complete independence, and chiefly through the influence of Euler, D'Alembert, Danie Bernoulli, Lagrange, Laplace, and Legendre.

Professor Cantor's own contribution is largely bibliographical, consisting of a list of the most important works mentioned by his collaborators. He makes the same assignment of date to Ruffini's first work on equations of the fifth degree as made by Professor Cajori. It is evident that the printing of the work began in 1798 and was completed in 1799.

It is too early to enter into critical details of such a work. That numerous errors will be found is certain, as witness the partial list given by Müller, and the fact that the *Bibliotheca Mathematica* gives every month a list of corrections extending back to the first edition of the first volume, published nearly thirty years ago. On the other hand it will be a long time before any one will attempt to treat so exhaustively this remarkable period in which the genius of Euler, D'Alembert, Lagrange, Laplace, and Legendre showed at its best, and in which Gauss was beginning the labors that placed his name among the leaders of modern times. DAVID EUGENE SMITH.

SHORTER NOTICES.

High School Algebra. By H. E. SLAUGHT and N. J. LENNES. *Elementary Course,* 1907, vii + 297 pp., $1.00; *Advanced Course,* 1908, vii + 194 pp. $0.65. Boston, Allyn and Bacon.

THE common weakness of our college students in elementary algebra shows a great need for improvement in the teaching of this subject. The appearance of these admirable texts, constructed after a new model, marks a distinct advance in the teaching of algebra in our high schools. Recent discussions have shown that it is quite generally agreed that high school algebra should be divided into a first course of elementary algebra during the first year, and a second course of review and advanced algebra during one half of the third or fourth year, preceded by one year of plane geometry. The authors have met this demand of teachers by dividing their text into two parts, and in doing so have succeeded in presenting the subject of high school algebra in the most concrete and teachable form we have yet seen.

As stated in the preface, "the main purpose of the Elementary Course is the solution of problems rather than the construction of a purely theoretical doctrine as an end in itself." Throughout the first course the method is inductive. The interests, ability, and needs of the first year high school students seem to demand this selection of the inductive method. In writing elementary texts, mature mathematicians too often let their delight in the subject as an elegant deductive system obscure the ability of the student at this particular stage in his development. The teachableness of this book is greatly increased by the fact that the authors never lose sight of the standpoint of the *pupil.*

The illustrative problem is used throughout the book to show the need of new processes and to secure real insight into the principles involved. This enables the authors to build upon the arithmetic knowledge and experience of the pupils, to proceed from the known to the unknown, and to make the work more concrete. Illustrative problems are made to take the place of long abstract descriptions and explanations. New notions and processes thus first made evident by illustrative exercises are always followed up by a careful and precise statement of the *principle* involved. By this method eighteen principles used in the Elementary Course are introduced. These eighteen principles stand out prominently as a body of facts which each student must know and be able to use. It is somewhat of a surprise that the authors have been able to reduce the subject of elementary algebra to such a small number of fundamental principles.

New ideas are introduced one at a time and only when needed in the solution of problems. This, with a desire to secure a maximum of concrete matter in the early parts of the book, has led to a slightly new order of topics. The most noticeable change from the conventional order is in the position of factoring, long division, and literal fractions. Factoring follows simultaneous linear equations, where first needed as an introduction to the quadratic. Long division comes quite late in the course because it is first needed in the square root process required in the general solution of the quadratic. Literal fractions come in the last chapter because the knowledge of arithmetic fractions and the use of simple operations are sufficient for earlier problems. This new order of topics is justified by the gain in more early concrete work and in placing the complicated work later in the course. Of course such an order of topics requires greater care in the grading of problems, and this has been exercised.

Since the chief purpose is the solution of problems, the equation as an instrument for this purpose is early introduced and occupies a prominent place throughout the book. The analogy drawn between a balance and an equation is good. Equations are solved for many different letters, which is excellent preparation for later work in physics. The real test for a solution, namely that it satisfies the equation, is properly emphasized by many exercises in checking. The form of arrangement in the illustrative problems requires a justification of each operation by an explicit reference to some one of the eighteen fundamental principles. This tends to overcome the common fault of blind juggling of symbols and lays an excellent foundation for the deductive proofs of the Advanced Course. On nearly every page appear evidences of a purpose to develop ability to translate English into the forms of algebra; many parallel statements of the same fact in English and algebra are found; formulas are restated in words; numerous exercises in translation are given.

Negative numbers are well introduced and treated by means of concrete examples; emphasis is placed on the interpretation of the sign of the result of a problem. The term "less than zero" which confuses beginners is avoided. The term "absolute value" is introduced in this connection.

The justification for the introduction of graphs into algebra is their usefulness in illustrating the principles involved in simultaneous equations. The authors introduce graphs just *before* considering linear equations and then *use* them in their treatment of simultaneous equations throughout both books. The usefulness of graphs is made so evident that they become an integral part of the subject, instead of seeming to be an extraneous topic tacked on to algebra.

The emphasis placed upon elimination by substitution should be noted. Real insight into what is happening comes through the process of substitution. Elimination by addition and subtraction is given second place. Elimination by comparison is omitted in the Elementary Course but is found in the Advanced Course.

Factoring is introduced on page 172 — about the middle of the course. Complicated cases are omitted in the first course. The treatment of the type $ax^2 + bx + c$ is especially good. In each set of exercises for factoring there are some expressions which are not factorable by the case under consideration; this has the advantage of requiring the pupil to study the *form* of each expression.

The treatment of the quadratic illustrates the authors' use of the spiral method. The quadratic first appears on page 197 as an application of factoring; on page 240 as an application of square root; quadratic and linear simultaneous equations come on page 246; while the complete theory of the quadratic comes in the Advanced Course. This spiral method has the advantage of going from the simple to the complex and gives a good review of a topic by repeated considerations, each time in a more complicated form.

Literal fractions appear in the last chapter. It is surprising how many problems can be solved without the use of literal fractions. In the treatment of complex fractions the shorter method of multiplying both terms of the complex fraction by the least common multiple of all the minor denominators is used. This method is too frequently neglected.

Numerous definitions appear in the Elementary Course, but a tendency to avoid or postpone the introduction of technical terms is noted. The authors seem to feel that by repeating the description of technical terms, such as *transpose* or *equivalent equations*, each time they are needed a greater insight into real processes is obtained.

The reviewer was impressed by the number of concrete problems; in the first 150 pages there are about 65 pages of concrete problems and 25 pages of abstract exercises. The problems contain interesting information and are of such a nature as constantly to raise the question whether the result is reasonable. The desirable habit of appealing to one's common sense to justify results is thus strengthened. The inductive principle is even carried into the problems. General or literal problems which result in formulas are approached through a series of numerical problems. The close connection of algebra and arithmetic is brought out by many problems in Arabic numbers; nearly every principle is illustrated by Arabic numbers.

The Advanced Course presupposes a knowledge of the principles of elementary algebra, considerable training in deductive reasoning obtained in the preceding year of plane geometry, and greater maturity in the pupils; hence its point of view is entirely different from that of the Elementary Course. Its method is deductive, and instead of principles to be illustrated the algebraic facts are put in the form of theorems to be proved. This new point of view gives a fine review of the Elementary Course without degenerating into an uninteresting rehash of old work.

The first chapter is given to a careful statement of the axioms used. The nine axioms cover the assumptions made concerning the uniqueness of addition, subtraction, multiplication, and division and their commutative, associative, and distributive laws. The exclusion of division by zero is properly emphasized here; throughout both books many notes call attention to this exclusion of division by zero.

A good treatment of equivalent equations is found in Chapter III. A complete treatment of factoring, including the factor theorem, appears in Chapter V. The graph is continually used in simultaneous equations. The theory of exponents and radicals is completed in this course. The discussion of formulas as a preparation for later work in physics is emphasized. Radicals are well treated and followed by a good collection of problems involving radicals.

The topics covered in the two books fully meet the present college requirements with additional chapters on variation, logarithms, arithmetic and geometric progressions, and the binomial theorem for positive integral exponents.

The following definition of irrational number seems unnecessarily complicated and incomplete: "If a number is not the kth power of an integer or a fraction, but if its kth root can be approximated by means of integers and fractions to any specified degree of accuracy, then such a kth root is called an irrational number" (page 70). The incompleteness is willful, for the authors later remark, "There are other kinds of irrational numbers besides those here defined." No *explicit* definition of rational numbers was found. There seems to be a slight inconsistency between this discussion of numbers on page 70 and that of roots on page 11.

We regret to see no use of the symbol i for the complex unit, and the omission of the term *discriminant* in the discussion of the quadratic. An index of definitions would be a useful addition to both books. There is no appendix. The authors have had the courage to omit entirely topics which they believe have no place in a high school algebra, such as simultaneous equations of more than three unknowns (a single example in four unknowns is given) inequalities, complicated complex fractions.

Judged as a whole, these books are the best texts on high school algebra we have seen, and their use will produce stronger students than we are now receiving from the high schools.

ERNEST B. LYTLE.

Einführung in die Hauptgesetze der zeichnerischen Darstellungsmethoden. By Professor ARTUR SCHOENFLIES. B. G. Teubner, Leipzig and Berlin, 1908. iv + 92 pages. 98 figures.

DURING the last few years the need for more systematic and comprehensive instruction in geometric drawing has become much more keenly felt among school and college teachers. While the courses in descriptive geometry for technical students are frequently sufficiently extensive to claim nearly half the time of the student for a year or more, the corresponding training for prospective teachers is usually very inadequate. Among the attempts to supply this need, numerous recent publications of the firm of B. G. Teubner are worthy of careful consideration. The question may be fairly put, whether the emphasis should be placed on the technical details or on the geometric principles underlying them. The little book of Schütte* is of the first kind; that of Müller and Presler† and also that of Schüssler‡ are much more comprehensive and combine considerable instruction in geometry with the explanations of the constructions. But the book under review puts most of the emphasis on the geometric principles. It is meant for much more mature students than the other books mentioned. The aim is frequently not so much the acquisition of sufficient knowledge or skill to introduce all the geometric details into a figure, but rather the power of making a good illustration by means of a few lines drawn free hand.

After a good discussion of the principles of plane perspective the theorem that any two projective pencils have one right angle formed by corresponding lines is proved in great detail. This theorem is used later. The treatment of the methods of descriptive geometry is brief, but clear. A reader of fair ability and much patience could read these parts profitably

*See review in BULLETIN, vol. 14, p. 294.

†See BULLETIN, vol. 10, p. 207.

‡Reviewed in BULLETIN, vol. 12, p. 361.