

Die logischen Grundlagen der exakten Wissenschaften. By PAUL NATORP. (Wissenschaft und Hypothese, XII.) Leipzig and Berlin, B. G. Teubner, 1910. xx+416 pp.

Probleme der Wissenschaft. Part I: *Wirklichkeit und Logik.* Part II: *Die Grundbegriffe der Wissenschaft.* By FEDERIGO ENRIQUES. Translation by KURT GRELLING. (Wissenschaft und Hypothese, XI.) Leipzig and Berlin, B. G. Teubner, 1910. x+vi+599 pp.

Erkenntnistheoretische Grundzüge der Naturwissenschaften. By PAUL VOLKMANN. (Wissenschaft und Hypothese, IX.) Second Edition. Leipzig and Berlin, B. G. Teubner, 1910. xxiii+454 pp.

A PHILOSOPHER devotes the greater part of his book to the foundations of mathematics; a mathematician boldly attacks the problems of philosophy; a physicist lectures on the theory of knowledge. These phenomena, as well as the fact that the books in question are sent to a mathematical journal for review, may well be regarded as significant of our times. Recent progress in the foundations of mathematics and the revolutionary conceptions and theories of present-day physics have necessarily struck deep into the current of philosophical thought. The authorship of these books need then occasion no surprise.

An extended notice of books of this character in a mathematical journal seems hardly called for as yet, however, as they will probably be of interest only to a limited number of mathematicians. A brief indication of the nature of their contents, however, may be given.

Professor Natorp devotes the first 97 pages of his book to an exposition of what he regards as the fundamental problems of logic. He follows Kant in insisting that the primitive act of thought is synthetic and repudiates vigorously the attempt of some logicians to base logic on a meaningless symbolism. In fact, logic as such is not and can not be a deductive science at all. This point of view leads necessarily to a genetic theory of knowledge in which the process or method of thought is the determining factor of knowledge.

The author regards as a primitive faculty of the mind the power of conceiving any mental act to be repeated indefinitely. He thus obtains essentially what mathematicians would call the abstract form of an unlimited sequence. On this he

bases his development of the notion of integral numbers, and thence by changing his "unit" the notion of rational numbers, together with that of the fundamental operations on numbers (pages 98-159). Then follows (pages 160-224) a very careful and illuminating discussion of the problems involved in the notions of irrational numbers and continuity. The ideas of Dedekind, Weierstrass, Cantor, Pasch, and Veronese are considered in much detail. In the next chapter (pages 225-265) the concepts of direction and dimensionality are developed as attributes of the notion of number, which is followed (pages 266-325) by a discussion of time and space as mathematical concepts. The foundations of geometry receive attention here. In the last chapter (pages 326-404) the author deals with the time-space order of natural phenomena and the mathematical foundations of science. Here we find a discussion of the fundamental concepts and laws of mechanics, the principle of the conservation of energy, etc. The book closes with a discussion of the principle of relativity, in which the author finds a final justification of an idealistic philosophy.

Professor Enriques' two volumes will be examined with much pleasurable anticipation by any mathematician interested in philosophical questions. One of the most interesting features of the books is the fact that the author applies mathematical terminology to the formulation and discussion of his problems—he speaks a language, therefore, which is intelligible to a mathematician. His program is an ambitious one, nothing less in fact than to seek the common elements in the various branches of scientific activity, to seek a general point of view from which all science, in its broadest sense, may be unified with reference both to the formulation of its problems and the development of its methods. The reader cannot fail to be struck with the wide range from which the author has drawn his illustrations. The chapter headings will perhaps serve to give an idea of the general plan of the work: I. Introduction; II. Facts and theories; III. The problems of logic (pure logic, the applications of logic, the physiological aspects of logic); IV. Geometry (significance of geometry, psychological genesis of geometrical concepts); V. Mechanics; VI. Extension of mechanics (physics as an extension of mechanics, the mechanistic hypothesis and the phenomena of life). The first volume contains the first three chapters; the second, the last three. We may add that the translation from the Italian original seems exceptionally well done.

The first edition of Professor Volkmann's book was reviewed in this BULLETIN, volume 4 (1898), page 355. What was said there will apply very well to the new edition. It seems superfluous, then, to do more than indicate very briefly the changes that have been made in the new edition. What in the first edition was the first lecture has in the second been incorporated in the closing lectures. The new edition begins with a historical retrospect on the development of science and its conceptions. A section on the subjective and objective elements in knowledge has been added, as well as an appendix containing two earlier papers of the author giving an appreciation, from the philosophical point of view, of the Newtonian system of mechanics. The book closes with a complete bibliography of the author's philosophical writings and very complete author and subject indexes.

J. W. YOUNG.

Repertorium der höheren Mathematik. Zweite Auflage, erster Band, erste Hälfte, herausgegeben von P. EPSTEIN. xv+527 pp. M. 10. Zweiter Band, erste Hälfte, herausgegeben von H. E. TIMERDING. xvi+534 pp. M. 10. Leipzig, B. G. Teubner, 1910.

THIS work is called a second edition of Pascal's *Repertorium of Higher Mathematics*.* It is, however, in many respects, a new work. The text has been thoroughly revised and greatly augmented. The half which has already appeared is nearly equal in size to the whole of the first edition. We are also informed (volume I, page viii) of a change in the purpose of the work. The aim of this edition is to give the reader, "a systematic survey of the total field of mathematics, based on genuine understanding."

As a further conspicuous departure from the original edition, which was the work of a single author, the editors have secured, as authors of the individual chapters, men particularly interested in the subjects covered by them. In the part of the work that has appeared, the authors are: in analysis, Hans Hahn, Alfred Loewy, H. E. Timerding, Paul Epstein; in geometry, J. Mollerup, H. Liebmann, H. Timerding, L. Heffter, G. Guareschi, M. Dehn, F. Dingeldey, L. Berzolari, G. Giraud, E. Ciani, H. Wieleitner. It is interesting

* A review of the first volume of the original Italian edition will be found in this BULLETIN, vol. 5 (1899), pp. 357-362.