where $F(N)$ is the number of solutions of

$$N = (2n + 1)(2n + 4b + 3) - 4a^2 \qquad (a = 0, \quad \pm 1, \cdots \pm n)$$

and n, b are positive integers. On the other hand Hermite shows that $F(N)$ is nothing but Kronecker's function F defined above. Let us now set $x = 0$ in (5). The left side vanishes, and if we arrange the right side according to powers of q, Hermite finds, letting d', d'' be divisors of N such that $d' > \sqrt{N}$, and $d'' < \sqrt{N}$, that

$$A\Theta(0) = \tfrac{1}{2}\Sigma q^{\frac{1}{4}N}(\Sigma d' - \Sigma d'').$$

The coefficient of $q^{\frac{1}{4}N}$ on the right Kronecker calls $\Psi(N)$; the left side we see is the product of two infinite series in q. Performing the multiplication and equating coefficients of like powers of q gives finally

$$F(N) + 2F(N - 2^2) + 2F(N - 4^2) + \cdots$$
$$+ 2F(N - 4k^2) = \tfrac{1}{2}\Psi(N),$$

a relation between the number of properly primitive quadratic forms with the determinants $-N$, $-(N-4)$, $-(N-16)$, $\cdots$

If we have gone into some details in speaking of the papers 6), 12), and 15), it is partly because their importance demands more than a passing notice and partly with the hope that our remarks may awaken the interest of some reader of this BULLETIN to look farther into these matters.

JAMES PIERPONT.

SHORTER NOTICES.

Serret's Lehrbuch der Differential- und Integralrechnung. Dritte Auflage, dritter Band,* neu bearbeitet von GEORG SCHEFFERS. Leipzig, Teubner, 1909. xii + 658 pp.

THIS book on differential equations is the third and last volume of Scheffer's "Umarbeitung" of the second edition of Serret's Lehrbuch. In comparison with the first two volumes, there are many more alterations made in this third edition of the third volume. In fact one can hardly recognize any traces

*The first two volumes of this work were reviewed in the BULLETIN, vol. 15 (1908–09), p. 140.

of the old book in the new. The arrangement of material has been changed, some subjects have been omitted entirely, some have been developed from mere paragraphs into chapters, the real variable has been separated from the complex, the figures have been redrawn, and although the new edition contains fewer general topics than the old yet it is nearly one half larger. The worked out illustrative examples are left practically unchanged, but many of them have been shifted to more fitting sections.

The first chapter is a thirty-page general discussion of differential equations, ordinary, total, and partial, with a clear statement of the two directions which a course in differential equations might take:

I. "Die Bestimmung aller Lösungen bzw. Lösungensysteme soweit wie mittels Elimination, Substitution und Differentiation möglich auf blosze Quadraturen zurückzuführen."

II. "Unmittelbar aus den vorgelegten Gleichungen oder Systemen von Gleichungen die Eigenschaften der Lösungensysteme zu erkennen."

A broad interpretation of the first point of view dominates the book, but the second is not neglected, about a quarter of the volume being devoted to the function theory side of the subject.

The second chapter consists of the derivation of existence theorems for ordinary differential equations, systems of ordinary differential equations, and implicit functions of one or more variables. This chapter is parenthetical in character and "upon first reading" may be omitted. The real variable alone is considered. The third, fourth, and fifth chapters treat of ordinary differential equations of the first order, systems of ordinary differential equations of the first order, and ordinary differential equations of higher orders respectively. The rather brief discussion of Lie's integration methods based upon infinitesimal transformations which appeared in the second edition has been greatly expanded in these chapters and has been made one of the features of the book, notwithstanding the detailed treatment of the older methods. The sixth chapter, also parenthetical in character, extends the theory of the second chapter into the domain of the complex variable.

Partial differential equations are taken up in the next two chapters. The seventh treats of the linear partial differential equation of the first order, first reducing the problem to that of the homogeneous partial differential equation. A few pages at the end are devoted to Pfaff's equation in three variables. The eighth chapter takes up the general partial differential equation of the first order by two methods. First the emphasis is laid on the geometrical methods of Lagrange and Monge, which are made to serve as an introduction to the analytical method of Cauchy given in the second part. Partial differential equations of higher orders are not discussed in this edition.

Zermelo's chapter on the calculus of variations in the second edition has been replaced by a shorter chapter with practically the same content. The notation has been changed to conform to the notation of the more recent articles and treatises. There is no treatment of sufficient conditions, merely a discussion of Euler's equations for the simplest problem of the calculus of variations, with some extensions to isoperimetric problems and problems in three variables. The subject matter is illustrated by the usual examples, the catenary, brachistochrone, etc. We note that in this edition, "Die Eulersche Differentialgleichung" replaces "Die Lagrangesche Differentialgleichung," a result probably of Professor Bolza's championship of Euler's claim to priority.

Harnack's appendix on the integration of partial differential equations and the few pages of "Bemerkungen" have been omitted in this edition.

As a third volume in a course in calculus, intended for students in their first three semesters, the present volume will be found rather advanced, notwithstanding the footnotes pointing out paragraphs and chapters which may be omitted. Scientifically, however, the new edition is a vast improvement over the old. The arrangement of the subject matter, the clearness of the language, the precise statement of definition and theorem, the copious index, and the typography place this work among the best reference text-books on differential equations in German or English.

A. R. CRATHORNE.

Kreis und Kugel in senkrechter Projektion, für den Unterricht und zum Selbststudium. Von Dr. OTTO RICHTER. Leipzig und Berlin, Teubner, 1908. x + 187 pp., with 147 figures.

THE author's aim as set forth in the preface is to furnish a supplement to the numerous elementary treatises on descriptive geometry. He proposes to give general solutions of certain fundamental problems which are studied for special cases only in books on descriptive geometry, and to give the student a