curvature. A geometric interpretation of the third derivative is due to Abel Transon (1841) who introduced the notions abberrancy of a curve, and axis of aberrancy. This notion has been almost completely lost, and the author is to be commended for reviving it. Transon's term, however, was "axis of deviation," which just reverses the historical statement as given by the author in regard to these two names.

The book is carefully printed and none of the misprints noticed by the reviewer can give rise to any difficulty.

E. J. WILCZYNSKI.

Elliptische Funktionen. Von Professor Dr. KARL BOEHM. Erster Teil. Göschen (Sammlung Schubert XXX). Leipzig, 1909. xii + 354 pp. 8.60 Marks.

A TREATISE on any of the functions of analysis, the properties of which are well known, must rely for its usefulness upon the mode and style of presentation. The historical development may be followed, or the functions may be introduced through later discovered properties. There can be no doubt that the former is the natural and more easily comprehended introduction, especially to the higher functions. Professor Boehm has elected the latter course, in this first volume, with the understanding, however, that the reader may commence with the second volume which starts out with elliptic integrals and the inversion problem.

The present volume is occupied with the various infinite series which represent simply and doubly periodic functions, with related series and products, and with their mutual interdependence.

The beginner will probably do well to take the author's suggestion and commence with the second volume. Students who have had a good course in the calculus can easily appreciate the inversion problem and its close proximity to so-called applications, but would most likely become discouraged and cry cui bono if requested to assimilate the contents of this volume in order to become acquainted with elliptic functions. This is true even if the shorter course were followed which the author has carefully planned and indicated by footnotes at the proper places through the volume.

It must be said, however, that the volume contains all the necessary preparation for an understanding of the series representing the elliptic functions. Indeed we do not meet with these series until we have read nearly one-third the volume, or through the first four chapters. These introductory chapters are occupied with infinite series and infinite products in general and with the particular series and products which represent simply periodic functions — the latter for the purpose of analogy. The author takes advantage of these early pages to introduce ideas and a terminology peculiar to the theory of functions in general and to periodic functions in particular. Thus the reader learns about zeros and poles, periodicity and homogeneity, even and odd functions, polarized functions (after Méray), holomorphic and meromorphic functions, etc.

We begin the study of the series leading to the elliptic functions at the fifth chapter. The author avoids the Mittag-Leffler theorem (it is mentioned with a reference to the *Acta Mathematica* on page 34) and considers de novo the convergence of the series

$$\sum_{n=-\infty}^{n=+\infty} \sum_{m=-\infty}^{m=+\infty} [n + m\pi + n\theta]^{-h}.$$

This is also done for the analogous simply infinite series in the introductory chapters. The work seems unnecessarily long and tedious in consequence, but a deeper insight into the nature of the convergence is perhaps thereby gained. For example, in section 46 the above series for $h = 2$ is considered between the *finite* limits $-n_1, +n_1; -m_1, +m_1$. The limit toward which the series converges for infinitely increasing values of m_1 and n_1 is shown to depend upon the limit of the ratio $m_1 : n_1$ and is finite only when the latter limit is finite. It is doubtful, however, if the knowledge gained is worth the effort to master the longer, if more elementary, processes entailed as well as the sacrifice of a definite statement and proof of the Mittag-Leffler theorem as a point of departure. At the end of the chapter, which is forty pages in length, we are in possession of the conditions for convergency of the basal series for $h = 1, 2, 3$, together with some of the properties of the functions to which these series lead; i. e., ζu, pu, $p'u$.

Chapter six is given up to the consideration of the sigma function. The same general method is followed, beginning with the consideration of the convergency of the proper doubly-infinite product and ending with the development in trigonometric functions.

After an excellent chapter on congruent complex numbers with respect to a double modulus, we come in chapter eight to the first explicit definition of an elliptic function, viz., an elliptic function is a doubly periodic function which is meromorphic over the finite region. Then follow the Liouville theorems concerning the sum of the residues, the equality in number of zeros and poles, and the congruency of the sum of zeros to the sum of the poles in a period parallelogram.

We have now completed the groundwork of the theory and it remains to fill in the details. Some points in the arrangement of this detail may be noted. The development of the primary Weierstrassian functions in power series is put off to the eleventh chapter after a long discussion (chapter ten) devoted to the less readily comprehended Hermitian elliptic functions of the second and third kinds. Again the Jacobi theta functions, the sigmas with index, and the Gudermanian *snu*, *cnu*, *dnu*, are not introduced until the twelfth and last chapter.

One might naturally hesitate to adopt this arrangement in presenting the subject to students for the first time, even if the above series were elected as the point of departure. A more or less ready familiarity with the primary functions and the modes of reducing them to numerical computation would seem advisable before proceeding to the more complex functions which can be expressed in terms of them.

On the other hand, it must be said that there is a certain gain in elegance when all the elliptic functions of first, second, and third kinds are introduced by means of their formal developments in infinite series and the property of periodicity together with a proof that they can all be expressed in terms of a relatively small number of primary functions. The detailed discussion of these primary functions would then follow and complete the course. That Professor Boehm has, in the main, adopted this latter arrangement of material makes his book appeal more to the advanced reader than to the beginner.

The book is consistent throughout in adhering to the main line of thought above sketched and thus avoids confusing the reader by presenting a number of points of departure with their consequent developments. While one may not agree with the author as to arrangement of material, one must admire the style and general effect of the book. It is interesting and valuable to have consistent and parallel developments of the simply and doubly periodic functions brought together in a convenient form.

L. WAYLAND DOWLING.