"*Leipziger Magazin für reine und angewandte Mathematik;* herausgegeben von J. Bernoulli und C. F. Hindenburg, erster Jahrgang 1786. S. 13 ff." On account of the extraordinary interest of this article and the present great rarity of this magazine, the Leipziger Gesellschaft der Wissenschaften, at the suggestion of Dr. Staeckel, will reprint it in their *Abhandlungen*. Meanwhile, to show how wonderfully it anticipates Lobatschewsky, Bolyai, Riemann, Beltrami, in what it maintains, we need only cite the following:

(1) The parallel-axiom needs proof, since it does not hold for geometry on the surface of the sphere.

(2) In order to make intuitive a geometry in which the triangle's angle-sum is less than two right angles we need an "imaginary" sphere [pseudo-sphere].

(3) In a space in which the triangle's angle-sum is different from two right angles, there is an absolute measure [a natural unit for length].

The whole paper is another unexpected illustration of the words in a letter from Sir Robert Ball: "It is also noteworthy how many mathematicians, approaching the subject from very varied sides, have been led to the study of what mathematics would be like without the eleventh axiom."

AUSTIN, TEXAS, *November*, 1893.

THE TEACHING OF MATHEMATICS AT GÖTTINGEN.*

THE purpose of the following remarks is not to furnish to students who wish to prepare themselves to teach mathematics and physics in the higher schools, a detailed scheme of the lectures and exercises which they should attend during each semester. It would be impossible to do this on account of the great number of branches of mathematics and mathematical physics and the frequent changes necessary in the subjects and the arrangement of the courses of lectures. It is essential, however, that students should be acquainted with

* Translation of a circular, "Announcement of a scheme of study for those wishing to become teachers of mathematics and physics; together with an extract from the regulations of the seminary of mathematics and physics," issued by the University, and reprinted in the *Zeitschrift für math. und naturwiss. Unterricht*, vol. 24. pp. 540-546. Apart from its relation to the work at Göttingen, the circular is of general interest to teachers of mathematical science.—T. S. F.

certain general principles and requirements of their chosen subject in order to bring their course of study into conformity with them, and that they should have a general idea of the whole field to be covered in order to distribute their lectures and work conveniently through the several semesters. It is to be observed that in the theoretical development of mathematics and mathematical physics the advanced courses rest altogether upon foundations laid in the elements, so that after the introductory courses of the first two or three semesters have been attended and are properly understood, the advanced courses, so far as they are not immediate continuations of previous ones, may be taken in almost any order.

The following advice to our students relates in its "general part" to the

1. Introductory lectures of the first semester.
2. Elaboration of the lectures.
3. Number of courses to be attended.
4. Subordinate subjects.
5. Philosophical courses.
6. Work in the seminary and laboratories.
7. Private study.

In a second "special part" is given an account of the most important courses of lectures, with remarks upon several groups of them.

I. General Part.

1. *Introductory lectures of the first semester.*

Analytical geometry and the differential and integral calculus, including algebraic analysis, constitute the foundations not only of the higher mathematics in its narrower sense but also of mechanics and mathematical physics. They should therefore be completed, as far as possible, in the first two semesters of study. The same is true of the course in experimental physics, knowledge of which is required for attendance upon the practical work in physics and the study of mathematical physics. As a preparation for the latter, mechanics is also necessary, and we would recommend its study in the third semester. For further details we refer to the "special part" of this plan of study.

2. *Elaboration of the lectures.*

It cannot be stated with sufficient emphasis that mere listening to a course of lectures has no value in mathematics or mathematical physics, and but little value in experimental physics. The student can make the material presented to

him in the lecture his own property only by carefully elaborating at home the notes which he has taken down during its delivery. In this connection he is earnestly requested to consult the lecturer whenever in difficulty or in doubt. Such questions, which will materially lighten the labor of elaboration, are much desired, and will always be most willingly answered.

3. *Number of courses to be taken.*

The requirement just mentioned limits the number of courses in mathematics and mathematical physics which can be attended simultaneously. It is not possible to elaborate together, nor to attend therefore with profit, more than two, or at most three, four-hour mathematical courses; particularly as courses in natural science and other subjects are also to be attended. The necessity of choosing from the whole number of courses offered, those which are indicated by the special direction of the student's personal interest, will make itself more or less felt in the later semesters. The thorough study of some special branch of science, for which the University of Göttingen through its great number of special courses affords peculiarly favorable opportunities, is much to be encouraged on account of the resulting thoroughness of scientific training. Such a concentration of study in a special department can be accomplished without interfering with the demands of the examination, which requires for teachers of the higher classes in mathematics "a well grounded knowledge of the most important theories of higher geometry, higher analysis and analytical mechanics," and in physics "a general survey of mathematical physics, and an acquaintance with the fundamental mathematical investigations in one of the more important departments of theoretical physics."

4. *Subordinate subjects.*

No general rule can be given in regard to the subordinate subjects (natural sciences, geography, etc.), on account of the diversity of the students' aims and inclinations. It will generally be in the interest of the mathematical and physical studies to attend these subjects in the earlier semesters, in order to keep the time in the later semesters as free as possible for the seminary work and advanced lectures.

5. *Philosophical lectures.*

The philosophical lectures may be postponed with advantage to the second half of the period of studentship. They

should not present to the student a theoretical system of essentially dogmatic character, but rather should enable him to trace out, from the point of view of his own subject, the relations which philosophy bears to an extensive field of positive knowledge.

6. *Work in the seminary and laboratories.*

The lectures on mathematics and natural science are supplemented by courses devoted to exercises and practical work, from which the students gain, in addition to knowledge, a corresponding facility in its use. Most of the mathematical exercises are held in the seminarium of mathematics and physics. The student is earnestly recommended to enter the seminary during his earlier semesters, since even at this period it affords, through its reading-room, great assistance and encouragement to study. The conditions for admission to and the use of the reading-room are mentioned below.

The exercises in descriptive geometry should be taken, as a rule, in the first half of the period of studentship, which is less burdened with other practical work. Practice in geometrical drawing is so important a requirement for the teacher of mathematics and physics that no student should lose the opportunity offered for acquiring it.

The practical work in physics and chemistry attach themselves to the corresponding lectures, and should therefore be performed in general during the middle semesters. In physics and chemistry it has been long established by experience, that mere attendance upon lectures is insufficient for a thorough understanding, and that work of observation and measurement in the laboratories is indispensable. In mathematics, also the application of the general principles to particular examples and practice in the performance of numerical calculations form an important auxiliary to theoretical progress. In this connection, the attention of the students is called to the work in magnetism, astronomy, and geodesy, in which, quite apart from the general interest of the subjects treated, instruction is afforded in the systematic reduction of extended series of observations.

7. *Private study.*

The information acquired by lectures and practical work should be completed, primarily by private study of textbooks. Individual mental activity will be developed in a greater degree, however, by reading original works. Facility in appreciating new ideas will be thus obtained, and upon this

depends the ability to follow, in later years, the subsequent developments of science. The general library of the university and the reading-room of the seminarium of mathematics and physics, both of which are open during vacation, furnish the best opportunity for an extensive study of the literature.

We finally urge upon our students that they should not lose sight of their future occupations. The lectures attended by them afford many opportunities for reflecting upon the subjects of study in the gymnasia, and for appreciating the significance which mathematics and physics possess for the whole of our modern civilization. The breadth of view thus to be gained will be of great assistance in quickening the instruction to be imparted later, and in rendering it suggestive and productive. For the same reason, those courses are of the greatest importance to future teachers which relate especially to the subject of elementary mathematics. These courses are intended for the later semesters.

II. Special Part.

A. *Courses in mathematics and mathematical physics.*

The number of courses in mathematics and mathematical physics at Göttingen is unusually great. Not only are those subjects treated which, in the present state of science, have a recognized place in academic instruction; but numerous courses extend into those special departments of science which have only recently been established and are still actually in process of construction. It thus appears the more necessary to append to the general remarks of the preceding part, an account of the courses of lectures themselves. These are divided into introductory courses, advanced courses, and special courses. The introductory courses are given every year; the advanced courses are repeated every two or three years; while in the case of the special courses there is no regular rotation.

In the following table the title of the course first given is the most general for each subject. The other titles, added in parentheses, relate to courses which possibly have special aims, but are on the whole equivalent to that first specified.

Introductory lectures.

(1) Analytical geometry.
(2) Differential and integral calculus.
(3) Mechanics. (Elementary mechanics. Introduction to analytical mechanics, Introduction to theoretical physics.)

The differential and integral calculus extends at times

through two semesters. Occasionally a special course in algebraic analysis is also given, in which certain parts of algebra are treated by themselves.

Sometimes a course is given under the title "Introduction to the higher mathematics." This is intended for students of other subjects, who wish to acquire a limited knowledge of higher mathematics. Such a course does not afford a sufficient basis for a further study of mathematics.

Advanced lectures.

(1) Algebra. (Theory of algebraic and numerical equations, Determinants, Invariants.)

(2) Theory of numbers.

(3) Higher geometry. (Analytical and synthetic treatment of projective geometry.)

(4) Curved surfaces, and curves of double curvature. (Application of the differential and integral calculus to geometry.)

(5) General survey of the elementary mathematics.

This course, in which at one time the algebraic, at another time the geometrical side is put in the foreground, has for its object the elucidation of the principles of elementary mathematics from the point of view of the higher mathematics.

(6) Higher parts of the integral calculus. (Definite integrals, Differential equations, Calculus of variations.)

(7) Theory of functions.

(8) Elliptic functions.

The three courses immediately preceding are of fundamental importance, not only for all branches of pure mathematics, but for their applications to mathematical physics.

(9) Higher mechanics. (Analytical mechanics.)

Higher mechanics treats largely of analytical problems developed by the necessities of astronomy. The importance of the geometrical element is, however, constantly increasing on account of the technical applications. (Graphical statics, Kinematics.)

(10) Potential.

Potential will be given as a rule during two semesters, and introduces the applications of the theory of potential to gravitation, electrostatics, magnetism, electrodynamics, and hydrodynamics.

(11) Partial differential equations.

This course is devoted to the methods of integrating the differential equations of mathematical physics.

(10′) General theory of electricity.

(11′) Theory of elasticity, and acoustics.

Courses 10, 11 on the one hand, 10′, 11′ on the other, treat in great part of the same subjects, the former more from the

mathematical, the latter more from the physical, point of view They are therefore to a certain extent equivalent.

(12) Theory of light.

(13) Theory of heat. (Mechanical theory of heat, Chemical physics.)

In the courses in mathematical physics not only the theory of the physical phenomena presented in the course in experimental physics is elaborated, and the methods of their measurement given, but many important results, which to be understood require previous theoretical training, are stated for the first time.

(14) General astronomy. (Geodesy, Astrophysics, Geophysics.)

(15) Theory of probabilities. (Method of least squares.)

Apart from its general value in reasoning, the theory of probabilities possesses great importance, on the one hand for the method of least squares, indispensable to the sciences depending upon measurement, and on the other, for the scientific theory of statistics and life insurance.

Special courses.

The special courses are intended chiefly for those students who have inclination or occasion for independent scientific work, such as, among other things, is necessarily assumed in the case of candidates for the doctor's degree.

The subjects of these courses cannot be specified as they change with the state of science and with the predilections of the lecturers. In general, special parts of the preceding advanced lectures will be treated more in detail.

B. *The philosophical courses.*

1. History of Philosophy. 2. Logic. 3. Psychology. 4. Pedagogics.

In this place it is necessary to remind the students that they should not neglect their general culture during the period of their studentship. They should consider, therefore, in addition to the philosophical courses, those in history and in the history of literature.

C. *Courses in natural science and geography.*

1. Experimental physics.

The course in experimental physics extends through two semesters, and is so arranged that the second part of the course can be taken without previous attendance upon the first part.

2. Experimental chemistry.

Here the first part of the course, inorganic chemistry, is indispensable to the understanding of the second part, organic

chemistry. Attendance upon the practical work in chemistry is permissible during the course in inorganic chemistry.

3. Mineralogy. (Crystallography.)

Apart from its individual importance, this course contains important material complementary to certain parts of physics and chemistry.

4. Zoology. 5. Botany.

In regard to botany it may be stated that those students who wish to teach it in the middle classes must not fail to make themselves acquainted with the local flora by participating in the botanical excursions.

6. Geography. 7. Geology.

Physical geography bears close relations to all branches of natural science and depends, in a variety of ways, upon mathematics and physics. Geology is complementary to physical geography, and knowledge of it is necessary for understanding more fully the laws of morphology and of geographical distribution. Geological excursions are a necessary auxiliary to the course in geology.

Extract from the regulations of the seminary of mathematics and physics.

1. The province of the seminary is to lead the students to independent work and to instruct them in the application of what they have learned in the lectures.

2. The object of the seminary is accomplished by discourses on the part of the directors and by exercises performed under their guidance. The latter consist of lectures delivered by the students themselves, and the solution of problems assigned by the director to be discussed in writing. The students are given opportunities also to make physical, magnetic and astronomical measurements, in the performance of which the rooms and instruments of the physical institute and the astronomical observatory may be employed. The lectures and exercises of the seminary of mathematics and physics are regularly announced in the scheme of lectures.

3. Every matriculated student of mathematics and physics may become a member of the seminary. Those entering must make themselves known to the director in charge and, after admission, must present themselves to all the other directors whose work they wish to attend.

4. The reading-room of the seminary of mathematics and physics is designed to make the literature of these subjects as accessible as possible to the students in prosecuting their studies.

5. Those members of the seminary who wish to use the reading-room must make personal application to a director,

and must agree to observe the rules established for the management of the reading-room.

6. In accordance with a legal statute, a library fee, payable in advance, is collected from every member at the beginning of each semester. For ordinary members, to each of whom a private drawer in the reading-room may be assigned, the fee is five marks, and for other members it is three marks. In addition, there is a deposit of three marks required from each member receiving a key, and returnable upon the surrender of the key.

7. The collection of these fees and the maintenance of order in the reading-room are in charge of the librarian, under the supervision of the directors. Members are required to obey his instructions. Further regulations will be announced by means of notices in the reading-room.

The directors of the seminary of mathematics and physics: RIECKE, SCHERING, VOIGT, KLEIN, SCHUR, WEBER.

NOTES.

A REGULAR meeting of the NEW YORK MATHEMATICAL SOCIETY was held Saturday afternoon, November 4, at half-past three o'clock, the president, Dr. McClintock, in the chair. Mr. Isaac Hodges Turrell of Cincinnati, Ohio, having been duly nominated, and being recommended by the council, was elected a member. Professor Fletcher Durell read a paper entitled "Application of the new education to the differential and integral calculus," in which he advocated the presentation of the first principles of the calculus from an almost purely geometrical point of view. T. S. F.

THE National Academy of Sciences met in the Capitol at Albany, November 7–9. The papers presented included one by Dr. S. C. Chandler, entitled "Additional researches on the motion of the earth's pole." Dr. Chandler finds that the most recent observations obtainable (some still unpublished) confirm the law deduced by him. He showed that the two separate motions of the pole both take place from west to east. Dr. Chandler's paper was discussed by Professors Hall, Newcomb and Boss. They all expressed themselves as now favoring the truth of Dr. Chandler's law of variations. Professor C. S. Hastings read a paper on "A new form of telescopic objective, as applied to the twelve-inch equatorial of the Dudley Observatory." The principal characteristics of this instrument are: first, that one of the