(used to indicate determinants) have been misplaced. In the lists of reduced forms some small errors have caught my eye;* on p. 142 (footnote) in reference to Frobenius's paper in *Crelle*, vol. 86, we should read p. 146 for p. 20. Near the foot of p. 166, $(S+T)^{-1}$ $(S+T)$ should be $(S+T)^{-1}$ $(S-T)$; and in some places there are slight errors in the titles of papers quoted.

In conclusion I may say that Dr. Muth's book is of great interest and very useful in extending one's knowledge of certain branches of the subject. I hope that it may induce other readers to take up this part of invariant theory, which is important on account of its applications as well as for its intrinsic interest.

T. J. I'A. BROMWICH.

ST. JOHN'S COLLEGE, CAMBRIDGE,
January 1, 1901.

SHORTER NOTICES.

Kurzgefasste Vorlesungen über verschiedene Gebiete der höheren Mathematik, mit Berüchsichtigung der Anwendungen. Von DR. ROBERT FRICKE. Large 8vo. Pp. ix + 520. Leipzig, B. G. Teubner, 1900.

Two objects are sought in Dr. Fricke's timely book: first, to supply a defect in German mathematical literature, a handbook for students who have mastered the elements of analysis and are not yet qualified to read profitably the highly specialized treatises; second, to smooth the way for technical students who discover a taste for the more abstract branches of mathematics. The present volume is confined to analysis and theory of functions, a second is announced as in preparation, to treat of advanced portions of algebra and geometry.

The reader is presumed to have a pretty thorough acquaintance with integral calculus, though not with the calculus of imaginaries. Fourier's series are first introduced, with applications to vibrating strings and to diffusion of heat. A short chapter is given to spherical and cylindrical harmonics, with tables for the functions $P_1(\mu)$, $P_2(\mu)$, $-P_6(\mu)$ according to Byerly, and of $J_0(\vartheta)$ and $J_1(\vartheta)$

* The second form in each of the following needs correction: p. 91 (c. 3); p. 117 (5) and (8). The last of these has an x instead of a y; in each of the other two a suffix has been misprinted.

according to Lommel. The remaining chapters are based upon the third. Functions of a Complex Variable, and it is in this narrow space of 100 pages that the skill of the author in concise and transparent exposition is most strikingly evident.

In attractive, inductive sequence, mainly in geometric style, are presented linear functions and inversion with respect to a circle, stereographic projection, typical rational functions, exponential and hyperbolic functions, then the general concept of analytic functions of the complex variable. The style is that of the lecturer rather than that of the critic, and the requirements of rigor are met by making definitions and hypotheses amply inclusive. At once the motion of an incompressible fluid is taken up briefly, and the question of a function whose integral $\int f(x, y)ds$ from point to point of a domain shall be independent of the path s. Thus are brought in the potential equation, Cauchy's integral, and Green's theorem, while the notion of analytic function is still in the formative stage in the student's mind. Power series, convergence, irregular points, entire functions developed in products, and the gamma function are rapidly sketched, the chapter closing with a well illustrated and suggestive paragraph on Riemann surfaces and the problems arising from them.

Elliptic functions are treated first by the Weierstrassian method, then conversely after Lagrange and Jacobi, as based upon integrals and theta functions. Here, as earlier for spherical harmonics, tables are given: for the elliptic integrals of the first and second kinds and their periods, and for log q as a function of k. These tables are from Lévy's Précis élémentaire etc. Numerous applications of elliptic functions fill 60 pages, among which the short paragraph on confocal quadric surfaces and elliptic coördinates is the one that will be most useful to teachers.

Two concluding chapters are devoted to ordinary and partial differential equations in complex variables, the method and applications being principally drawn from Fuchs, Klein, Schwarz, and Jacobi. These chapters, though condensed, are not crowded, but are throughout readable. Most teachers to whom the purpose indicated by Dr. Fricke appeals as eminently desirable will find this book full of helpful suggestions, and will moreover find their own interest in the successive topics vigorously stimulated by the occasional reading of a chapter. The second volume will be awaited with not a little of curiosity.

Two extracts from the preface are deserving of consideration in this country at least. "— Ich darf wohl auf Zustimmung hoffen, wenn ich die Meinung äussere, dass zu einer tiefer gehenden Verständniss der Mechanik sowohl in ihren Grundproblemen als in ihren Einzelausführungen verschiedene mathematische Disciplinen erforderlich sind, die zur Zeit leider eine regelmässige Pflege an den technischen Hochschulen nicht finden. * * * In Amerika und England sehen wir vielfach ein Unterrichtssystem in Kraft, bei welchem mit relativ geringfügigen mathematischen Hilfsmitteln höchst ausgedehnte Gebiete der Anwendungen berührt werden. Der richtige Weg liegt auch hier gewiss in der Mitte. Um ihn mit Erfolg zu betreten, wird die Mathematik mit den technischen Wissenschaften unter gegenseitiger bereitwilliger Förderung Hand in Hand gehen müssen."

HENRY S. WHITE.

Ebene Geometrie der Lage. Von PROFESSOR DR. R. BÖGER. Leipzig, G. J. Göschen'sche Verlagsbuchhandlung, 1900. (Sammlung Schubert, VII.) 8vo. Pp. x + 289. Price 5 marks.

A NEW text book on the geometry of position is prepared by Dr. Böger, who is Professor at the Real-gymnasium in Hamburg. The book has some novelties, and distinctive merits. In the first place it is well arranged for reference. Section title and number and paragraph number stand at the head of alternate pages. Paragraph numbers appear black in the text, and are set also in smaller type in the outer margin. Definition, theorem, and corollary are distinctly marked as such. Thus the frequent references back and ahead are consulted with the least possible labor, a rapid review is easy, and the student is never left in doubt as to purpose or connection of a sentence. The table of contents is full and well divided, but there is no index. This lack however will not greatly hinder the student, for at every critical point a line of back references is struck which leads to every desired explanation.

As its title indicates, the work is confined to the plane. Beginning with the perspective relation, it concludes with the construction of a polar relation out of any five real or imaginary elementary data (pairs of conjugate points). Steiner and Cremona defined projective ranges to be any two members of a sequence, each member of which is in perspective with the next preceding and the next succeeding. This definition Böger adopts, and so proves as a theorem