

impossible to tell by any experiment whether one is at rest or in uniform motion of translation with respect to the ether; one cannot say that two events are simultaneous in an absolute sense; moreover, all bodies will undergo a shortening in the direction of their motion. It is with this fascinating subject, more especially with the modified mechanics that it implies, that Poincaré deals.

It is only in bodies that possess a very great velocity that one can hope to discern a deviation from the laws of the Newtonian mechanics. Now Mercury moves at the greatest velocity of any of the planets and it is precisely Mercury that possesses a small anomaly not yet explained. The new mechanics accounts for a part of this, as Lorentz has shown, and nowhere else produces a sensible modification in the motion of the planets.* After these facts have been presented, Poincaré concludes by observing that the Newtonian mechanics will remain forever the mechanics at velocities which are small with respect to the velocity of light, and thus will continue to preserve its fundamental importance.

G. D. BIRKHOFF.

THE THEORY OF ELECTRONS.

The Theory of Electrons and its Applications to the Phenomena of Light and Radiant Heat. By H. A. LORENTZ. Leipzig, B. G. Teubner, 1909. 332 pp.

Ueber Elektronen. Von W. WIEN. Zweite Auflage. Leipzig, B. G. Teubner, 1909. 39 pp.

THE physical hypotheses which are adopted for the formulation and discussion of the correlation of a certain restricted group of physical phenomena, and in particular for the development of their mathematical theory, depend to a large degree upon those phenomena themselves and are in no small measure independent of other groups of related phenomena and of the point of view adopted relative to physics as a whole. Thus in dealing with most problems in heat, the old idea of heat as a sort of massless substance, caloric, still serves as the simplest

* Newcomb and Seeliger have shown that the anomaly in the motion of Mercury and of the other inner planets can be explained, for the most part, by the attraction of matter, diffusely distributed about the sun, of plausible mass, and of such distribution as to give rise to the known phenomenon of the zodiacal light.

hypothesis, and is and should be used despite the facts that for some phenomena in heat the kinetic theory is necessary and that for the proper point of view relative to physics as a whole heat is regarded as a mode of motion. A similar, though more highly diversified, state of affairs exists in regard to electricity. There is still ample field for the application of the old hypothesis of one or two electric fluids, there is still occasion to avail oneself of the hypothesis of action at a distance as well as of that of action through a medium, and furthermore there are now coming more and more into the foreground some phenomena which are best treated by adopting the hypothesis of atomic electricity—at any rate as far as negative electricity is concerned. It is with this latest hypothesis, this atomistic view of electricity that Lorentz's Theory of Electrons deals.

The application of the atomic hypothesis to electricity is by no means of very recent date. It was foreshadowed already in Faraday's work on electrochemical equivalents; it was stated with clearness and precision by Helmholtz in his Faraday Lecture nearly thirty years ago; it has for a long time been the subject of numerous theoretical and experimental researches on the part of many physicists of whom J. J. Thomson and Lorentz are perhaps the most eminent. In the last fifteen years the development and application of the atomic view has reached such proportions as to constitute of itself a vast science. Some years ago the publication by H. Abraham and P. Langevin of their large collection of memoirs by various authors on the general subject of ions, electrons, and corpuscles* did much to render the literature easily accessible; but, as a connected, theoretical, and detailed account of electrons and their applications has for some time been sadly needed, it is particularly fortunate that we have now just such a work from the pen of one who is himself the founder and builder of so great a part of the whole theory. The lectures which formed the foundation of the book under review were delivered in this country at Columbia University in 1906. The years between the delivery of the lectures and their publication saw many advances in the atomistic theory of electricity and of these advances those which are most germane to the point of view of the lectures have been incorporated in the volume.

* *Les Quantités élémentaires d'Électricité—Ions, Électrons, Corpuscules—mémoires réunis et publiés par H. Abraham et P. Langevin, Paris, Gauthier-Villars, 1905. xvi + 1136 pp.*

Chapter I deals with general principles and the theory of free electrons. The author adopts the same system of units and the same notation as in his articles in the fourth volume of the *Encyclopædia*; he uses vector analysis. It is interesting to notice the attitude he takes toward vector analysis—about the same attitude as he takes toward the choice of units—one of convenience. Nearly two pages are required to construct his system of vector analysis. The next thing to do is to set up the equations of the free ether. Then, after mentioning the possibility of deriving the equations for ponderable bodies (at rest) by the introduction of certain constants, the author settles down to showing the necessity of the electronic or atomistic point of view and to setting up the desired equations for the electrons. The integration of the equations by means of the scalar and vector retarded potentials is given. With this groundwork everything is ready for a rapid development of such vital points in electromagnetic theory as the Poynting vector and the flow of energy, stresses in the medium, the pressure of radiation, electromagnetic momentum, (approximately) uniform motion of a charge, electromagnetic mass, the vibration of a charge and the radiation of energy, the electromagnetic equations referred to moving axes, and the kinetic theories of Drude. The arrangement of the text is noteworthy. The details of mathematical analysis are relegated to notes at the end of the book (where they occupy 100 pages) and thus the main text is kept free for the presentation and discussion of the results. The same method is pursued in the later chapters. There is nothing dogmatic about the handling of the material, the author discusses his problems frankly and with every attempt to explain the why as well as the how.

It is but natural that the first chapter should be rather plain sailing and that its chief interest should lie in the artistic presentation of the necessary introductory material. With the second chapter, on emission and absorption of heat, we come to the consideration of some of the most troublesome questions of modern physics, lying in the widely different fields of kinetic theory, thermodynamics, and electromagnetism. The author treats Kirchhoff's law, the Boltzmann-Stefan law, and Wien's displacement law. He then turns his attention toward the further determination of the law of distribution of energy in the spectrum of a black body. Here three different theories are presented and discussed—that of Planck, which leads to

the most complete determination of the function but is not treated in detail because it is hardly germane to the method of this book, that of the author himself, which is a beautiful combination of kinetic and electric theories but is valuable only for large wave lengths, and that of Jeans, which is more dynamic in nature but does not admit a strict state of equilibrium. The discussion and comparison of these theories is clear and illuminating. Despite the excellencies of the theories of Jeans and of the author, our own inclinations are strongly toward that of Planck; its foundation upon the rather vague mechanism of the resonator may be hardly electromagnetic, its use of the theory of probability may be scarcely canonical, its implied quasi-atomic structure of energy may be distinctly annoying, but somehow it does get very close to the facts and in time its difficulties may evaporate.*

Chapter III on the theory of the Zeeman effect is a very pretty piece of work. After discussing the simplest case of the Zeeman effect the question of series of spectral lines is treated. There then follows an examination of the general properties of a vibrating system of electrons and of the special properties of certain simple systems. The difficulties of a complete explanation of the phenomena involved are thoroughly presented. The author is bent on stating the importance of the problem and the need of its solution much more than on over-emphasizing the value of work already accomplished; he even concludes that "after all, we are rather at a loss as to the explanation of the complicated forms of the Zeeman effect." There can be little doubt, however, that he is on the right track and has contributed largely to the ultimate resolution of the difficulties of the analysis of the spectrum. The very diversified spectra of different elements, the astonishing simplicity of some and the bewildering complexity of others, will occupy the attention and perhaps thwart the endeavors of mathematical physicists for a long time to come.

Intimately connected with the foregoing and an immediate extension of it is the work of Chapter IV on the propagation of light in a body composed of molecules and the theory of the

* It may be remarked that Larmor has recently given an exposition of Planck's theory which apparently removes the difficulty as to the structure of energy. Another most remarkable fact is that, although Planck by his method found a value for e (4.7×10^{-10}) about fifty per cent. larger than that which had long been accepted, some recent investigations seem to corroborate his theoretical value. A showing like that means a great deal.

inverse Zeeman effect. The author passes from his equations for electrons to those for ponderable bodies by a system of averages executed over large numbers of electrons and he is able to give the theory of dispersion of light with the particular hypothesis that the electrons, which are now so well known to be fundamental, are those small sensitive particles which occur with no specializing hypothesis in the theories of Sellmeyer and others. Although the author makes no mention of Gibbs, it may be well to recall that Gibbs, in a series of papers, remarkable for generality rather than specialization of hypothesis and culminating in his deduction of the general equations of monochromatic light in media of every degree of transparency, constructed an electromagnetic theory of optics on a foundation of averages. In a problem like this it is decidedly advantageous to distinguish those results which may be obtained by general consideration from those which follow from special hypotheses. The author's work with electrons leads to a number of formulas which may be compared with experimental evidence to show their accuracy and to emphasize the value and essential truth of the physical model which he uses. After touching upon the relation of Faraday's rotation of the plane of polarization to the Zeeman effect and upon some of Voigt's results in magneto-optics, the author again closes with his characteristically modest warning that the theories he has set forth can hardly be regarded as more than a start toward the final solution of the problems. A perusal of the chapter will, however, convince the reader that he is reading some very real up-to-date vital physics.

The last chapter is devoted to optical phenomena in moving bodies. Here again, as constantly throughout the volume, we are in the presence of theories which are essentially due to the initiative of Lorentz. Here again there is no haste to assume any particular hypothesis without due consideration; the points of view of Stokes and Fresnel are examined before passing to the strict theory of electrons. The experiments of Michelson and Morley and of Rayleigh and Brace are discussed. The significance of the transformations of what Poincaré has called the Lorentz group is made clear. Some remarks relative to the rigid electron of Abraham and the electron of constant volume of Bucherer and Langevin are inserted. Finally the work closes with a brief reference to the principle of relativity as formulated by Einstein. At present the principle of relativity,

owing to pioneer work of Einstein and Minkowski and the enthusiastic cultivation of their followers, is receiving a great deal of attention. Lorentz speaks somewhat regretfully of the fact that he cannot give it greater space in his book. But we must remember that the methods employed by Einstein, Minkowski, and others in developing this principle are Maxwellian rather than Lorentzian, that they are not essentially concerned with electrons, and that consequently they are not really a proper part of the present work.

In fact, what some students of relativity would do with electrons, especially with vibrating electrons emitting trains of waves, is hard to imagine; for there are not a few who seem intent on abolishing the ether and on disregarding the phenomena which were the prime cause for its introduction. We may all come to this — one never can tell. If Lorentz's ether-electron model of the universe is so highly valuable as a basis for some domains of physics, there may well be other nearby domains for which the etherless kinematic space of some of the relativists is equally useful. And it will not do to overlook J. J. Thomson's Faraday-tube model! To the mathematical purist it may seem frivolous to take such a broad, smiling attitude toward the coexistence of such different models in a single scientific field; that is because the pure mathematician is an esthesiogenist rather than a scientist. So careless is the real scientist in the presence of divers models in the same field that we may safely venture the guess that even among specialists on heat, thermodynamics, and kinetic theory there are few who from the kinetic point of view can readily establish the equations for the flow of caloric in a marble column; they have more important and interesting things to do.

To pass from Lorentz's work to the little essay of Wien is but a step. It was in 1906, we believe, that Wien delivered an address before the *Versammlung deutscher Naturforscher und Aertze* on electrons. The address was printed, and has now appeared in a second edition to which some material of recent date has been added. As the pamphlet is itself a very succinct review of a wide field, it would be futile to review it here further than to mention its popular and yet scientific nature which must recommend it to all who are interested in what is going on in live physics and who have not yet acquired Wien's broad view over the field of electron theory. For those

really unfamiliar with the subject it would be well to read Wien before Lorentz and then again afterward. It is an excursion that is worth while for anyone. Come, drop an epsilon just for once and pick up an electron; it is a deal larger, even if its radius is only 10^{-13} cm.

EDWIN BIDWELL WILSON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS., August, 1910.

SHORTER NOTICES.

Vorlesungen über Differentialgeometrie. By R. v. LILIENTHAL.
Erster Band: *Kurventheorie.* Leipzig, B. G. Teubner, 1908.
8vo. vi + 368 pp. 12 Marks.

It is undoubtedly one of the achievements of the mathematical development of recent years to have shown that mathematical rigor is not the same as pedantry; that a theory may be presented in an irreproachable manner and still be attractive and interesting. There can be little doubt from a pedagogic point of view that if, in a treatise intended for students, a choice must be made between a comprehensive but tedious, and a less comprehensive but interesting treatment, the latter should be given preference, provided of course that its limitations be properly indicated. Both of these general principles seem to have been ignored completely in the construction of the book under review.

The author wishes to be rigorous and general. For this purpose he excludes all considerations involving infinitesimals. One might begin to quarrel with him on this score, since a properly formulated notion of infinitesimals is not altogether unknown. He then confines his attention exclusively to the case in which all of the functions which occur are analytic, a restriction of generality which seems extraordinary from the author's point of view, since all of his developments require only the existence of a finite number of derivatives. But even then, also for the sake of rigor, he excludes (as a matter of course, he says), the consideration of questions of order of contact, although all of these questions are easily treated by power series methods and would fall most naturally into the theory as developed by him.