Orthographic, stereographic, and allied projections of the spherical surface form the subjects of Chapter XII, and the next three deal with the principles of shade, shadow, and perspective.

While it is not written expressly for mathematicians, no student of geometry, especially of surfaces, can review this book without profit. In the main it is firmly grounded on mathematical principles accompanied with illustrations of the best practice of modern draftsmen.

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PASCAL'S REPERTORIUM OF HIGHER MATHEMATICS.

Repertorio di matematiche superiori. I. Analisi. Per ERNESTO PASCAL, Professor ordinario nella R. Università di Pavia. Milan, U. Hoepli, 1898. 16mo, xv + 643 pp.

THAT the increasing need of encyclopædic literature in mathematics is being promptly met is indicated by the appearance during the last decade of Carr's synopsis, Láska's compendium, the first two volumes of Hagen's synopsis, and most recently the first volume of Pascal's repertorium and the first part of Burkhardt and Meyer's encyclopædia. The body of doctrine of mathematics is expanding at such a rapid rate that the difficulty of securing a liberal education in the science is becoming wellnigh insurmountable, a difficulty that is obvious on contemplating the marvelous development centering about the notions function and group, to cite particular examples. In no subject is special specialization growing more imperative than in mathematics ; in the midst of difficulty and demand the student should hail with delight the valuable services of a work so admirably adapted to purposes of orientation as Professor Pascal's repertorium promises to be.

The author's plan, adhered to without deviation, is to present with regard to each theory of modern mathematics the fundamental definitions and notions, the characteristic necessary theorems and formulæ, and citations to the principal works of its bibliography. The definitions are clear and unequivocal; the statements of the theorems, always given without demonstration, are concise and unambiguous; and the bibliographical references* are sufficiently full to supply the needs of the general mathematical reader.

The subject matter of the first volume, devoted to analysis as announced in the title, is arranged in chapters in the following order: 1° introductory theory of higher algebra, thirty-six pages; 2° theory of substitution groups, sixteen pages; 3° theory of determinants, twenty pages; 4° theory of series, infinite products, and continued fractions,* twenty-six pages; 5° theory of algebraic equations, thirty-two pages; 6° differential calculus, twenty-six pages; 7° integral calculus, thirty-four pages; 8° differential equations, thirty-eight pages; 9° theory of transformation groups, thirteen pages; 10° calculus of finite differences, fifteen pages; 11° calculus of variations, eighteen pages; 12° theory of invariants, forty-three pages; 13° functions of complex variables, thirty pages; 14° the theory of functions in relation to the theory of groups; periodicity and automorphism, twenty-six pages; 15° algebraic functions and abelian integrals, thirty pages; 16° theory of elliptic functions, fifty-four pages; 17° hyperelliptic and abelian functions, nineteen pages; 18° special functions, fifty-eight pages; 19° analytical representation of functions, fourteen pages; 20° theory of integral numbers, rational or complex, forty-three pages; 21° theory of algebraic and transcendental numbers, thirteen pages; 22° calculus of probabilities, thirty-two pages; 23° analytical instruments and apparatus, sixteen pages.

There is space to specify the contents of no more than a few of the chapters. Those commented upon have been selected either because their subjects, as stated in the paragraph above, give but a vague idea of their material or because their presence calls for special notice. The introductory theories of the first chapter comprise those of irrational numbers, complex numbers, quaternions, groups of points, aggregates and ensembles, general concept function, rational

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* The extent of the bibliographical notes may be seen by observing that, in the chapter on differential equations, page 192, the existence theorem is accompanied by references relative to its demonstration to Moigno's differential calculus, vol. 2 (1844), for Cauchy's proof; to Briot and Bouquet, *Jour. de l'Éc. Poly.*, vol. 36; to Lipschitz, *Ann. di mat.*, vol. 2, *Darboux's Bulletin*, vol. 10; to Volterra, *Battaglini's Journal*, vol. 19; to Peano, *Acc. Torino* (1886), *Math. Annalen*, vol. 37; to Arzelà, *Acc. Bologna* (1896).

* To the list of corrections given on page 643 should be added that in the head-lines of pages 93 and 94 "funzione" should be replaced by "frazione."

integral functions of one variable, limits, superior and inferior limits of the values of a function, continuous and discontinuous functions, combinations, and binomial coefficients.

In passing we remark as noteworthy features the sections on functions appertaining to substitution groups, and on the analytical representation of substitutions in the second chapter; the collection of special determinants including Pfaffians, Wronskians, Jacobians, and Hessians of the third chapter; the convergence criteria and particular series of the fourth chapter; the résumé of Galois's theory in the chapter on the theory of equations; the discussion of Rolle's theorem and of the Taylor-Maclaurin formula, together with the tables of integrals, in the chapters on the infinitesimal calculus. The chapter on differential eqnations is well written, but the difficulties of the theory of partial differential equations have severely taxed the author's powers of exposition and condensation.

The recent numbers of the Italian journals give evidence of the favorable reception and productivity of Lie's theories at the hands of the Italian mathematicians, and it is equally refreshing to find a chapter of this book devoted to Lie's work, even though the chapter be compressed to thirteen pages. The author considers in turn groups of point transformations, finite and differential invariants of groups of one parameter,* contact transformations, and differential invariants and parameters. Relative to the theory of differential invariants and parameters both categories of research are presented, namely those which spring directly from Lie's theory of groups, and those whose origin is in the theory of differential forms. The bibliographical references include Lie's latest work on the geometry of contact transformations.

The interpolating function and approximate quadratures are adequately treated in the chapter on finite differences; among the interesting details of the eleventh chapter is a a collection of the historic problems of the calculus of variations. The chapter on invariants, one of the most complete in the book, recapitulates the theory in seven sections: binary forms, symbolic representation; invariants and covariants; Clebsch-Gordan formula; vocabulary of the various terms employed in the theory of forms; complete systems of invariant forms; typical representations of binary forms, associated forms; canonical representation of forms.

* The term "*r*-gliedrige" of Lie's terminology, which usually appears in English and French as "*r* parameter" or "of *r* parameters," Pascal translates "di classe *r*."

The thirteenth chapter presents the elements of the theory of functions of a complex variable in a most logical manner and the discussion of the definition of an analytical function is one of the most useful digressions in the book. It was a happy thought of the author's that led him to include a chapter on the theory of functions related to the theory of groups in which he has incorporated linear substitutions, groups of linear substitutions, anharmonic groups, polyedral functions and groups, periodic functions, modular functions, and the functions of Fuchs and Klein. The chapter on algebraic functions and abelian integrals, after generalities on algebraic functions and Riemann surfaces, presents the properties of abelian functions of the first, second, and third species and concludes with Abel's theorem. The ϑ function of Jacobi, elliptic functions of Legendre, the σ and p functions of Weierstrass, and rational functions of p and p' constitute the material of the chapter on elliptic functions which terminates with the theory of the transformation and the multiplication of these functions. In addition to the theorem of inversion of Jacobi and the general properties of abelian functions, the chapter on hyperelliptic functions has sections on the ϑ functions and their properties, the ϑ functions whose arguments are abelian functions of the first species, and the σ of Klein in the hyperelliptic case. Under the heading special functions the author has constructed the largest chapter of the book; it includes the discussion of the exponential, logarithmic, circular, hyperbolic, and hypergeometric functions and the functions of Euler, Bernouilli, Legendre, Laplace, Bessel, and Lamé. The series of Wronski, of Lagrange, and of Fourier, together with developments of functions in series of functions of Legendre, Laplace, and Bessel, make up the chapter on the analytical representation of functions which is the last chapter devoted to the theory of functions.

The twentieth and twenty-first chapters have to do with the theory of numbers. The author's arrangement is as follows: Divisibility of rational integral numbers, prime numbers, the numerical function $E(x)$, generalities upon congruences, congruences of the first degree, congruences of the second degree, quadratic residues, binominal congruences, residues of the third and higher orders, exponential congruences, primitive roots and indices, quadratic forms, representation of numbers by means of forms, integral complex numbers of Gauss, integral cubical complex numbers, generalities on algebraic numbers, divisibility of algebraic numbers, ideal numbers of Kummer, transcendental numbers, the number π (the number e is treated in connection with the exponential function in the eighteenth chapter).

The usual elements of the theory of probabilities and its relation to the theory of errors are the principal subjects of the twenty-second chapter. The last chapter consists of three sections occupied respectively with the mechanical instruments of arithmetic, of algebra, and of the integral calculus; among the last are Lord Kelvin's harmonic analyzer and the integraph of Abdank-Abacanowicz.

The author undoubtedly has had many perplexing problems to settle relative to the admission or rejection of numerous details that suggested themselves, but the reader is occasionally at a loss to explain omissions among which may be mentioned for example that of Horner's method of approximation of the real roots of an algebraic equation, that of examples of derivativeless functions, and that of the theory of integral invariants which might have been introduced in its proper place as a corollary of Lie's theory of differential invariants or presented under Poincaré's treatment. The absence of an analytical index of notions and names is perhaps to be accounted for by limits previously set on the size of the book; it is to be regretted that the publisher did not use for this purpose the part taken up by a catalogue the majority of whose entries possess only a very secondary interest to the student of mathematics.

Relative to this modest volume it is observed finally that it is an unusual piece of work as a product of the bookmaker's art. To be sure the margins are narrow, but the type is large and clear, the pages are not wide enough to tire the eye, and, although there are more than seven hundred pages (a catalogue of the Hoepli manuals is appended) in substantial cloth binding, the perusal of the book is accompanied by no more muscular fatigue than attends that of an ordinary paper bound volume of half the number of pages. These mechanical points will contribute in no small degree to the usefulness of the book and they are worthy of consideration by those who are contemplating the making of mathematical text books.

The companion volume on geometry will be awaited with impatience by all who have availed themselves of the volume on analysis. Its early appearance is promised in a preliminary announcement which proposes the following list of chapters: 1° geometry of the fundamental forms; 2° conics; 3° quadrics; 4° general theory of curves and surfaces; 5° plane cubics; 6° plane quartics; 7° space cubics; 8° space quartics; 9° surfaces of the third order; 10° surfaces of the fourth order ; 11° geometry of the straight line ; 12° ruled surfaces, complexes, and congruences ; 13° differential geometry ; 14° non-euclidean geometry ; 15° geometry of space of n dimensions ; 16° kinematical geometry ; 17° theory of connexes.

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D'OCAGNE'S DESCRIPTIVE AND INFINITESIMAL GEOMETRY.

Cours de Géométrie descriptive et de Géométrie infinitésimale. Par MAURICE D'OCAGNE, Ingénieur des Ponts et Chaussées, Professeur à l'École des Ponts et Chaussées, Répétiteur à l'École Polytechnique. Paris, Gauthier-Villars et Fils, 1896. 8vo, xi+428 pp.

THIS work is an expansion of the course in pure geometry given by the author at the École des Ponts et Chaussées. It proposes to give an exposition of all the geometrical notions which are of interest to engineers, the material not required in the course appearing in small print. As the title indicates, the construction of the book is unique. The author deems it necessary to separate completely that which has to do with the representation of geometrical bodies from that which treats of their intrinsic properties. Accordingly his course is divided into two parts represented in the work by the two distinct divisions : *Géométrie descriptive* and *Géométrie infinitésimale.* The author believes also that the exposition of general doctrines should precede that of the details of a subject and prefaces each chapter with a body of essential principles before examining any particular case ; thus for example, in the theory of surfaces he presents an ensemble of general properties before studying surfaces of a special nature, such as the *surfaces gauches* ; this is contrary to the custom prevailing in similar courses.

1. The first part (two hundred and forty-six pages) of the work includes the first four chapters. In the first chapter, *Projections cotées*, we have the usual details relative to the representation of the ordinary relations between right lines and planes, together with certain problems concerning the round bodies, and the theory of topographical surfaces and profiles.