by him in the choice of his university, or he must have good fortune in writing a thesis whose weak points are not evident on a superficial examination, but his task is, on the whole, not a difficult one, and he gets at least the advantage of a period of foreign residence.

For another class of men foreign study may be recommended without qualification, namely, for able students who have already had a substantial training in one of the better American graduate schools, or who have even taken the doctor's degree at such a school. Such men will naturally go either to one of the great mathematical centers like Paris or Göttingen, where they will have the opportunity to hear lectures by several of the leading mathematicians of the day, and, perhaps, to see some of them occasionally outside of the lecture room; or they will select some mathematician of eminence in a particular field with whom they may hope to gain direct personal contact, and go to the university where he happens to be. Thus of late years a small but steady stream of American students has gone to Italy.

To the students just considered, and to some extent to their weaker comrades mentioned above, the period of residence at a great European mathematical center or of contact with an eminent mathematician at a less important European institution brings with it a realization of what high scientific ideals in mathematics are, and to what an extent they prevail abroad. Such ideals prevail also, it is true, at the strongest American institutions; but it is hard for the young American to appreciate their great diffusion in a ripened civilization until he has experienced it by personal contact.

SHORTER NOTICES.

Shop Mathematics. By EDWARD E. HOLTON, Head of the Department of Machine-Shop Practice, the Technical High School, Springfield, Mass. Springfield, The Taylor-Holden Company, 1910. xi+211 pp.

THE present rapid development of secondary vocational schools and their competition with general secondary schools have set in operation forces which tend to modify materially the character of secondary mathematical teaching in this country.

Since the mathematical work of the higher institutions rests upon the foundation laid by the secondary schools, it is important that any tendency to alter that foundation should be fully appreciated.

The book under review is one of the first published results of the attempt to develop a course in mathematics adapted to the needs of the secondary technical school, and as an indication of those needs, and possibly of the character of the mathematical instruction in such schools, it deserves serious attention.

The pioneers of the movement of protest against the traditional curriculum and methods of the secondary school have set high standards for the work of their own institutions. Witness the statement of Dr. C. M. Woodward:* "No blow is struck by him" (the pupil), "no line drawn, no motion regulated by mechanical habit. The only habit acquired is that of thinking. The quality of his every act springs from the conscious will, accompanied by a definite act of judgment."

Let us test the book before us by this standard, remembering that its object is to teach the pupil "what the shop problems are"† and "how to apply mathematical principles, rules, and formulas to the solution of such problems" and that "no attempt is made to teach mathematical theory or principles."

Consider the following problem (Problem 17, page 28). "What length of bar will be required to raise a building of 100 tons weight with 10 lbs. pull each on 100, 1/4 in. lead jackscrews?" Result (page 191), 7.957 inches. If the reader, or better, the author, will try a simple experiment with a spring balance he will find that a force of ten pounds can easily be exerted by merely bending the little finger. The length found for the bar is about an inch more than that of an ordinary penholder. Now just picture a hundred laborers at work raising a building, each pulling with the little finger of one hand on a bar an inch longer than a penholder. Then picture what happens to the "shop foreman of twelve years' experience" when the contractor who is paying the gang of one hundred laborers, comes around and sees what they are doing! Again, consider the computed result: seven and nine hundred fifty-seven thousandths inches. Remember this is a problem on the

* Report of the U. S. Commissioner of Education, 1893–4, part I, p. 896.

† Introduction, p. viii. This introduction is by Dr. C. F. Warnier, principal of the Springfield, Mass., Technical High School and of the Evening School of Trades.

jack-screw, and the power is applied, not by means of a knife edge, but by the hand. The author gives the length of the bar to the thousandth of an inch!

This is not an isolated but a typical example of the "definite act of judgment" which accompanies the author's consideration of the data and results of his problems. For example, in problem 12, page 51, the pupil is asked to find the weight which can be raised by a force of 60 lbs. applied to a somewhat complex combination of screw and gears. The answer is given (page 195) as 203,575.68 lbs., i. e., to one part in twenty million. If the given force of 60 lbs. be altered by an amount equal to the weight of a single drop of water the computed result will be affected by more than the 0.68 lbs. appearing in the answer. This sort of thing is not a result of mere inadvertence in writing down the answers; it appears in the illustrative examples which are intended to serve the pupil as models. For example, in a problem concerning the friction of a lathe (page 54), where the coefficient of friction is given as .08, we have the following work:

$$400 \times .08 = 32, \quad \frac{32 \times 1\frac{1}{2}}{12} = 4,$$

$$\frac{50 \times 2\pi r}{12} = 50 \times 6.2832 = 314.16,$$

$$\frac{314.16 \times 4}{33000} = \frac{1256.64}{33000} = .038.$$

The coefficient of friction is known here only to one part in 8 but π is used to five significant figures, i. e., to one part in thirty thousand.

The pupil accustomed to books "written almost entirely from the point of view of the teacher of pure mathematics with little reference to concrete problems of life and having no reference whatever to the actual problems of the drafting room and the shop"* may be pardoned if he asks for information concerning the concreteness and actuality of problems, results, and methods of which the above are samples. With such models before him is the pupil likely to acquire the habit of thinking?

The author seems not to have realized that the standards of efficiency governing the mathematical work of the shop are

* Cf. Introduction, p. vi.

entirely analogous to those by which the mechanical work is judged.

In a little Handbook for Apprenticed Machinists issued by a prominent firm of manufacturers primarily for the use of their own apprentices the following statement appears: "He" (the apprentice) "should know the parts that are to be accurate; if a part is machined only to make it smooth, there is no need to take time to size it accurately." Had the author caught the spirit of this and applied it to the construction of his book, he would not have given the length of a jack-screw bar to the thousandth of an inch or a weight of a hundred tons to the hundredth of a pound.

The author's mathematical methods exhibit the same lack of appreciation of the essential features of a problem as is displayed in his data and results. For example, a problem in spiral gearing (page 45) involves the following relations:

$$P_1 = \frac{\pi}{P}, \quad P = \frac{P_1}{\cos Y}, \quad N = \frac{\pi D}{P}.$$

The data are $P = 16$, $D = 1\frac{1}{2}$, $Y = 60°$; the required quantity, N. The author's solution is

$$P_1 = \frac{3.1416}{16} = .19635,$$

$$P = \frac{.19635}{\cos 60°} = \frac{.19635}{.5} = .3937.$$

Then

$$\text{No. of teeth (i. e., } N) = \frac{1\frac{1}{2} \times \pi}{.3937} = \frac{4.7124}{.3937} = 12.$$

Could a better example be found of how *not* "to apply mathematical . . . formulæ in the solution of such problems"?* In the three pages of explanation of the principles of spiral gearing which the author has prefixed to this solution it has not occurred to him to collect the formulas bearing upon it and to deduce the simple relation $N = DP \cos Y$, which gives at once, on inserting the data, $N = \frac{3}{2} \cdot 16 \cdot \frac{1}{2} = 12$.

This particular performance is especially provoking in that it violates the cardinal principles both of efficient and of accurate shop practice. Efficiency calls for the avoidance of un-

* Introduction, p. viii.

necessary operations, accuracy, the restriction as far as possible of the number of operations of adjustment, e. g., the centering of a piece of work in the lathe. In the example quoted the author has violated the first principle by executing processes which are equivalent to multiplication followed by division by π; he has violated the second in that both these operations are necessarily effected with approximate numerical values and are consequently unnecessary operations of adjustment.

The mathematical content of the book consists largely in the statement of formulas and the substitution of numerical data therein. As indicated in the problem on spiral gearing, the author does not make efficient use of even the small amount of elementary mathematical knowledge which may reasonably be supposed to be in the pupil's possession. The notation used is frequently made unnecessarily cumbersome by the use of two or more letters where one would be sufficient, e. g., *Wa* for weight arm (pages 8, 10), *CP* for circular pitch (page 33). This last notation seems a wholly unnecessary and undesirable complication. Not only do both C and P appear in the problem, but their product is not the circular pitch which the author calls *CP*. Moreover the notation used in practice, in so far as the reviewer can judge from the publications of the manufacturers, and from a widely used Mechanical Engineers' Pocket-Book, to both of which the author acknowledges obligation, is not that adopted by him.

The introduction contains the following statement* "What is needed is to purge the old books of useless material and put in place of it practical mathematical work distinctly planned to make up for the shortcomings of the old methods when measured by the practical demands of modern times." It would be impossible to state more accurately the process to which this book should be subjected.

The vocational schools have already attained a prominent position in our system of education. For the interest of these schools themselves as well as for that of the other secondary and the higher schools it is important that their mathematical instruction shall be of a high standard. As to the precise content of the curriculum there is room for much honest difference of opinion. When however we consider that the problems of the shop demand besides the elementary operations such things as geometric progression (cone pulleys and hoisting

* Introduction, p. viii.

tackle), cube root (belting problems), Euclid's algorithm of the greatest common divisor and continued fractions* (gearing and screw problems), it would appear that if the course in mathematics is to do nothing more than provide a secure foundation for the work in the shop its abstract content is not likely to be much less extensive than has been usual in secondary schools. The nature of the concrete problems will naturally vary with the requirements of the different schools. But whatever their nature they must be *real* shop problems, that is they must be such that in solving them the pupil is compelled to consider not only the purely mathematical element but also the significance and reasonableness of the numerical data and results and the appropriateness of the algebraic and arithmetic processes used in their solution. Excessive formalism has been the bane of the teaching of abstract mathematics. It is just as common and just as pernicious in the shop as in the class-room.

CHARLES N. HASKINS.

Die Mathematik in den physikalischen Lehrbüchern. Von H. E. TIMERDING. Band III., Heft 2. Leipzig, Teubner, 1910. vi + 112 pp.

IN the systematic study of the teaching of mathematics in Germany which is being made under the auspices of the International commission of the teaching of mathematics, the present volume covers the field of the mathematics required and used in the physics of the "Höhere Lehranstalten" and "Hochschulen." The principles of mathematics found in the text-books on physics in use in Germany to-day form the basis of the discussion.

While the author states that the mathematics of physics is mainly of a geometric nature, it is easily seen that he considers the main problem to be concerned with the amount and quality of the infinitesimal analysis used in the texts investigated. Fundamentally, the problem may be stated as follows: The exact theory of most physical phenomena in its development requires a use of the principles of infinitesimal analysis. A scientific attitude toward these problems on the part of the instructor will not allow him to be satisfied with mere formulas or even a confused word picture, or the skillful manipulation of a "near calculus" which may interest but not convince the

* Practical Treatise on Gearing. Brown & Sharpe Mfg. Co., pp. 130–134.