SHORTER NOTICES.

Corso de Analyse Infinitesimal. F. GOMES TEIXEIRA. 3 vols., 8vo. Porto, 1892–96.

WHILE perusing the present book it was a constant source of regret to me that Portuguese is not better known in our country. Otherwise this admirable work on the calculus would enjoy widespread popularity among us. Its author, the distinguished director of the Academia Polytechnica at Porto, has been uniformly successful in the difficult task of selecting from the immense material available. The manner of presentation leaves nothing to be desired. The style is lucid and elegant, and the whole work bears in a refreshing manner the imprint of an original mind. In many places the author has incorporated parts of his own prolific and valuable writing on the subject. In regard to rigor, it seems to us that Professor Teixeira has very happily chosen the golden mean. The excessive rigor of a Weierstrassian has been wisely avoided; at the same time the author has given this matter due attention. An occasional slip will doubtless be corrected in later editions. Altogether the work has so favorably impressed us that we should prefer to see it translated into English rather than any other work on the subject we know of. It is a deplorable confession that the English language does not to-day possess a work on the calculus of this class. We indicate very briefly its contents.

Differential Calculus, pp. 383.—The first hundred pages are devoted to a careful exposition of the fundamental notions and processes of analysis; numbers, limits, infinite series and products, continued fractions, continuity. At page 112 the differential calculus proper begins. We touch only on one or two matters. Th esubject of implicit functions, ordinarily treated so carelessly, is given here with rigor and elegance. In this connection the elementary properties of functional determinants are given. The exact form of higher derivatives is given with more than usual fullness. Application is made to Bernoulli's numbers and formulæ of Waring. The treatment of maxima and minima of functions of two variables is incorrect. The error which the writer of these lines criticised in the BULLETIN of July, 1898, has also been made in this place. After the usual application of Taylor's series to geometry follows an interesting chapter on functions defined by series.

Hankel's principal of condensation of singularities is given, and functions without derivatives are illustrated by Weierstrass's classic example. The volume closes with a remarkably rich chapter on functions of a complex variable. Weierstrass's and Mittag-Leffler's theorems are given, and existence of lacunary functions illustrated.

Integral Calculus, Part I., pp. 339.—The first half of the volume is devoted to the usual topics. We notice as praiseworthy in particular the author's treatment of the reduction of hyperelliptic integrals, the theorems of Green, Cauchy, and Stokes. Among the applications of definite integrals we mention Gauss's proof of the fundamental theorem of algebra, Sturm's theorem, and Fourier's series.

The second half of this volume contains a good elementary account of differential equations. Without giving the reader the disagreeable feeling that unduly much has been forcibly compressed into a very small space, a sufficient orientation is given not only in ordinary but also in partial differential equations of the first and second orders.

Integral Calculus, Part II., pp. 348.—The first eighty pages treat of Cauchy's theory of integrals between complex limits with application to the calculation of definite integrals and to the development of functions into power series, into series of rational functions, into Fourier's series, and into series according to the powers of sine and cosine.

About 125 pages are devoted to the elliptic functions. The starting point is the definite integral, first for real, then for imaginary, and finally by virtue of the addition theorem for complex argument. From a second point of view they are defined as infinite series in terms of their poles. The development of the sigmas in Fourier's series makes it necessary to multiply them by exponential factors. This leads very naturally to the thetas and the relations existing between these two classes of functions. Instead now of studying the elliptic functions of Jacobi by means of the thetas, the author prefers to do this through $\wp$ and σ (by means of the integral definition). The reader gains a good idea of the way to use the elliptic functions by well chosen applications to geometry. A chapter on many valued functions and a sketch of the elements of calculus of variations terminate this excellent work.

JAMES PIERPONT.