While beginning with the elements of the calculus, it carries the reader to the point where he is prepared to use original sources and extracts from ϵ-proofs the underlying thought. When the future historian inquires how the calculus appeared to the mathematicians of the close of the nineteenth century, he may safely take Professor Goursat's book as an exponent of that which is central in the calculus conceptions and methods of this age. W. F. OSGOOD.

HARVARD UNIVERSITY,
February, 1903.

SHORTER NOTICES.

Niedere Zahlentheorie. Erster Teil. By Dr. PAUL BACHMANN. B. G. Teubner's Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Band X, 1. Leipzig, 1902. x + 402 pp.

IN view of the ambitious series of volumes by Bachmann, giving a comprehensive exposition of number theory, a series not yet completed, the appearance of a new volume on the elements of the subject, quite independent of the series mentioned, will doubtless cause some surprise. When the invitation came to the author to contribute to Teubner's Sammlung a text upon the subject on which he is so eminent an authority, he hesitated long, fearing that a text on the elements of number theory ran the risk of conflicting with his Elementen. The author has attempted to avoid this conflict in two directions: first by the addition of much important material; second, by employing a method of construction different at least in essential points. The author believes that the present book, both in contents and in foundation, may well be considered as a supplementary volume to his former series. As indicating in detail parts differing essentially from the Elementen, there may be mentioned the chapter on the different euclidean algorithms, including Farey's series, the theory of binomial and general congruences, the exhaustive treatment of the known proofs by elementary number theory of the quadratic reciprocity law and the interrelations of these proofs. The theory of higher congruences is appropriately introduced, even in the Niedere Zahlentheorie, both by way of climax to the elementary parts and to afford a satisfactory insight into the means employed by Gauss in his seventh proof of the reciprocity law.

The introduction (15 pages) gives a brief history of number theory. Chapter I is a rather formal discussion of the concept number, the idea of sequence being taken for the foundation. The 38 pages of Chapter II are devoted to the divisibility of numbers. On pages 34 and 35 is given a simple proof of the existence of integers ξ and η satisfying the equation $a\xi + b\eta = \delta$, where δ is the greatest common divisor of a and b, use being made of the *modulus* of Dedekind and Kronecker. But this proof of the fundamental theorem gives no algorithm for finding ξ and η as does the proof based on Euclid's algorithm. A series of special recent theorems due to Weill, André, de Polignac, Catalan, Landau, and Liouville show that certain expressions in fractional form have integral values. The 33 pages of Chapter III are devoted to the usual theorems on residues and congruences. The 54 pages of Chapter IV are devoted to the euclidean algorithm, continued fractions, Farey's series. The 27 pages of Chapter V relate to the theorems of Fermat and Wilson and their generalization. Chapter VI, on the theory of quadratic residues, is the longest in the book, containing 138 pages; it is devoted chiefly to the various proofs of the reciprocity law, a chronological table of which appears on pages 203 and 204, with an addition on page 402. The 82 pages of the final Chapter VII gives the theory of higher congruences. The theory of primitive roots is treated at length with two distinct proofs of the existence of primitive roots modulo p, a prime. A general theory, including that of circulating decimals, is then developed. Finally follows the chief results of Galois, Schönemann, Dedekind, and Serret on higher congruences. On pages 373–375 is given Gauss's elegant and simple method of determining the number of incongruent primary irreducible functions (mod p) of degree n; following this is the usual purely arithmetic development.

The author has certainly succeeded admirably in the task he has undertaken. It will appeal especially to the student who wishes a complete account of the results and methods of the elementary parts of the theory of numbers.

L. E. DICKSON.