

integrals, their definition as the limit of a sum, their general properties, and their evaluation in a few simple cases. The work thus far covers a little more than half the volume; the remaining portion is given to various applications.

The remarkable thing about all these applications is the complete omission of any ideas concerning limits of sums. The method is always to find a derivative, and then integrate. For example, the result for ds/dx has been found; hence s may be obtained. In like manner $dA/dx = y$ may be established, and hence A is the integral of y . And so on, to arcs and areas in polar coördinates, to surfaces and volumes of revolution, to centers of gravity, centers of pressure, moments of inertia, and attractions. All are treated by differentiation. Why not? Why not eliminate the troubles connected with limits of sums? The author has made the presentation clear and rigorous, and has shown conclusively that we do not need to bother with the integral as a limit of a sum in elementary calculus. His method is worthy of our most serious consideration—if we desire to be rigorous instead of suggestive, and we can hardly be both in a first course on calculus.

The remaining applications are to the dynamics of a particle, prefaced by a few sections on the integration of the simpler differential equations. There are notes on the integration of infinite series, on Riemann's discontinuous integrable function, and on Fourier's series.

These texts merit our special consideration because they are different from those we are used to. It would be interesting to see them tried on American classes both for the effect on the students and for the effect on the teachers.

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SHORTER NOTICES.

Mémoires Scientifiques. By PAUL TANNERY. Publiés par J. L. HEIBERG and H. G. ZEUTHEN. I. *Sciences Exactes dans l'Antiquité*, 1876-1884. Toulouse, Edouard Privat; Paris, Gauthier-Villars, 1912. xix+465 pp. Price 15 francs.

IN our time there have been three men whose love for ancient science and whose perfect command of the Greek language

fitted them above all their contemporaries to write upon the subject of the mathematics of Greece. These men are Paul Tannery, Sir Thomas Heath, and Professor Heiberg. If to this trio another should be added, it might well be Professor Zeuthen. It is, therefore, very proper that the scientific memoirs of Paul Tannery should be edited by Professors Heiberg and Zeuthen, for no others are better fitted to undertake the task *con amore*, nor could any scholars be found more adequately prepared for the labor.

Paul Tannery was a rare genius, one to whom Greek was a second language, and one who lived his intellectual life in the contemplation of the science of antiquity. Never properly recognized by his country, employed in the government manufacture of tobacco instead of being offered a chair in the university, he nevertheless produced a series of memoirs that would have brought honor to any higher institution of learning and that will give him an enduring place among the scholars of France. He was a prominent member of the third International Congress of Mathematicians at Heidelberg, in 1904, and seemed at that time to be in his prime; but it was only a few weeks later that the news of his sudden death appeared. He passed away too soon for his greatest scientific success, and his loss was a sad one for the scientific study of the history of mathematics in France.

When it was planned to bring together his memoirs, the talented Madame Tannery secured the coöperation of Professors Heiberg and Zeuthen in editing the work, and the promise of his brother Jules Tannery to write an introduction to the first volume. The premature death of the latter, in November, 1910, deprived the *Ecole normale supérieure* of its leader, and at the same time prevented the carrying out of this plan. Thus within the short space of six years these two brothers passed away, each in the prime of life, and each seeming to leave a great work unfinished.

Paul Tannery began his contributions in 1876, and the present volume contains such memoirs as appeared on the exact sciences of antiquity from that date until 1884. The next two volumes will continue this topic, bringing the memoirs down to the time of his death. The subsequent volumes will relate to the exact sciences among the Byzantines (volume IV), the exact sciences in the middle ages (volume V), pure mathematics (volume VI), philosophy (volume VII), classical philology

(volume VIII), and reviews (volume IX). There will also be a volume (volume X of the series) containing his biography, a bibliography, and selections from his correspondence.

In editing the memoirs advantage has been taken of manuscript notes on the margins of the author's copies, and these have been marked in such a way as to show that they did not form part of the original publications. The editors' notes have also been marked in a special manner so that the responsibility for them is easily placed. In general, however, the articles and footnotes stand as they were written, a plan of editing which every student of the history of mathematics will commend.

Tannery's memoirs form a basis for a history of Greek mathematics, although in themselves they do not form a connected narrative. The mind of the author did not work along the lines of continued narration; he preferred to attack isolated problems and solve them. His is the material out of which lesser writers make their histories, and no one has done more than he to furnish material on Greek mathematics for the use of some future Montucla.

The memoirs in the first volume are twenty-nine in number. They include a wide range of subjects, beginning with the astronomical system of Eudoxus and ending with the "modius castrensis,"—the *Italicus modius* or *Italicus sextarius* mentioned in Diocletian's edict *De pretiis rerum venalium*. Some of the more interesting and important topics considered are the nuptial number of Plato, which Tannery takes to be 2,700; the lunes of Hippocrates; the solution of the Delian problem by Archytas and Eudoxus; the question of the date of Diophantus, based in part upon a study of the varying cost of the wine mentioned in the problems of the *Arithmetica*; the Greek arithmetic as set forth by Pappus and Heron; the Eudemian fragments; the origin of the proof of nines, which he assigns to the Greeks instead of the Orientals; the method of Archimedes in measuring the circle; the solution of the quadratic before Euclid; and the stereometry of Heron.

It is unnecessary to speak further of the contents of these memoirs, since the latest of them was published thirty years ago and all are therefore known to historians of mathematics. It may suffice to say that the volume is a credit to the editors and publishers as well as to the distinguished scholar whose articles it contains.

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