

Monographs on Topics of Modern Mathematics Relevant to the Elementary Field. Edited by J. W. A. YOUNG. New York, Longmans, Green and Co., 1911. viii + 416 pp.

THIS book contains nine monographs by as many authors, as follows:

1. The Foundations of Geometry, by Oswald Veblen. Pages 1-51.
2. Modern Pure Geometry, by Thomas F. Holgate. Pages 53-89.
3. Non-Euclidean Geometry, by Frederick S. Woods. Pages 91-147.
4. The Fundamental Propositions of Algebra, by Edward V. Huntington. Pages 149-207.
5. The Algebraic Equation, by G. A. Miller. Pages 209-260.
6. The Function Concept and the Fundamental Notions of the Calculus, by Gilbert Ames Bliss. Pages 261-304.
7. The Theory of Numbers, by J. W. A. Young. Pages 305-349.
8. Constructions with Ruler and Compasses; Regular Polygons, by L. E. Dickson. Pages 351-386.
9. The History and Transcendence of π , by David Eugene Smith. Pages 387-416.

The authors of these monographs were of the opinion that there is room for a serious effort to bring within the reach of secondary teachers, college students, and others of a like stage of mathematical advancement, a scientific treatment of some of the regions of advanced mathematics that have points of contact with the elementary field. They felt that a great need of our secondary instruction in mathematics is the enlargement of the mathematical horizon of the teachers themselves; and they believed that there is a large body of earnest teachers and students that are eager to extend their mathematical knowledge if the path can be made plain and feasible for them.

The object of these monographs is to make a contribution toward meeting this need. That the topics are well selected for this purpose may be seen from the foregoing list. The aim of each monograph is to bring the reader into touch with some characteristic results and viewpoints of the topic considered, and several of them point out the bearing of these matters on elementary mathematics.

The material in each of the monographs falls more or less definitely into three parts. First, there is a considerable body of results proved in full. Next, there is a statement without proof of some of the further leading methods and results, so as to give in minimum compass a bird's-eye view of the whole subject. Finally, in connection with most of the monographs there is a small number of references indicating what the reader may profitably take up after he has mastered the contents of the monograph itself.

Naturally, the amount of technical mathematical knowledge presupposed on the part of the reader varies with different subjects. For the reading of a large part of the book, a knowledge of elementary algebra and geometry, together with a certain measure of mathematical maturity, is sufficient. Since the various papers are written by men well qualified to speak on the several subjects, there is much in them that will repay careful and detailed study by advanced students.

There is not space here to go into an analysis of the separate monographs of the book. It seems desirable, however, to indicate a few specific criticisms.

The symbol $\{ABC\}$ is used in two senses in the first monograph (cf. assumption II and theorem 7); it would seem better to avoid this. The proof of theorem 19 in the same paper is incomplete.

The statement on page 265 that the word function was originally used to denote any power of a number seems to be inaccurate. Compare *Encyclopédie*, tome II, volume 1, page 3.

In the seventh monograph we have the definition, "A prime number (or briefly, a prime) is a number having no other factors than itself and unity." According to this definition, unity is a prime number. But on page 311 we find $\varphi(1) = 1$, $\varphi(p) = p - 1$, where p is a prime—an obvious inconsistency if 1 is a prime. To the reviewer it seems better to exclude unity from the list of primes.

The proof reading on some of the monographs was not carefully done; see, for example, the inconsistent use of italics on page 11. But the errors of this sort will usually cause the reader no serious inconvenience, and consequently no list of them is supplied here.

In conclusion I should like to say with emphasis that this book makes a step in the right direction. It looks forward to the time when the secondary teacher will know some of the

most modern notions concerning fundamental mathematical disciplines, a precursor of that day when the undergraduate curriculum will contain, in their more elementary aspects, many of those subjects and ideas which make mathematics a thing of esthetic delight to those who are now laboring in its development.

Another valuable contribution to the same end would be a treatise on elementary geometry written from the point of view of the first monograph of the present book. How this may well be done can be seen from the nature and arrangement of this monograph.

R. D. CARMICHAEL.

Higher Algebra. By H. E. HAWKES. Boston, Ginn and Company, 1913. v + 222 pp.

THE subjects treated in a course in algebra designed for freshmen and advanced secondary students constitute almost a fixed unit; as to the manner of presenting these subjects there is some difference of opinion. Some teachers believe in carefully formulating a few assumptions and building upon these with absolute rigor. From the standpoint of the scientist this is possibly the only correct view. Some have asserted that this thoroughly rigorous method of proving every step is practical as well as theoretically elegant; but by far the greater number of teachers have found by experience that an entirely different method of procedure is preferable. The average freshman does not have the intensive interest of the scientist in the subject; he is looking for general rules rather than the exceptions with which the scientist is vitally concerned; the interest of the student should be awakened and stimulated by frequent appeals to his intuition and by giving the subject a real and tangible basis; any long series of purely logical steps should be avoided if possible; hence, it has been found desirable in presenting the subject to this type of student to make bold and explicit assumptions as they become necessary in the development, and to postpone proofs of a severely logical character to a later and more critical study.

Professor Hawkes has written his book consistently from the second of the viewpoints just described. The book has been prepared to meet the needs of the student who will continue his mathematics as far as the calculus. The author has adapted the book both to the engineer and to the student