in observation), make the transformation $x = \pm \frac{s}{\sqrt{2}}(1 + i)$, where $i = \sqrt{-1}$, and it is readily found that $\frac{1}{\sqrt{2}}\int_0^\infty e^{-x^2}dx = \int_0^\infty \sin s^2\, ds = \int_0^\infty \cos s^2\, ds = \pm \frac{1}{2}\sqrt{\frac{\pi}{2}}$, integrals studied by Euler as early as 1781.* Now if we consider Jacob Bernoulli's problem,† to determine the curve whose curvature is proportional to its arc, we are led (on taking the constant of proportion as unity) to the equations

$$x = \int_0^s \sin s^2\, ds, \quad y = \int_0^s \cos s^2\, ds$$

which define a double spiral curve,‡ turning about the asymptotic points, determined by the Euler integrals above, and hence named by Cesàro the Clothoïde.§ It would also be interesting to remark that the curve is associated with the name of Fresnel, who was led to it in discussing the diffraction of light.||

R. C. ARCHIBALD.

BROWN UNIVERSITY,
December, 1913.

SHORTER NOTICES.

Archimedis Opera Omnia. Volume II. By J. L. HEIBERG. Leipzig, B. G. Teubner, 1912. xviii + 554 pp. 8 Marks.

IT may seem strange that a new Latin-Greek edition of the works of Archimedes should be deemed necessary, the first one under the editorship of Professor Heiberg having appeared as late as 1880–1881. We expect new translations into modern

* "De valoribus integralium variabilis $x = 0$ usque $x = \infty$ extensorum." "M. S. Academiae exhib. d. 30 Aprilis, 1781." Euler here evaluates the integrals by means of gamma functions. Published in Inst. Calculi Integr. IV (1794), pp. 339–345.

† "Invenire curvam cujus curvedo in singulis punctis est proportionalis longitudini arcus; id est, quae ab appenso pondere flectitur in rectam," Opera, Geneva, 1774, vol. 2, pp. 1084–1086.

‡ Cf. Picard, Traité d'Analyse, tome 1, 2e ed., 1901, p. 357.

§ *Nouv. Ann. Math.* (3), vol. 5 (1886), p. 512.

|| Œuvres complètes, tome I, p. 319, Paris, 1866; "Mémoire sur la diffraction de la lumière," presented to the Academy of Sciences in 1818, crowned in 1819 and first published in 1826. *Mém. de l'Acad. Fran.*, tome V, for 1821–22, Paris, 1826.

languages, and new interpretations of classical treatises from time to time, but new editions of the text alone, when the work has so recently been done by a scholar of such ability as Professor Heiberg, are cause for some surprise.

It is, however, a pleasant indication of the steady advance in the history of mathematics that such a state of affairs should exist. For the first Heiberg edition was a work of the highest order of critical scholarship, so that the present edition means that later discoveries have been drawn upon for the new material which is here given.

It was four years after the publication of Volume I of the first Heiberg edition that Rose discovered the Latin translation of Archimedes made by the "notorious William Flemming," as Roger Bacon designates William of Moerbecke, or Guilielmus Brabantinus, the chaplain to Clement IV. Cantor thought that Tartaglia (1543) took the translation of Archimedes, of which he so boastfully speaks, entirely from this work.* This manuscript dates from the thirteenth century, and was faithfully made from a Greek codex older than any now extant. Its importance, therefore, in restoring lost or imperfect passages is quite apparent.

The second noteworthy discovery which makes this edition necessary is the one made by Professor Heiberg in 1906, the Method of Archimedes† found in a manuscript of the tenth century in a monastery at Constantinople. Until the time of this discovery the work was known to the world only through brief extracts in the works of Heron and Suidas, and while it is now familiar through various publications,‡ its great importance added to the necessity for this present edition.§ The manuscript is also important because it contains parts of the work on Floating Bodies (Ὀχουμένων)|| of which no Greek copy had been supposed to be extant, and also certain passages from the Stomachion (Στομάχιον).¶ Still more material from this manuscript is promised for Volume III.

* Cantor, Geschichte, vol. II, p. 514. The question is, however, fully discussed in Heath's Works of Archimedes, p. xxvii, and in the Kliem edition, p. 14.

† De Mechanicis propositionibus ad Eratosthenem Methodus.

‡ *Hermes*, 1907, vol. XLII, p. 235; *Revue générale des Sciences*, 1907; *The Monist*, vol. XIX, p. 202, 1909; and in pamphlet form by Sir Thomas Heath (1912) to accompany his work on Archimedes.

§ Vol. II, pp. 426–507.

|| Vol. II, pp. 318–413.

¶ Vol. II, pp. 416–424.

It is unnecessary to say that Professor Heiberg was the first to give a critical and modern study to the works of Archimedes, and that, with the exception of Sir Thomas Heath, there is no one now living who combines such a perfect knowledge of Greek, Latin, and the mathematics of the classical period.

DAVID EUGENE SMITH.

Archimedes' Werke. Mit modernen Bezeichnungen herausgegeben und mit einer Einleitung versehen von Sir THOMAS L. HEATH. Deutsch von Dr. FRITZ KLIEM. Berlin, 1914. xii + 477 pp.

As stated in the preceding review, there was abundant reason to justify Professor Heiberg in preparing a new edition of the works of Archimedes, and the same may of course be said concerning a work like that of Sir Thomas Heath. In the case of the latter's well-known treatise the lack of reference to the Methodus in the first edition has been overcome in part by the publication* of a pamphlet giving an English translation from the original Greek text. There is, however, good reason for a second edition of the work by Sir Thomas Heath, and it is to be hoped that he will find time to supply this need. Meantime the German edition by Dr. Kliem is most welcome. While the translator, in preparing this edition, has, in the main, followed the English text with fidelity, he has not hesitated to amplify it, with the author's permission and assistance, so as to include all the recent discoveries, and to add a number of footnotes which are calculated to assist the student. Thus in Chapter I we have a reference to Förster's article on Pheidias the astronomer, and some mention of the Stomachion as in the Heath supplement of 1912; in Chapter II, a reference to the codices used by Heiberg in his second edition, with information concerning the finding of the Codex rescriptus Metochii Constantinopolitani and the nature of the text, and reference to the recent literature on el-Biruni's knowledge of the work of Archimedes on the circle; and in the following chapters the same policy has been pursued.

In Chapter VII, for example, Dr. Kliem has amplified the treatment somewhat, particularly with reference (page 149) to the new matter found in the Methodus. He has, however, omitted Chapter VIII, on the terminology of Archimedes. This is perhaps justifiable from one point of view,

* Cambridge, 1912.