drawing the curve $y = \tan \frac{x}{2}$ and rolling the paper into a cylinder of radius 1, so that the asymptotes coincide.]

(10) The characteristics of all planes which pass through a point are at any instant secants of a circular cubic space curve.

(11) All points of Σ which, in three positions, lie in a line of Σ' form a cubic space curve through the points at infinity on the 3 axes of displacement.

(12) All points of Σ which in 4 positions lie in a plane of Σ' form a cubic surface.

(13) All points of Σ which in 5 positions lie in a plane form a sextic space curve.

(14) There are in general 10 points which in 6 positions lie in a plane.

(15) The points which in 5 positions lie on a sphere form a quartic surface. The centres of the spheres form a quartic in Σ'. In the inverse movement the quartics interchange their rôles.

(16) The points which in 6 positions lie on a sphere form a space curve of order 10.

(17) There are in general 20 points which in 7 positions lie on a sphere.

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THE NEW EDITION OF WEBER'S ALGEBRA.

Lehrbuch der Algebra. By HEINRICH WEBER. Second edition. Braunschweig, Vieweg und Sohn, 1898–99. 8vo. Vol. I, pp. 703. Vol. II, pp. 855.

MOST of the readers of the BULLETIN have been aware that a new edition of Weber's Algebra was in progress. Some time ago the first volume appeared; the work is now complete, the second volume of the new edition having just come out. The many excellencies of this great work have been so generally appreciated that Professor Weber has experienced the very unusual pleasure of seeing a new edition required in less than two years after the publication of the first.

The first question that those who already have purchased the first edition will wish answered is: Are the changes so great as to make it desirable to secure the new edition? To this we would reply : University and seminary libraries and all those who are especially interested in algebraical speculations should not fail to have the new edition. For such it will be comforting to know that the price of the new edition is reduced by almost half. On the other hand, we do not consider it necessary for those who have only an occasional or incidental interest in algebra to put themselves to the expense of purchasing new copies.

We turn now to indicate rapidly some of the more important changes. The first thing that strikes the reader is that the work has increased considerably in size, in spite of the manifest effort of its author to keep it within bounds. The first volume has increased 50 pages, the second 61 pages, so that the work has now 1558 pages. The task of mastering such bulky volumes may well at first intimidate the prospective reader. He will, however, generally be interested in only some of the many subjects treated ; the remainder will form a convenient store for reference. The size is, thus, no particular objection, since the work is so arranged that it is not necessary to read it consecutively.

The next feature that one observes in comparing carefully the two editions is the great number of slight changes that have been made. Certain paragraphs have been rewritten; others left out or transposed ; still others are wholly new. The whole work has been subjected to careful scrutiny and no pains have been spared to make the style clearer and the demonstrations simpler or more rigorous. The cumulative effect of all this is to enhance the value of the work very materially.

Let us illustrate this by looking over chapter XVI on cyclotomy. It opens by discussing the irreducibility of the polynomials X_n. In the old edition this was considered in chapter XII when treating of the roots of unity. The form of the proof is changed ; in the old edition a proof of Kronecker was followed, here one by Dedekind is given. In discussing the sums of Gauss Σr^a, Σr^b, the new edition follows Kronecker, while the old followed Gauss. Finally a new paragraph on the polynomials

$$\prod_a (x - r^a), \qquad \prod_b (x - r^b)$$

is inserted. Such changes run more or less throughout the book.

Let us note now some of the changes of greater extent. In volume I these are limited to two chapters, the fourth and the eighth. In chapter IV the theory of elimination has been treated with much more detail and this is a most welcome addition. The additions of chapter VIII incorporate some of Hurwitz's researches relating to Sturm's theorem.

In volume II the first important addition we meet is an account of Dedekind's researches on Hamiltonian groups in chapter IV. Chapter VI has been completely remodeled; the part relating to Klein's Formenproblem and the invariants of linear substitution groups has been thrown into a new chapter and in its place we find the first properties of matrices given. The theory of matrices which was not mentioned in the old edition has been inserted in order to give an account of Frobenius's remarkable papers on groups which have appeared recently in the *Sitzungsberichte of the Berlin Academy.* These treat of the characters and determinants of finite groups and their representation by linear substitutions. The fourth book, which treats of algebraic numbers, has been altered more than any other part of the whole work; still its general character has been maintained. We call attention to a number of points: First, the insertion of Kronecker's method of breaking up a form into its irreducible factors for a given domain. A more extensive use of Minkowski's ideas relating to the geometry of numbers has been made. In the present edition a special chapter is devoted to them. The chapter on the relation of a body to its divisors gives a somewhat fuller account of Hilbert's researches. Finally, a remarkable application due to Dedekind of the theory of the Classenzahl is made to prove Legendre's postulate that every arithmetic progression

$$ax + b$$

where a and b are integers and relative primes contains an indefinite number of primes.

We rejoice to learn, by remarks made in two places, that the author still contemplates writing a third volume, which is destined to treat of the application of the general theory of algebraic numbers to the complex multiplication of elliptic functions. We are sure we express the unanimous sentiment of the whole mathematical community in hoping that Professor Weber will resolutely persevere in this arduous undertaking.

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YALE UNIVERSITY,
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