plain statement of the truth to add that as a scientific treatise the work cannot be regarded as an authority.

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GRAVITATION AND ABSOLUTE UNITS OF FORCE.

ABSTRACT OF A PAPER READ BEFORE THE NEW YORK MATHEMATICAL SOCIETY AT THE MEETING OF APRIL 7, 1894.

BY PROF. W. WOOLSEY JOHNSON.

THE writer held that the conflict between gravitation and absolute units was irrepressible because of the impossibility of reconciling the practical necessities of the engineer with the scientific needs of the physicist. Accordingly most modern text-books admit both kinds of units. The history of the matter was briefly summarized. Weighing being the inevitable manner of comparing masses, the same terms have always been used to describe masses and the pressures produced by their gravitation. With the rise of mechanical science the conceptions of force and of mass must be differentiated. The older writers were content to write $P \propto mf$; force, mass, and acceleration might be expressed each in its own unit; but the modern method is to write such a relation in the form $P = kmf$, and, first establishing fixed units to be employed, to proceed to determine k. Since no occasion had hitherto arisen for a unit of mass as distinguished from a unit of weight, no difficulty was at first felt in adopting for m such a unit that $k = 1$, and hence $P = mf$, while the pound, the foot, and the second were the units of force, length, and time. In other words, in using $W = mg$ no inconvenience was felt from the fact that in assigning a numerical value to m its unit was not a mass weighing one pound, but a mass weighing g pounds. There would rarely be occasion to employ the numerical value of m, W/g being substituted for it in final results.

But g is found to be variable, and since our standards furnish us with an invariable mass, it is seen that we have been using a variable unit of force. The engineer and practical man, however, while admitting that mass and not force is the third primary unit, still finds it more convenient for his purpose to use this variable, or rather let us say 'local,' unit of force, in spite of the fact that in using the formula $W = mg$ this implies also a variable or 'local' unit of mass.

This variable unit of mass seems intolerable to a certain class of writers who object *in toto* to gravitation units. With these writers "the British unit of mass is the Imperial

Pound"; "the weight of a body is the force (a variable) with which the earth attracts the body"; and "the unit of force is the poundal, in which unit all forces must be expressed in order that $P = mf$ may be universally true." Accordingly "we ought to speak of the weight of a pound," and in examples we find a resistance specified as "204 lbs. weight" a force as "equal to the weight of a ton," etc. These writers of course put m for the number of pounds and bring out every force in poundals (or in dynes if C. G. S. units are used); on the other hand, those who freely admit gravitation units, while using $W = mg$, put W equal the number of pounds in dynamics just as most writers do in statics. The argument of the former class is this: In $W = mg$, W stands for weight and is a force, m stands for mass; surely m is the constant and W the variable, when g varies. Granted, say their opponents, that the mass is constant and the force variable. A force is the product of a numerical measure and a corresponding unit. If, in the case of a force varying with g, it be convenient (as it distinctly is) for certain purposes to throw this variableness upon the unit, and not upon the numerical measure, why not do so, as well as *vice versa* for other purposes which make the reverse process desirable?

The confusion which sometimes arises, and of which instances were quoted, is due to the double meaning not so much of the general term "weight" as of the special term "pound," now that it is customary to say that the pound is the unit of mass and to put m for mass. By an incautious use of language we are sometimes left in doubt as to whether a certain numerical value is to be assigned to m or to W.

In the opinion of the writer, those who employ gravitation units make a mistake in trying to erect the pound into the semblance of an absolute unit by referring to a conventional locality and to the illustration of a spring-balance graduated at such a locality. Thus Williamson says: "In this system the unit of force is the weight at some definite place [London] of the pound mass," and later adds: "When scientific accuracy is required we must correct for the change in the value of g due to any difference in altitude or latitude from those of the place to which the standard was originally referred." It is assumed by Thomson and Tait in the Natural Philosophy that: "In all cases where great accuracy is required the results obtained by such a method have to be reduced to what they would have been if the measurements of force had been made by means of a perfect spring-balance graduated so as to indicate the force of gravity on the standard weights in some conventional locality."

Let it rather be understood that, in the use of gravitation units, the pound when used as a unit of force is always local, and that in the formulæ the numerical value of m also implies a local unit of mass. But to avoid confusion it is best that in formulæ for application m should be replaced by W/g. And, in any comparison of results involving different values of g, let each force be expressed in poundals by simply multiplying by the local value of g.

Professor Greenhill has made a marked innovation in this matter as compared with the usage of recent English writers. Calling the pound the unit of weight, and refusing to regard weight as a force but rather as "the quantity which is determined by the operation of weighing," he practically makes the pound a unit of mass; and, abandoning the formula $W = mg$, puts W for the number of pounds, so that in formulæ it appears where we are accustomed to see m. Then, with regard to force he says "it is convenient to take the attraction of the earth on a pound weight as the unit of force, and to call it the *force of a pound;* this is the British unit of force in universal use in all practical problems of architecture, engineering, mechanics, and artillery."

As contrasted with the usual notation supposing absolute units to be employed, W is thus merely put in the place of m so that Wg instead of mg is the expression in poundals for the force of gravity acting on the body. At the same time, however, Professor Greenhill uses F, R, and other symbols of force as co-dimensional with W, so that they are the numbers of local pounds of force, and it must be remembered that the expressions for the same forces in poundals are Fg, Rg, etc. With regard to gravitation and absolute units, he says: "The attraction of the earth in any locality provides such a convenient and invariable measure of force that all instruments, great and small, for measuring force and work are calculated and graduated originally in gravitation measure; the reduction to absolute measure if required being made subsequently by means of the local value of g; presumably determined previously with the greatest attainable accuracy by means of pendulum experiments."

NOTES.

A REGULAR meeting of the NEW YORK MATHEMATICAL SOCIETY was held Saturday afternoon, April 7, at half-past three o'clock, the president, Dr. McClintock, in the chair. Mr. Pomeroy Ladue, of the University of Michigan, having been duly nominated, and being recommended by the council, was elected to membership. The president announced the resignation on account of ill health of the treasurer, Mr.