It is easily seen that this theorem plays the same rôle in the theory of infinite products that is taken by Gauss's rule of convergence, to which Professor Cajori refers on the second page of his note, in the theory of infinite series. Stirling attempted to make use of this theorem in order to establish the convergence of certain infinite series, but his efforts in this direction met with little success. More complete information in regard to Stirling's theorem and its applications to series are contained in my note "Ett konvergens kriterium från bôrjan of 1700-talet" (A rule of convergence of the beginning of the eighteenth century) inserted in the Bulletin (Öfversigt) of the Academy of Sciences of Stockholm, 1879, No. 9, pp. 71–84.

STOCKHOLM, *March* 31, 1894.

MEYER'S REPORT ON THE THEORY OF INVARIANTS.

Bericht über den gegenwärtigen Stand der Invariantentheorie von Dr. FRANZ MEYER, Professor a. d. kgl. Bergakademie, Clausthal. Contained in Vol. I. of the *Jahresbericht der deutschen Mathematiker-Vereinigung*, pp. 79–292. Berlin, Reimer, 1892. 8vo.

BOTH as regards its problems and as regards its methods, the theory of invariants presents a more heterogeneous mass of materials for the summarizer of its history to deal with than do most of the mathematical disciplines which have undergone so rapid a development during the second half of our century. Dr. Franz Meyer undertook, therefore, a less attractive task in preparing his "Bericht über den gegenwärtigen Stand der Invariantentheorie" than will fall to the lot of those who will assume similar labors for the German mathematical society in other fields. With infinite patience and comprehensive knowledge, Dr. Meyer has given us, in a volume of 200 pages, a condensed statement, not only of all the methods, but of a large part of the results, of research in the field of invariants, from its beginning in the discoveries of Boole, in 1841, to the very date of the preface to the present work, August, 1892. The period 1841–67 is, indeed, passed over more summarily than the last quarter-century, the memorable work done by Cayley, Sylvester, Hermite, Brioschi, and others in the earlier period being surveyed in a brief "Rückblick," though to some extent also discussed in the body of the report.

It is quite impossible to summarize in a few pages what is already in 200 pages a condensed summary; we shall therefore content ourselves with pointing out a few features of the Report. Dr. Meyer looks upon the matter—rather unfortunately, we think—as naturally and historically dividing itself into two main parts: the first having reference to the problem of equivalence, that is, the linear transformability of algebraic forms into each other, and the substitutions whereby these transformations are effected; the second to the algebraic relations between invariantive forms. The report is correspondingly divided into two parts, of which the first is naturally very much the smaller, occupying less than 30 pages. This first part would have been still more brief had the author confined himself to what we generally associate with the theory of invariants in connection with the question of equivalence. But the section on the equivalence-problem for quadratic and bilinear forms, and the section on the same problem for higher forms, are followed by a section entitled "Forms with linear transformations into themselves," relating to researches on the general problem of finding all possible groups of substitutions of n variables and the rational entire invariants of these groups. Here are mentioned in the first place the discoveries of Schwarz and Klein, which arose from researches in the theory of functions; then the connection with the theory of equations of the fifth degree, as developed by Klein in the "Ikosaeder"; then many other developments, connected with the transformation of elliptic functions, the theory of linear differential equations, etc. Of purely algebraic researches in this field, perhaps the most notable mentioned are Gordan's investigation of binary forms with linear transformations into themselves, Jordan's general discussion of finite groups of substitutions for n variables from the point of view of the theory of substitutions, and the generation of the "regular-polyhedron" equations given in Gordan's lectures on invariants.

The second part, which comprises the great body of the researches in the theory of invariants, consists, besides a few pages devoted to "Irrationelle Fragestellungen,"—which pages include the question of canonical forms and of the primitive forms corresponding to given covariants,—of three large divisions: "Endlichkeitsfragen," "Methodik," "Specielle Substitutionsgruppen und Formen." Of course the "questions of finiteness" are the questions relating to the constitution of the complete system of ground-forms pertaining to a given system of algebraic forms; and our author includes under this head not merely the question whether the system of ground-forms is finite or not, but also the methods of constructing the complete system, the question of the syzygies to which the ground-forms are subject, and the methods of denumeration, by which the types of the ground-forms and syzygies, and the number of each type, are determined without the actual construction of them. Thus the question of finiteness, strictly speaking, occupies only one of the five sections into which the division on "Endlichkeitsfragen" is subdivided. In this is sketched the beginning of the question in the early work of Cayley and Sylvester; then we have an outline of Gordan's proof that the complete system actually is finite in the case of any given system of binary forms, of subsequent improvements of this proof, and of Peano's extension of the proof to the case of systems of binary forms involving different pairs of variables, subject to independent linear substitutions. Of an entirely different kind are the proofs given by Mertens and Hilbert, some sixteen years later, depending, as they do, on the finiteness of the number of fundamental positive solutions of a system of linear diophantine equations ; these proofs are vastly simpler than Gordan's, especially Hilbert's, which is so brief as almost to be designated as instantaneous; on the other hand, they do not, like Gordan's, present any process by which the complete system may actually be obtained; moreover, they go outside the natural domain of rationality of the problem. All these investigations relate to binary forms only; and it was not until 1890 that the finiteness of the complete systems for forms in n variables was demonstrated. This was done by Hilbert in his great work in vol. 36 of the *Mathematische Annalen,* the distinguishing feature of which (so far as it bears on our subject) is that it operates in a far wider field than that of invariants, and deduces the theorem of finiteness for invariants of forms in any number of variables—and even for the syzygies of every rank and for the number of such ranks—from far more general theorems. The Report gives an outline of Hilbert's proof ; the proof has been simplified by Story (*Annalen,* vol. 41) since the appearance of the Report.

Passing over the intervening sections, we shall only mention further, in regard to the division on "Endlichkeitsfragen," that we have in the last section of this division ["Abzählende Richtung"] a very intelligible account of the Cayley-Sylvester methods of denumeration by means of generating functions ; a mention of the remarkable results arrived at by Jordan for the superior limits of the degree and order of the covariants of a system of binary forms ; and an account of the work of Capelli and Deruyts on the extension of the Cayley-Sylvester theorem for the number of linearly independent covariants of binary forms to the case of forms in several sets of n variables.

In the division on "Methodik," we have, under the subhead of "Symbolik und graphische Darstellung," not only an account of the great symbolic method initiated by Cayley, developed into a thoroughly organic system by Aronhold and Clebsch, and used with so much power by Gordan, but also a notice of the chemico-algebraic method of graphs devised by Sylvester and extended by Clifford, and of MacMahon's remarkable transformation of the question of seminvariants into a question of symmetric functions. Under the other heads of this division, and under the last division of the Report, that on "Specielle Substitutionsgruppen und Formen," the number of points that come up for treatment is so great that a continuation of even such cursory notice as we have been giving would be fatiguing. Suffice it to say, therefore, that whoever consults the Report will be impressed by the fact that the development of the Theory of Invariants in recent years, while overshadowed by the brilliant conquests made in the domain of the Theory of Functions, has been by no means at a standstill. Not to speak of the excursus into the field of differential invariants made by Sylvester and his followers, MacMahon, Hammond, and others, signal advances have been made in the central theory, especially by Capelli, Stroh, Study, and Deruyts. Dr. Meyer, in his preface, expresses regret that he found it impossible, except in a few instances, to include the geometrical applications of the theory in the scope of the Report. With this exception, the student of the theory of algebraic forms and invariants will find in the Report a remarkably full abstract of researches in this domain, accompanied by accurate bibliographical references, and will feel under great obligation for the assistance rendered by this result of Dr. Meyer's great learning and painstaking industry. It ought to be especially useful to any one undertaking to present, in a systematic work, the body of doctrine which is the outcome of the varied and often heterogeneous researches outlined in this compendious report.

F. FRANKLIN.

BALTIMORE, *April*, 1894.

CAJORI'S HISTORY OF MATHEMATICS.

A History of Mathematics. By F. CAJORI. New York, Macmillan & Co., 1894. 8vo, 14 and 422 pp.

IT is a long time since an American work has been awaited with so much anticipation by readers of mathematics as Professor Cajori's recent history. The book had been extensively advertised, there was and is a growing demand for such works, and the supply of material was well-nigh inexhaustible. But while few books have ever enjoyed such advantages, few books have ever so seriously failed to improve them. This is a harsh statement and should neither be lightly made nor lightly accepted. It is based upon the following