in the group of isomorphisms of the Hamiltonian group. When $a = 3$ this reduces to the case which has just been considered.

We proceed to consider the group of isomorphisms of a group G which is the direct product of a series of subgroups G_1, G_2, G_3, $\cdots$, G_a such that each of these subgroups corresponds to itself in every simple isomorphism of G to itself. We may suppose that each operator of these subgroups with the exception of unity is represented by a particular letter. Each operator of G will then be represented by a certain combination of these letters. Since each of the given subgroups can be made simply isomorphic to itself in every possible manner without affecting the isomorphism of the other subgroups, the group of isomorphisms of G may be represented as the product of the substitution groups corresponding to the simple isomorphisms of each of these subgroups to itself. Hence

Theorem III. *If a group is a direct product of a series of subgroups such that each of these subgroups corresponds to itself in every possible simple isomorphism of the group to itself then the group of isomorphisms of this group is the direct product of the groups of isomorphisms of the given subgroups.*

Corollary. The group of isomorphisms of a Hamiltonian group is the direct product of the groups of isomorphisms of its subgroups of orders 2^a, $p_1^{a_1}$, $p_2^{a_2}$, $\cdots$.

CHICAGO, *August*, 1898.

GALOIS'S COLLECTED WORKS.

Oeuvres mathématiques d'Évariste Galois; publiées sous les auspices de la Société Mathématique de France, avec une introduction par M. ÉMILE PICARD. Paris, Gauthier-Villars et Fils, 1897. 8vo, x + 63 pp.

MATHEMATICIANS will welcome this new edition of Galois's works published under the auspices of the French Mathematical Society. They were originally collected and published in 1846 by Liouville in the *Journal de Mathématiques.* The present edition is a reprint of the first. It is accompanied by a portrait* of Galois and an introduction by Picard.

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* A note stating that this portrait was made *d'après nature* when Galois was fifteen or sixteen years of age should have been added. It was in the possession of the family when discovered by Dupuy.

Readers of this BULLETIN† are already familiar with the tragic destiny of Évariste Galois. A malevolent combination of extraordinary circumstances gave him neither time to mature his theories, nor leisure to write down the first fruits of his profound meditations. Only fragments have come down to us; but these suffice to place their author among the few great inventors of all times. In the present edition his writings occupy sixty-one pages and embrace eight papers. The first five of these, occupying fifteen pages, were published during his lifetime; they may be grouped into two classes of marked difference. In the first class we place the following three memoirs:

I. "Démonstration d'un théorème sur les fractions continues périodiques."

II. "Notes sur quelques points d'analyse."

III. "Note sur la résolution des équations numériques."

They are mere notes and have no particular scientific value. Historically they are of interest as they show us not only that Galois at a very early age assimilated the works he read but also that they inspired him to original efforts. We proceed to indicate in a few words their contents. Memoir I has reference to the development of the roots of an equation in continued fractions. Memoir II contains two short notes; one deals with the radius of curvature of curves in space, the other strangely enough is an attempted proof that every continuous function has in general a derivative. While this demonstration is of course false, it is interesting as an example of Galois's bent to deal with the foundations of a subject. At the same time we must not forget that the first mathematicians of the age entertained no doubt as to the truth of the theorem and that some years before Ampère had made an elaborate attempt in the same direction. Memoir III is an improvement of an algorithm of Legendre's for the solution of numerical equations.

In the second class we place the two remaining memoirs. These are papers which deal with the two inventions indissolubly connected with his name, viz.:

IV. "Analyse d'un mémoire sur la résolution algébrique des équations."

V. "Sur la théorie des nombres."

The first of these is a mere statement of some of the remarkable results concerning primitive equations and the equations of transformation in the theory of elliptic functions which he had obtained by the aid of his theory. As

† 2 Ser., vol. 4 (April, 1898), p. 332.

this paper was published in the April number of the *Bulletin des Sciences mathématiques*, 1830, it shows that Galois, then only 18 years of age, had already the essential principles of his theory of equations well in hand. This paper is remarkable as being the only one he himself published on this subject. It occupies a page and a half in the present edition. Memoir V sketches the theory of imaginaries introduced by him and named after their illustrious inventor. The theory is rapidly and concisely sketched; still, it is more readable than any other of his important papers. It closes with an application to the group of equations of degree p^ν soluble by radicals. The few lines he devotes to this he thinks sufficient, so that "les personnes habituées à la théorie des équations le verront sans peine." Curious delusion; how many of his misfortunes are directly traceable to this source!

We turn, now, to the posthumous papers, three in number. The first of these is the celebrated Lettre à Auguste Chevalier, written the night before the duel which was to terminate his career. It gives in a few pages a rapid account of his meditations on the solution of equations and on the new theory of ultraelliptic integrals founded by Abel. The two other papers are a "Mémoire sur les conditions de résolubilité des équations par radicaux" and "Des équations primitives qui sont solubles par radicaux." The first of these two had nearly acquired a final form at the time of Galois's death; the second is a fragment.

These two papers are the source of our knowledge of Galois's theories relative to the solution of equations. They occupy twenty-eight pages. Only eight pages are devoted to explain such fundamental ideas as reducibility, domain of rationality, adjunction of new irrationalities, substitution groups, the resolvent function of $n!$ values and the corresponding resolvent, its reduction for a given domain of rationality, and the resulting resolvent, the group belonging to the equation, and its reduction on adjoining the roots of auxiliary equations. The rest are devoted to applications. Is it wonderful that Poisson declared his paper unintelligible and Liouville applied to him Descartes's dictum, "Quand il s'agit de questions transcendantes, soyez transcendentalement clair."

In closing this notice we wish to express our disappointment that in editing these Oeuvres a body of notes and comments was not added similar for example to that in the new edition of Abel's works by Sylow and Lie, or in the edition of Grassmann's works, which is appearing now under the care of Engel. We rejoice, to be sure, that we can all have the writings of Galois in the *original* text at a low price. In an excellent German translation,* also without critical notes, they have been accessible to any one for nearly ten years. It seems to us worth while to tell the reader who is becoming interested in Galois's theories and wishes to consult the original papers, the exact sense of many obscure passages, to indicate how some of the more important gaps may be filled in and to point out some of the principal differences between Galois's exposition of his theory and that received to-day. All this is left to the reader to do, who, no doubt, if sufficiently versed in Galois's circle of ideas, and possessed of the requisite time and patience, will do so. That this will not usually be the case seems to us patent. Another point : we read with particular interest that part of the introduction, not quite two pages in length, which M. Picard devotes to an account of Galois's researches in the theory of abelian integrals. The theory of these integrals and that of algebraic equations hang so intimately together that Galois, like Abel, was irresistibly drawn toward these new transcendents. All we know of Galois's researches is contained in his letter to Chevalier. To us it seems worth while to take up this part of the letter and study it as a historian does a precious document or inscription which, injured by the ravages of time, is, by patient weighing and comparing, to be reconstructed or interpreted. Some of Galois's statements are false, so, for example, the existence of a function with one logarithmic discontinuity, or again, the degree of the equation for the division of the periods. Others, on the contrary, are startlingly accurate and profound, so his conception of the problem of transformation. While doubtless many details in an estimate of Galois's achievements in this field and the power of his methods would be uncertain and so give occasion to a difference of opinion, still it seems to us a duty we owe to this unfortunate youth that a conscientious and exhaustive attempt be made in this direction. The claims of Galois to our admiration for his theory of equations and for his imaginaries are admitted by all, but let us not forget that his powerful intellect had, long before Riemann, penetrated into a still higher region and discovered some of its rarest gems.

A final remark. It is a pardonable weakness which

* "Abhandlungen über die algebraische Auflösung der Gleichungen," von N. H. Abel und É. Galois. Deutsch herausgegeben von H. Maser, Berlin, J. Springer, 1889. The price of this little work is only 4 marks.

mathematicians share in common with all mankind to wish to know something of the personality and the surroundings of the men whose works have influenced the progress of civilization. In accordance with this general wish we find more and more the collected works of great mathematicians accompanied by biographical notices except when they form special works accessible to all, as in the case of Abel or Hamilton. Who has not read with pleasure the touching Lebenslauf of Riemann which Dedekind wrote for the Gesammelte Werke, or the biographical sketches attached to the Works of Henry Smith and the Lectures and Essays of Clifford?

For a long time the details of Galois's life were surrounded with mystery. Like a blazing comet his genius appeared to the mathematical world only to disappear with equal suddenness into the gloom of impenetrable obscurity. The introductory remarks of Liouville to his works and possibly the affecting sketch by his friend Chevalier in the *Revue Encyclopédique* comprised all that most of us knew of him. Today, thanks to the patient and loving labors of M. Paul Dupuy, the details of his life are quite well known. Appearing, however, in the *Annales de l'École Normale Supérieure*, this valuable monograph cannot be purchased separately and so is not readily accessible to all. Would not a short sketch containing the principal features of Galois's life have been a valuable addition?

We close here our suggestions; they have been offered solely in the hope that when a new edition becomes necessary it may be enlarged along the lines here indicated and so spread still more widely the appreciation of Galois's genius and the usefulness of his theories.

JAMES PIERPONT.

YALE UNIVERSITY,
November, 1898.

THREE MEMOIRS ON GEOMETRY.*

Thèse sur la géométrie non-euclidienne. Par M. L. GÉRARD, Professeur au Lycée Ampère (Lyon). Paris, Gauthier-Villars et fils, 1892. 4to, 110 pp.

Lezioni di Geometria intrinseca. Per ERNESTO CESÀRO, Professore ordinario della R. Università di Napoli. Naples, Alvano, 1896. 8vo, 264 pp.

* The order in which these memoirs appear above is that given in the official announcement and presumably determined by lot, since the regulations of the Lobatschewsky Prize specify that precedence shall be so determined in case of a tie.