear to the reader as it is to the writer, and that is all that can be reasonably expected.

As nearly as we have been able to make it out, the thesis may be broadly stated to be that, owing to a kind of reaction of language and especially of highly symbolic language upon

thought and belief, mathematicians tend to acquire a state of mind in which many purely fictitious things appear to be actual or real things. The thesis, which is very elaborately discussed, is approached from several points of view. The book falls into three parts.

In Part I the method is a priori. The Pythagoreans raised mathematics to the level of a science and the Pythagoreans were "enthusiasts a d mystics." The conjunction is suspicious. This and kindred considerations lead Mr. Berkeley to devote about 50 large pages to a very interesting psychological and linguistic dissertation designed to show that we are in constant danger of "the vicious reaction of words upon the process of thought," a reaction having a kind of hallucinatory effect, producing a "tendency to mysticism," a tendency to fancy that all symbols and modes of expression represent actualities instead of representing, as frequently they do, merely conventional or fictitious things.

The argument in Part I is general; it applies to all language. But the more symbolic the language, the more vicious is "the vicious reaction." And Part II (94 pages) is devoted to showing, on the one hand, that mathematicians by constantly talking about imaginaries and infinites in algebra and geometry lose sight of the fictitious and convential character of such things, and, on the other hand, to showing how algebra and geometry may be thoroughly purged of mystical elements.

Part III (113 pages), which deals with metageometry, arrives at the interesting conclusion that it "is an illusion" to suppose "that we can conceive different kinds of distance" or different kinds of space, as euclidean and non-euclidean. The illusion is "engendered by an unrealized reaction of symbolic forms, verbal or algebraic, on the thinker's judgment of his own process of thought."

Stylistically the book is well done. Its message to mathematicians—we mean its caution about treating such fictions as the circular points, for example, as real entities in space—might have been more effectively delivered in a score of pages. If it were shorter, the work would be more useful even for philosophers and other laymen.

CASSIUS J. KEYSER.