VON HELMHOLTZ.

Hermann von Helmholtz. Von L. KOENIGSBERGER. 3 Bde., 8vo. Braunschweig, Vieweg und Sohn. Bd. 1: 1902, xi + 375 pp., 3 portraits; Bd. 2: 1903, xiv + 383 pp., 2 portraits; Bd. 3: 1903, ix + 142 pp., 4 portraits.

Vorlesungen über theoretische Physik. Von H. VON HELMHOLTZ. 6 Bde., large 8vo. Leipzig, J. A. Barth. Bd. 1_1: *Einleitung zu den Vorlesungen über theoretische Physik,* hrg. von KÖNIG und RUNGE, 1903, 50 pp., 1 portrait; Bd. 1_2: *Vorlesungen über die Dynamik discreter Massenpunkte,* hrg. von KRIGAR-MENZEL, 1898, x + 380 pp.; Bd. 2: *Dynamik continuirlich verbreiteter Massen,* hrg. von. KRIGAR-MENZEL, 1902, viii + 247 pp.; Bd. 3: *Vorlesungen über mathematische Principien der Akustik,* hrg. von KÖNIG und RUNGE, 1898, x + 256 pp.; Bd. 4: *Elektrodynamik,* "soll bald erscheinen"; Bd. 5: *Vorlesungen über die elektromagnetische Theorie des Lichtes,* hrg. von KÖNIG und RUNGE, 1897, xii + 370 pp.; Bd. 6: *Vorlesungen über Theorie der Wärme,* hrg. von F. RICHARZ, 1903, xii + 419 pp.

HELMHOLTZ died on September 8, 1894. Although he was at that time considerably past seventy years of age, he was still in the midst of his researches, still as enthusiastic as ever, still as piercing of intellect. On July 9, just before his last illness, he had been busy correcting the proof of his pupil Hertz's Mechanics, but nevertheless had stated that it had been a lucky day for him because he had discovered what he and many others before him had long been looking for. What this was we shall never know; for before he had had the necessary time to write the matter out and present it to the Academy he was mortally ill. His own researches and those of his pupil Hertz, who had died so shortly before, were by no means the only things that occupied his mind. He was director of the Reichsanstalt and had come to the decision two years previous to publish his lectures on mathematical physics. Indeed the fifth volume, on the electromagnetic theory of light, was already in the press with the proofs two-thirds read. This was a fitting end to the activity of one who for fifty years has been astonishing the world with the fecundity of his investigations

in widely different fields of science. As Lord Kelvin said in an address before the Royal society in November 1904: "Of the whole of Helmholtz's great and splendid work in physiology, physics, and mathematics, I doubt whether any one man may be qualified to speak with the power which knowledge and understanding can give."

Upon some of Helmholtz's pupils devolved the task of seeing his lectures through the press; and at the deathbed of the great physicist's youngest son, when the name of von Helmholtz was slipping away from the world forever, his old friend and long time colleague at Heidelberg, Koenigsberger, determined to write his biography. The biography and the lectures (with the exception of the fourth volume) are now complete; and with the books which Helmholtz himself published they form a readily accessible monument which some of us may believe a more impressive memorial to his genius than that unveiled amid such royal pomp in the garden of the University of Berlin.

Koenigsberger's biography is not merely an account of Helmholtz's private life, it is not an analysis of his scientific achievements, it is not the memoirs of an intimate acquaintance nor the critical history of an outsider; it is all of these together. On one page the reader will find some family correspondence, perhaps concerning the health of Robert or Fritz; on the next, will be a critique of a series of articles on the foundations of geometry or sometimes an unprinted public address. As one reads, he follows the whole life of the master.

The work is divided into sections corresponding to the different positions which Helmholtz occupied — a student, a naval surgeon, teacher in an academy of fine arts, professor of physiology at Königsberg, professor of anatomy at Bonn, of physiology at Heidelberg, of physics at Berlin, and president of the Reichsanstalt. The titles alone should seem to justify Kelvin's remark. It should be remembered that Helmholtz did not have a free choice as to his profession. He always said that he was a born physicist. But his father was poor; being a physicist was too dear; the boy must choose medicine as his work even though both he and his father regretted it. The result was one of the world's greatest physiologists, with the Lehre von den Tonempfindungen and the Physiologische Optik as chief witnesses. He could not have been a greater physicist even if he had not been a physiologist, perhaps not so great.

It would be ungracious to draw any moral concerning the elective system of studies and the necessity of doing what one pleases too early in life.

Helmholtz had a very marked ability for keeping out of ruts. A predisposition for getting snugly into some rut is perhaps one reason for the opinion current in some quarters to the effect that one's work is done at the advanced age of forty. If Helmholtz had lived no longer, we should have had the Erhaltung der Kraft, the Integrale der hydrodynamischen Gleichungen welche den Wirbelbewegungen entsprechen, and the Handbuch der physiologischen Optik; but what of the Lehre von den Tonempfindungen and indeed the last third of the Physiologische Optik, what of the work on the foundations of geometry which led to Lie's researches on that subject, and the investigations on electromagnetism which culminated in Hertz's discoveries, or the Thermodynamik chemischer Vorgänge which, written at the age of sixty, would have been the great landmark in physical chemistry if Gibbs's then unknown work had not appeared half a dozen years before? And this is not all. It is necessary to read the whole biography to appreciate how Helmholtz retained his spontaneity and power into the last days of his life. A rare parallel to this is seen in the life of his close friend, Kelvin.

In its general make-up as a book the biography is almost de luxe. The paper is heavy and the type, which fortunately is Roman, is clear. The reproductions of portraits, for the most part by Lenbach, add to the artistic appearance of the work, and to its personal interest. The scientific world has reason to thank the publisher for the form in which the volumes have appeared just as it should be very grateful to the author who, though by no means a young man, has devoted so much time to accomplish religiously his self-appointed task.

Helmholtz's Lectures on theoretical physics are particularly interesting and instructive because he himself was personally responsible through his own researches for so large a part of their contents. It is this which gives them their life and individuality. Moreover the acquaintance of the author with so great a realm of physics, mathematics, and biology gives a virility to his lectures and a powerful variety to his illustrations which it would be difficult to find elsewhere.

The introduction of fifty pages which forms the content of the first part of the first volume is a sort of epistemology or

methodology of the physical sciences. This subject was of great interest to Helmholtz and upon it he had printed a large number of papers which may be found admirably analyzed in the biography. The author was too much of a physiologist to be won over without restriction to that abstract formalism which was represented by Thomson and Tait and by Kirchhoff. He doubtless could understand their point of view when they undertook to demonstrate that force did not exist; but he preferred to retain force as a fundamental idea. In the treatment of the number system the same practical point of view appears. Note the statement: "That diagonals of squares exist in nature there is no question; and so, too, definite physical investigation may thrust other irrationalities upon us. The difficulty or the impossibility arises here, not in regard to the reality of such quantities or ratios, but in the incompleteness of our methods. . . . Similarly with the much discussed question of the existence of continuous quantities." This and other statements which the author makes may furnish a sounder basis for metaphysical speculation than many a more complete theory developed by those less in touch than he was with the real living world.

Such careful attention to the underlying conceptions is everywhere typical of the lectures. The discussion of the dynamics of discrete masses begins with a detailed account of such things as the material point, coördinates, continuity and differentiability, velocity, acceleration, Newton's laws, and so forth. It would be difficult to find a clearer and more careful statement of these fundamental matters. As might be expected from the director of the Reichsanstalt, the theory of pendular vibrations is treated in great detail, sixty-five pages being given to it. To illustrate the way in which the author gives the student the necessary mathematical methods as he goes along we may mention that the questions of least squares and of linear non-homogeneous differential equations of the second order find their treatment at the points where each is needed. Stress is laid on the importance of the general integrals of motion, the integrals of momentum, of moment of momentum, and of energy. Work and energy receive the attention which one might expect to be given them by the author of the Erhaltung der Kraft. The various fundamental principles such as that of virtual velocities, of d'Alembert, and of Hamilton, and the lagrangian equations of motion are all taken up. It may

be noted that Appell in his Mécanique rationnelle follows this same plan of introducing into dynamics at a somewhat early stage these higher principles. It would be difficult to find an elementary treatment of mechanics which could better serve as an introduction to mathematical physics than this of Helmholtz.

Before taking up the theory of continua, the author gives a short discussion of the relation of the theory to the molecular hypothesis, and of the difference between ordered and unordered motion in the interior of the continuum. He then establishes a connection with the homogeneous strain by developing the displacement into a Taylor's series and examining the effect of the terms of higher order. After a preliminary treatment of strain, he makes the statement that the internal forces in a continuum follow the law of action and reaction and hence are conservative. This leads the way to Green's method of developing the theory of elastic bodies from the existence of a potential for the forces of deformation. The analytic theory of elasticity is beset with massive formulas. These the author largely avoids by turning his attention to the more physical side of the question. He sets about determining the function whose space integral gives the potential energy in a variety of different types of elastic bodies. The following section of the book treats of the equilibrium of elastic bodies. Those cases are solved which are of use in actually determining the elastic constants of the bodies. Finally the question of the propagation of plane or spherical waves, whether longitudinal or transversal, is taken up and solved. Helmholtz was about seventy-three years old when he prepared and delivered these lectures and must have been, if the lectures may be taken in evidence, an admirable teacher; for he shows a genius in avoiding analytic difficulties wherever he can, and in resolving them clearly for the student when to avoid them is impossible or undesirable.

With the third volume, which is on sound, there begins to appear a considerable amount of repetition. The amount of attention paid to the motion of a pendulum was noted in speaking of the first volume; here the matter must needs come up again. So different, however, is the method of procedure and so much is the original problem generalized that the repetition seems advantageous rather than otherwise. It is a serious mistake to avoid altogether the repetition of matters and methods which are so fundamental that they should become second nature

to the student. It is not possible to be old friends at first sight. Of course Fourier's series are the foundation of a large part of the mathematical work. Instead of assuming that they are known or may be learned by reference to some standard text, the author patiently and in all detail develops the method of Fourier's series, enters upon the necessary proofs of convergence and even goes so far as to show that the convergence is toward the right value. This development of the mathematical algorithms as they are needed goes a long way toward giving the reader that confidence which is so important. In treating waves in space, Green's theorem and Huyghens's principle are taken up in the same manner. In discussing vibrations, each different type, whether a plucked string, a stretched string, a membrane, free air, or air confined to a tube, is developed separately and in natural order.

The fourth volume, on electrodynamics, has been announced as "soll bald erscheinen" for almost a record breaking number of years. It is even rumored that the volume may never appear. This is probably due to the great changes which have taken place in electromagnetic theory since the lectures were delivered. The editors may feel that there would be little use in printing a book which would be out of date before it was written. It seems to us that this reason should not prevail here. Helmholtz's researches on electricity and magnetism were by no means insignificant. A large part of the time of his later years was spent in this field of physics. It was he who developed the theory of an ether which included Maxwell's as a special case. His lectures on electrodynamics could not fail to be of interest as a matter of history if in no other way. Furthermore it is inconceivable that what Helmholtz might have to say on the subject, his methods of attack, and his points of view should not be highly important and instructive even after the world has moved somewhat away from his position. We hope the volume may still appear.

The fifth volume, that on the electromagnetic theory of light, was the first to be published. Fortunately it is independent of the preceding one. With characteristic straightforwardness an introduction on some points in electrical theory and on waves in a continuous medium comes before the introduction of the Maxwell equations. In their proper places the elements of the theory of the potential function, Green's theorem, plane and spherical waves, and Huyghens's principle reappear, although

each has been taken up at least once before. After a long presentation of diffraction and the establishment of the laws of geometric optics from the fundamental dynamical equations, there follows a detailed treatment of the customary questions of geometric optics. Polarization and dispersion are not neglected; but the fact that the work is one on general physical optics rather than on special mathematical problems in optics is clearly indicated by the meagre attention paid to crystalline media — only eight pages. This is but another instance of the author's avoidance of complicated mathematics and insistence upon that which will be of most value to the practical physicist.

The lectures on the theory of heat are a charming close to the whole series. The introduction on "warmness" and temperature reminds one irresistibly of the author's philosophical speculations with which the series began. The treatment of problems in conduction with the repetition of the necessary mathematical tools, Green's theorem and Fourier's series, not omitting even the proofs of convergence, recalls the general style of the other volumes. The applications of these results are, however, new and lead up to the introduction of Fourier's integral with a full explanation of what this new tool does for one. Throughout the volume, the illustrative examples have even more than their accustomed variety — cooling of the earth, diurnal and annual variations in subterranean temperature, the vital theory of the black body, and Kirchhoff's law. The next large division of the volume concerns thermodynamics. As might be expected, the underlying concepts, the first law or conservation of energy, the second law or dissipation of energy, the entropy, the "free" energy, and so on, are set forth with especial pains. It would be difficult for the reader to fail to get the right idea. It is here and in the following section on cyclic systems that we find some account of the researches which took the major part of the author's free time after passing the age of sixty. The work appears to be still new to him. There is a freshness about the style which is not so much in evidence in the volumes which deal with his earlier work.

It has not been our intention here to give an account of Helmholtz's life and works on mathematical physics — this were too great a task — but to indicate as briefly as might be what may be found in the eight or nine volumes which treat of these subjects and which appeared during the decade after his

death. Of the lectures Koenigsberger says: "(Sie) bilden jetzt die ausgezeichnetsten Lehrbücher einzelner Theile der mathematischen Physik." Nothing could be truer. And may we not in all seriousness ask the question whether these live works of a master genius may not be more inspiring and really better text-books for the student than the work of some lesser person, no matter how careful and industrious he be? Of the biography let us add that it ends with consummate inspiration in the words which Helmholtz himself applied to Goethe and Beethoven: "Wir verehren in ihnen einen Genius, einen Funken göttlicher Schöpferkraft, welcher über die Grenzen unseres verständig und selbstbewusst rechnenden Denkens hinausgeht. Und doch ist der Künstler wieder ein Mensch, wie wir, in welchem dieselben Geisteskräfte wirken wie in uns selbst, nur in ihrer eigenthümlichen Richtung reiner, geklärter, in ungestörterem Gleichgewichte, und indem wir selbst mehr oder weniger schnell und vollkommen die Sprache des Künstlers verstehen, fühlen wir, dass wir selbst Teil haben an diesen Kräften, die so Wunderbares hervorbrachten."

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PIERPONT'S THEORY OF FUNCTIONS.

The Theory of Functions of Real Variables, Volume I. By JAMES PIERPONT, Professor of Mathematics in Yale University. Boston, Ginn and Company, 1905. xii + 560 pp.

SINCE the time of Weierstrass the so-called rigorous style in mathematical writing has increased constantly in favor until in recent years it has become a commonplace instead of a rarity. Such myriads of microscopic ϵ's and δ's have penetrated our mathematical thinking that it would be impossible to rid the system of them entirely even if it were desirable to do so. A return to the externals of the intuitive style of writing would, however, enable any one individual to obtain a conception of a much wider range of investigations, and it seems possible that such a return may be made after mathematicians have come to an understanding acknowledged by all of the precise meaning