

found that, in the process of revising and adding desirable new material, the second volume was becoming too large. At the suggestion of the publishers this volume was divided and the second part appears as a separate book. In comparing this new edition with the old, it is to be noted that the general character of the text has not been changed. It is still a first introduction to differential geometry, aiming to present the fundamental facts and principles in a simple and concise manner. Brevity, however, is no longer a striking feature, for there are 531 pages in the three volumes of the new edition.

The American student beginning the study of differential geometry now will probably not use this second edition so frequently as his predecessor employed the first one. When the latter appeared, four years after the German edition of Bianchi's book, there was no text in the English language. But now we have Professor Eisenhart's excellent work.

In the present volume the first 90 pages are devoted to the "special surfaces," including  $W$ -surfaces, minimal surfaces, surfaces of constant curvature, ruled surfaces, and triply orthogonal systems of surfaces. Here there are various minor alterations and additions, especially under the first three headings. But the greatest changes are to be found in the next 60 pages, which deal with rectilinear congruences. The sections on isotropic congruences are perhaps the most noteworthy, not only because of the fact that the treatment here is fuller than in most of the texts; but, also, because these ten pages contain material from a recent article\* by one of the authors. Of especial interest are the remarkably simple formulæ for the middle surface of the most general isotropic congruence.

The book is concluded by a collection of thirty-one problems.

E. B. COWLEY.

*Einführung in die Theorie der partiellen Differentialgleichungen.*

Von Dr. J. HORN, Professor an der Technischen Hochschule zu Darmstadt. Leipzig, Göschen, 1910. vii+360 pp.

THIS work may be considered as the third volume of the course in differential equations published in the "Sammlung Schubert." The first volume is the well known work by Schlesinger, *Einführung in die Theorie der Differentialgleich-*

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\* *Math. Annalen*, vol. 70, pp. 143-160.

ungen mit einer unabhängigen Variablen.\* The second volume is *Gewöhnliche Differentialgleichungen beliebiger Ordnung*, by the author of the book under review. The task which Dr. Horn seems to have set himself was to produce in one small volume a course in partial differential equations which was to be readable for the mid-course student, rigorous in treatment and such as to bring out a number of points of view of both the older and the more modern theory. In other words the book was to fill up the gap between the rather meager discussion of partial differential equations in the last chapters of treatises on analysis and the works on special topics or special methods of treatment.

After the first chapter, which takes up existence proofs for linear partial differential equations of the first order and with  $n$  independent variables, the author restricts himself to equations of the first and second orders with two independent variables. In the chapter on equations of the first order he gives a special existence proof and then takes up Cauchy's method of integration by means of the characteristics, using Darboux's geometrical language. The methods of Lagrange and Monge are used in the discussion of the complete integral. In the third chapter, after giving Goursat's existence proof for partial differential equations of the second order and discussing in general the characteristics and their relation to the integral surfaces, the particular equation of Monge and Ampère

$$Hr + 2Ks + Lt + M + N(rt - s^2) = 0$$

( $H, K, L, M, N$  functions of  $x, y, z, p, q$ )

is taken up with the linear equation of the second order as a special case. Laplace's method of solution is also used to introduce the subject of invariants. The fourth chapter is devoted to hyperbolic partial differential equations of the second order, in particular to the discussion of the integral by means of Green's theorem. The methods of Riemann are also briefly studied.

In the fifth chapter begins the characteristic half of the book, which up to this point differs only in conciseness and arrangement from other texts, say parts of the fifth and sixth volumes of Forsyth. Before introducing the reader to the

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\* See review in the BULLETIN, vol. 8, p. 168.

elliptic partial differential equation, the author includes a good introduction to the integral equation according to Fredholm. The chapter is clear and not too brief for the reader for whom the book is intended, though the following chapter on boundary problems in ordinary linear differential equations of the second order might have been shortened with profit in a work of this type and title. One quarter of the book is devoted to the introduction to integral equations. In chapter seven the results of the two previous chapters are applied to particular equations of the elliptic type. Properties of the solutions of

$$\Delta u = 0,$$

in particular two boundary value problems, are treated at some length, following Fredholm and Hilbert in treatment and notation. A few pages are devoted to special points connected with the solutions of

$$\begin{aligned}\Delta u + 2\pi\varphi(x, y) &= 0, & \Delta u + \lambda u &= 0, \\ \Delta u + \lambda k(x, y)u &= 0 \quad (k > 0).\end{aligned}$$

The volume closes with a short chapter on some partial differential equations of physics.

The book is clear and logical. Generalities are illustrated by well-chosen special examples. After deciding upon the content the author keeps to the point and does not forget the student for whom he is writing. As to the content, of course there will be differences of opinion as to the choice of topics from such a wide field. For example it would not have been a difficult task to give some notion of Lie's methods without an increase in size. This volume is well worthy of a place in a series which includes Schlesinger's little work.

A. R. CRATHORNE.

*Lehrbuch der Differentialgleichungen.* Von A. R. FORSYTH. (Mit den Auflösungen der Aufgaben von HERMANN MASER.) Zweite autorisierte Auflage, nach der dritten des englischen Originals besorgt und mit einem Anhang von Zusätzen versehen von WALTHER JACOBSTHAL. Braunschweig, Vieweg und Sohn, 1912. xxii + 921 pp.

FOR many years Forsyth's Treatise on Differential Equations has held a place of importance among physicists and