12. The formulas of Euler (and Rodrigues), which give rationally in terms of three parameters the nine coefficients of an orthogonal transformation in space, are available for expressing the vertices of any polar triangle of the conic

$$x_1^2 + x_2^2 + x_3^2 = 0.$$

This gives a means of discussing curves of the second and third orders, at least, which contain infinitely many inscribed polar triangles of a conic. Professor White's preliminary communication exhibited the method as applied to conics. The further question was raised, what sort of curve is the locus of points whose coördinates are the eulerian parameters of polar triangles inscribed in a single conic, or of orthogonal transformations which rotate the axes through the surface of an orthogonal quadric cone.

THOMAS F. HOLGATE,
Secretary of the Section.

EVANSTON, ILLINOIS.

SOME GROUPS IN LOGIC.

BY PROFESSOR E. W. DAVIS.

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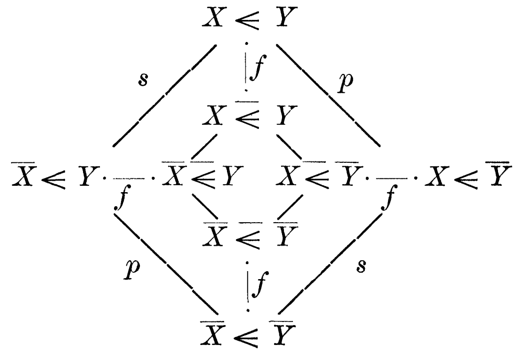
DE MORGAN has pointed out * that his eight forms of proposition identical with the A, E, I, O, and their contranominals of the older logic, can be derived from any one by the three operations of reversing the subject, reversing the predicate, denying the copula. If, in fact, we denote the operations in question by s, p, and f respectively, we have

$$Ap = E, \quad Af = O, \quad Afp = I;$$

while sp changes any proposition to its contranominal $X \prec Y$ to $Y \prec X$; or, what is the same thing to $\bar{X} \prec \bar{Y}$. Here $\prec$ is the sign of implication and the bar written over a letter or sym-

* Formal Logic, p. 63 et seq.

bol reverses or denies the meaning of the same. All the relations can be simply expressed by a diagram



This scheme differs from De Morgan's in that he assumes always the existence of both a subject and its contrary, whereas our assumption, following Mr. C. S. Peirce,* is that $X \leqslant Y$ means 'not X or else Y,' while $X \mathrel{\overline{\leqslant}} Y$ means 'X but not Y.'

There is also a group of order 32 which operating upon any one of 24 forms of valid syllogism will produce the rest, together with 8 non-valid syllogisms. To see this, start with *Barbara*

$$(X \leqslant Y)(Y \leqslant Z) \leqslant (X \leqslant Z),$$

or say

$$A_1 A_2 \leqslant A_3.$$

Let now a_1 mean the operation of performing sfp upon A_2 and A_3, with similar significations for a_2 and a_3. Then, B standing for *Barbara*, Ba_1 and Ba_2, are valid, while Ba_3 is not.† Moreover

$$G_4 = \{1, a_1, a_2, a_3\}$$

is a 4-group. Were B, instead of *Barbara*, any valid syllogism it would still be true that of Ba_1, Ba_2, and Ba_3, two would be valid while one was invalid. If B were invalid the operations a might lead to none that were valid, could at most lead to one that was.

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* *Amer. Journ. of Math.*, vol. 3, p. 18.

† Conversion of the propositions with negative copula reveals the fact that B, Ba_1, Ba_2, Ba_3, are in the four figures of the older logic.

Suppose x to be the operation of interchanging X and $\overline{X}$, with like significations for y and z. These 3 operations generate a group G_8, such that when performed upon any syllogism B the resulting 8 syllogisms are valid or invalid according as B is.

The group is commutative with G_4 and the product of the two is the G_{32} referred to above.

Let s_i, p_i, f_i be the operations of performing s, p, f upon A_i. It is evident that

$$x = s_1 s_3, \quad y = p_1 s_2, \quad z = p_2 p_3, \quad a_1 = s_2 p_2 f_2 s_3 p_3 f_3.$$

These 9 operations generate a group G_{512}, the product of G_{32} by the group

$$G_{16} = \{x_1 s_1,\ p_1,\ p_2,\ f_1\}.$$

The new syllogisms got by operating with this upon the 32 syllogisms derived from *Barbara* above are all invalid.

The operation called conversion interchanges the subject and predicate of a proposition, at the same time reversing each. The meaning is, of course, left unchanged. Immaterial to the argument likewise is it whether the order of the propositions be $A_1 A_2$ or $A_2 A_1$. Let the operation of converting A_i be denoted by c_i while the interchange of A_1 and A_2 is denoted by t. Then $\{c_1, c_2, c_3, t\}$ is of order 16. We call it H_{16}. Finally

$$G_{512} \times H_{16} = G_{8192},$$

whereof

$$G_{32} \times H_{16} = H_{512}$$

contains the operations which performed upon *Barbara* lead to all the valid syllogisms of BG_{8192} taking account of conversion and the order of the premises, viz., to 384.

The same sort of group building can of course be applied to more complicated sets of statements, and is a very simple consequence of the binary character of deductive logic.

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