

most modern notions concerning fundamental mathematical disciplines, a precursor of that day when the undergraduate curriculum will contain, in their more elementary aspects, many of those subjects and ideas which make mathematics a thing of esthetic delight to those who are now laboring in its development.

Another valuable contribution to the same end would be a treatise on elementary geometry written from the point of view of the first monograph of the present book. How this may well be done can be seen from the nature and arrangement of this monograph.

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*Higher Algebra.* By H. E. HAWKES. Boston, Ginn and Company, 1913. v + 222 pp.

THE subjects treated in a course in algebra designed for freshmen and advanced secondary students constitute almost a fixed unit; as to the manner of presenting these subjects there is some difference of opinion. Some teachers believe in carefully formulating a few assumptions and building upon these with absolute rigor. From the standpoint of the scientist this is possibly the only correct view. Some have asserted that this thoroughly rigorous method of proving every step is practical as well as theoretically elegant; but by far the greater number of teachers have found by experience that an entirely different method of procedure is preferable. The average freshman does not have the intensive interest of the scientist in the subject; he is looking for general rules rather than the exceptions with which the scientist is vitally concerned; the interest of the student should be awakened and stimulated by frequent appeals to his intuition and by giving the subject a real and tangible basis; any long series of purely logical steps should be avoided if possible; hence, it has been found desirable in presenting the subject to this type of student to make bold and explicit assumptions as they become necessary in the development, and to postpone proofs of a severely logical character to a later and more critical study.

Professor Hawkes has written his book consistently from the second of the viewpoints just described. The book has been prepared to meet the needs of the student who will continue his mathematics as far as the calculus. The author has adapted the book both to the engineer and to the student

of pure mathematics by using the topics which must be emphasized by the engineer, such as numerical computations, checks, graphical methods, the use of tables, and the solution of specific problems, as a basis on which the foundations of the more theoretical portions of algebra might be suitably and advantageously laid.

In the introductory review the usual topics are taken up, but in such a way that much may be brought out which has not permeated the mind of the student in passing over the work for the first time. Considerable attention is paid to the substitution of values of  $x$  in polynomials by a selection of a number of good exercises in this work. To factor an expression of the type  $ax^2 + bx + c$  the author separates  $b$  into two parts whose product is  $ac$ ; i. e.,  $6x^2 - 13x - 5 \equiv 6x^2 - 15x + 2x - 5 \equiv 3x(2x - 5) + (2x - 5) \equiv (2x - 5)(3x + 1)$ . This is an improvement upon the method of multiplying through by the coefficient of  $x^2$  and changing the variable, after which it must be changed back to  $x$ .

Probably the most salient feature of the book is the fact that the author insists upon the student's acquiring a knowledge of the fundamental theory of the equation; this aim appears not only in the chapter on the theory of equations, where it is woven into the very texture of the material, but also in the treatment of other topics. Some intimation of the theory of the quadratic is given on the first page of the book, where attention is called to the fact that the factors of  $x^2 + bx + c$  are  $(x + p)(x + q)$ , where  $p$  and  $q$  are two numbers whose sum is  $b$  and whose product is  $c$ . Later, to find the condition that one root of a quadratic shall be  $n$  times the other is given as an exercise; the condition that the two roots of the quadratic shall be equal follows as a special case, but the discriminant is later given the prominence that it deserves. Some emphasis is laid upon the parameters which occur in quadratics: application is made of this in finding algebraically the maximum and minimum of a quadratic function; i. e., in the expression  $y = ax^2 + bx + c$ , the  $y$  is transposed and considered a parameter, and that value of  $y$  is obtained which makes the discriminant of the resulting expression equal to zero.

The notion of function, the graph of a function and especially the slope of the graph are clearly explained and attractively illustrated. Much use is made of graphical

methods. An excellent example of the use of graphs is given on page 58 in discussing the behavior of a quadratic function when the coefficient of  $x^2$  approaches zero as a limit. It is easier to remember the method of expanding a three-rowed determinant by re-writing its first two columns, taking the right hand diagonals for positive terms and the left hand diagonals for negative terms, than the usual but more complicated method of forming these diagonals given by the author. The changing of  $x$  to  $y - 2$  in separating into partial fractions expressions of the type  $(3x^2 - 4x + 3)/(x + 2)^3$  is of so much advantage that one wonders why so many books fail to suggest it. That the fraction  $a/n$  approaches a limit when  $a$  is a constant and  $n$  a variable which becomes infinite is brought out in a dialogue between two speakers and serves as a relief from the hackneyed expressions which usually occur in that connection.

I believe that the author may be fairly criticized for not having given a more formal discussion of undetermined coefficients. If one is trying to find topics which must be used by the applied scientist and which may be used as a medium in which the foundations of theoretical algebra might be laid—the expressed intention of the author—I know of no subject which could be better used to advantage in this connection than undetermined coefficients.

The book is exceptionally free from typographical errors. I have noticed only one; on page 21, in the example at the bottom of the page, 3 75 should be 3.75.

All of the subjects taken up in this book except probability and infinite series have been treated in the author's former book on *Advanced Algebra*. Criticism of these topics over a period of about seven years has resulted in much improvement. It is my belief that the teacher will find the *Higher Algebra* a good text to follow closely in courses designed to give the student a thoroughly good workable knowledge of this portion of algebra.

JOSEPH EUGENE ROWE.

*The Teaching of High School Mathematics.* By GEORGE W. EVANS. Boston, Houghton Mifflin Company, 1911. x+94 pp. Price 35 cents.

THE little book under review is one of the *Riverside Educational Monographs*. When the reader meets in the pre-