

tiation and integration with fractional indices, 11° transformations of series into definite integrals and reciprocally, 12° expression of the sums of certain general series as definite integrals, 13° integration of equations, 14° determination of a particular integral of every linear differential, difference, or partial differential equation, having constant coefficients and second member, 15° inverse calculus of definite integrals, 16° integration of linear differential or difference equations with constant coefficients, 17° integration of linear partial differential or difference equations with constant coefficients, 18° integration of certain partial differential equations with variable coefficients, 19° integration of equations of mixed differences, 20° integration of simultaneous equations, 21° integration of certain equations which can be transformed into linear equations.

One of the principal advantages which the author claims for the method is the application which can be made of it to the integration of differential equations. Its rôle in higher analysis he likens to that of logarithms in numerical reckoning. It makes the integration of equations, difference and partial, single or simultaneous, depend on the solution of algebraic equations; moreover, it permits of determining in general the maximum to the number of arbitrary functions entering an integral in order that it be complete, and in particular cases the process can determine the exact number of these arbitrary functions.

The general process employed by this calculus for the integration of equations consists in representing the known and unknown functions under the generalized forms, substituting these in the equations, and deducing the integrals by the aid of generalization.

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#### SHORTER NOTICES.

*Jacob Steiner's Vorlesungen über synthetische Geometrie. Zweiter Theil: Die Theorie der Kegelschnitte gestützt auf projective Eigenschaften.* Auf Grund von Universitätsvorträgen und mit Benutzung hinterlassener Manuscripte Jacob Steiner's. Bearbeitet von HEINRICH SCHRÖTER. Dritte Auflage, durchgesehen von RUDOLF STURM. Teubner, Leipzig, 1898. xvii + 537 pp. Price, 14 marks.

THE fact that this new edition of Steiner's lectures, edited and published by Schröter in 1866, has been prepared by

Professor Sturm at the request of the publishers, shows that the demand which called for a second edition in 1876 has not been satisfied by the appearance of more modern books on the subject. It is a curious coincidence that Reye's "*Geometrie der Lage*" also made its first appearance in 1866 and went into a new edition in 1876.

To those not familiar with the scope of the work a short sketch of the contents may be of interest. It was Steiner's intention to publish his investigations in synthetic geometry in five parts, and the first, the important work: "*Systematische Entwicklung der Abhängigkeit geometrischer Gestalten von einander*" was published in 1832. The remaining four parts never appeared, but much of the subject matter that was to have formed the fifth part was included in a course of lectures which Steiner delivered several times at the University of Berlin. In 1852 Schröter attended a course of lectures by Steiner "*Ueber die neueren Methoden der synthetischen Geometrie*" and the notes he took of these lectures supplemented by manuscript notes left by Steiner form the basis of the present volume. In the "*Systematische Entwicklung*" (1832) Steiner, to avoid being thought too radical, had followed the old method, considering a conic as a section of a right circular cone, although, as he says in 1836, he fully felt that this method was unsuitable for his purpose; in his later unwritten work, part 5, he intended to recapitulate briefly the properties of projective ranges and pencils and from these to deduce his definition of the conic and the theories of involution and reciprocal polars. This plan Schröter has carried out in the first two sections of the work under discussion, following Steiner's arrangement almost exactly. Steiner had come to the conclusion that the use of trigonometry might be avoided in the development of the projective properties of lines and pencils and that this new geometry might, with great advantage, be incorporated with the most elementary geometry. Trigonometrical expressions are used, therefore, only in so far as they are needed for the further development of the subject, but the use of metrical properties is not avoided nor is an endeavor made to build up a logical structure on the ground work of fundamental geometrical axioms. Parallel lines and circles, are introduced without definition, algebra and trigonometry are used freely, the principle of duality is adopted without discussion, and the axes, area, curvature, and other metrical properties of conics are investigated.

The third section, of 178 pages, is devoted to an elemen-

tary treatment of pencils and ranges of conics ; the different kinds of conics are discriminated for cases of real and imaginary points and the pencil and range are investigated separately, little use being made of the principle of duality.

In the fourth section, of 112 pages, the polar system of the plane and the net of conics are discussed in some detail.

The book, as a text book, has the defect so often found in books compiled from lecture notes—namely, a lack of continuity in notation. This is especially noticeable in the treatment of fundamental points, which are denoted by four or five different sets of letters in as many articles. Professor Sturm has improved the edition greatly in this respect and has modernized the terminology. There are one or two misprints, for instance on page 132, and there is some slight confusion in the proof given on page 60, but the edition seems on the whole remarkably free from errors.

ISABEL MADDISON.

*Compositions d'Analyse, Cinématique, Mécanique et Astronomie, données depuis 1889 à la Sorbonne pour la licence ès sciences mathématiques.* Par E. VILLIÉ. Paris, Gauthier-Villars, 1898. 8vo, x + 299 pp.

AN American reader will probably peruse this volume more for the insight it gives as to the standard required from French students just finishing their courses in mathematics, than for the matter it contains. The questions are those set in the final examinations for students who have finished the training which corresponds to a college course here, and who expect to go out immediately and teach. The candidate who can successfully grapple with such problems in a limited time must have had a good preparation and be well equipped, as far as knowledge goes, for the profession of teaching mathematics.

M. Villié has previously published two similar volumes of worked-out problems ; the present one contains the solution of questions set since 1889. To American and English students, these volumes will be perhaps superfluous, our text books generally doing what is needed in that direction ; in France, so far as may be gathered from their mathematical publications, this part is left to the instructor and the volume before us doubtless serves a very useful purpose. In analysis we have chapters containing solved problems in integration, differential equations, orthogonal