THE TRANSITIVE SUBSTITUTION-GROUPS OF NINE LETTERS.

BY DR. F. N. COLE.

The list of these groups published by Mr. Askwith in the present volume (26) of the Quarterly Journal of Mathematics contains only 22 of the 34 actually existing types. The groups identified by Mr. Askwith are the more obvious forms—the symmetric and alternating groups and the non-primitives—together with two primitive types of order 72, while the omitted cases—5 non-primitives and 7 primitives—include among others such specially interesting forms as the triad group of order 216 identified with the theory of the points of inflection of the plane cubic curve, etc., and a *simple* group, apparently heretofore unrecognized, of order 504.

I give here a complete list of the transitive groups of this degree, together with very brief explanations of the processes by which I have obtained them.

A. *The Non-primitive Groups of Nine Letters.*

In respect to these groups the nine letters are distributed in three systems of three letters each, such that every substitution of the group replaces every system either by itself or by one of the other systems.

Those substitutions of the group which replace every system by itself form a self-conjugate subgroup, i.e., a subgroup which is transformed into itself by every substitution of the group. This subgroup obviously affects the three systems symmetrically.

Of the intransitive groups of nine letters with three transitive systems of three letters each only the following 8 satisfy the last requirement :

$H_{216} =$ (abc) all (def) all (ghi) all,

$H_{108} =$ $\{(abc)$ all (def) all (ghi) all$\}$ pos.,

$H_{54} =$ (abc) pos. (def) pos. (ghi) pos. $+$
(abc) neg. (def) neg. (ghi) neg.,

$H_{27} =$ (abc) pos. (def) pos. (ghi) pos.,

$H_{18} =$

1*,	1,	$abc\,.\,def$,	$acb\,.\,dfe$,
ghi,	abc,	$acb\,.\,def$,	dfe,
gih,	acb,	$abc\,.\,dfe$,	def,
gh,	$ab\,.\,de$,	$ac\,.\,df$,	$bc\,.\,ef$,
gi,	$ac\,.\,de$,	$bc\,.\,df$,	$ab\,.\,ef$,
hi,	$bc\,.\,de$,	$ab\,.\,df$,	$ac\,.\,ef$,

* I use this notation to indicate that every substitution of e, f, g is multiplied by the 3 several substitutions opposite it.

$$H_9 = \begin{array}{l} 1, \\ ghi, \\ gih, \end{array} \left| \begin{array}{llll} 1, & abc \,.\, def, & acb \,.\, dfe, \\ abc, & acb \,.\, def, & dfe, \\ acb, & abc \,.\, dfe, & def, \end{array} \right.$$

$$H_6 = 1, \quad \begin{array}{l} abc \,.\, def \,.\, ghi, \\ acb \,.\, dfe \,.\, gih, \\ \end{array} \quad \begin{array}{l} ab \,.\, de \,.\, gh, \\ ac \,.\, df \,.\, gi, \\ bc \,.\, ef \,.\, hi, \end{array}$$

$$H_3 = (abc \,.\, def \,.\, ghi) \text{ cyc.}$$

The non-primitive groups are formed by adding to the preceding groups H substitutions which permute the three systems a, b, c; d, e, f; g, h, i. These substitutions either permute the three systems cyclically, or they leave one system unchanged and interchange the other two. In order that the nine letters may be transitively connected, the cyclical permutations of the three systems must be present, and if there are any other permutations of the systems, all the six possible ones occur.

If H_r is the subgroup which in any particular case leaves all the systems unchanged, and if any substitution σ of the group permutes the systems in any way, then all the substitutions σH_r permute the systems in the same way, and these are all the substitutions of the group which produce this effect. Accordingly, the order of a non-primitive group of nine letters is either three or six times the order of the corresponding H_r.

Given the group H_r, a substitution of the required non-primitive group which shall permute the systems of non-primitivity cyclically may be selected in any way consistent with this requirement, subject only to the two conditions 1) that it must transform H_r into itself and 2) that its third power (which leaves the systems unchanged) must occur in H_r. To the resulting transitive group G of order $3r$ a further substitution may then be added which interchanges two of the three systems, which transform G into itself, and whose second power occurs in H_r.

I find the following 23 distinct types of non-primitive groups with nine letters:

Order

1296 $\{H_{216},\ adg \,.\, beh \,.\, cfi,\ ad \,.\, be \,.\, cf\}$.

648_1 $\{H_{216},\ adg \,.\, beh \,.\, cfi\}$.

$_2$ $\{H_{108},\ adg \,.\, beh \,.\, cfi,\ adbe \,.\, cf\}$.

$_3$ $\{H_{108},\ adg \,.\, beh \,.\, cfi,\ ad \,.\, be \,.\, cf\}$.

324_1 $\{H_{108},\ adg \,.\, beh \,.\, cfi\}$.

$_2$ $\{H_{54},\ adg \,.\, beh \,.\, cfi,\ ad \,.\, bf \,.\, ce \,.\, gh\}$.

162_1 $\{H_{54},\ adg\,.\,beh\,.\,cfi\}$.

$_2$ $\{H_{27},\ adg\,.\,beh\,.\,cfi,\ ad\,.\,bf\,.\,ce\,.\,gh\}$.

$_3$ $\{H_{27},\ adg\,.\,beh\,.\,cfi,\ ad\,.\,be\,.\,cf\}$.

108 $\{H_{18},\ adg\,.\,bfi\,.\,ceh,\ ad\,.\,be\,.\,cf\,.\,gh\}$.

81 $\{H_{27},\ adg\,.\,bfi\,.\,ceh\}$.

54_1 $\{H_{18},\ adg\,.\,bfi\,.\,ceh\}$.

$_2$ $\{H_9,\ adgbficeh,\ ad\,.\,be\,.\,cf\,.\,gh\}$.

$_3$ $\{H_9,\ adg\,.\,bfi\,.\,ceh,\ ad\,.\,be\,.\,cf\,.\,gh\}$.

$_4$ $\{H_9,\ adg\,.\,bfi\,.\,ceh,\ ad\,.\,bf\,.\,ce\}$.

36 $\{H_6,\ adg\,.\,beh\,.\,cfi,\ ad.\,bf\,.\,ce\,.\,hi\}$.

27_1 $\{H_9,\ adgbficeh\}$.

$_2$ $\{H_9,\ adg\,.\,bfi\,.\,ceh\}$.

18_1 $\{H_6,\ adg\,.\,beh\,.\,cfi\}$.

$_2$ $\{(abcdefghi)$ cyc., $bi\,.\,ch\,.\,dg\,.\,ef\}$.

$_3$
$$\left\{\begin{matrix} abc\,.\,def\,.\,ghi, \\ adg\,.\,beh\,.\,cfi, \\ aec\,.\,bfg\,.\,cdh, \\ afh\,.\,bde\,.\,ceg, \end{matrix}\quad ad\,.\,bf\,.\,ce\,.\,hi\right\} = \{(abcdefghi)_9,\ ad\,.\,bf\,.\,ce\,.\,hi\}.$$

9_1 $(abcdefghi)$ cyc.

$_2$ $(abcdefghi)_9$.

B. *The Primitive Groups of Nine Letters.*

In a primitive group the subgroup which leaves any letter unchanged affects all the remaining letters.* These subgroups may therefore in the present case be selected from the table of groups of eight letters. The number of possibilities is greatly reduced by the consideration that a primitive group cannot contain a substitution of two or of three letters, or of four letters, if it affects more than eight letters,† and that, if it contains a transitive subgroup of lower degree, it is at least doubly transitive.‡

With these restrictions the only groups of eight letters, except the alternating and symmetric groups, which can serve as bases for primitive groups of nine letters are readily found to be the following:

Order $336,\ 168_1,\ 168_2,\ 56,\ 48,\ 24,\ 24,\ 16,\ 16,\ 8_{20},\ 8_{21},\ 8_{22},\ 8_{23},\ 8_{24},\ 8_{25},\ 6_2,\ 6_3,\ 6_4,\ 4_6,\ 4_8,\ 2.$

The orders of the corresponding primitive groups will be these multiplied by nine.

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* Cf. Netto, Theory of Substitutions, American Edition, p. 94.

† Ibid., p. 138. ‡ Ibid., p. 95.

The several possibilities have now to be considered separately.

Order 9 . 336. This group would contain 36 conjugate subgroups of order 7, each of which would therefore be transformed into itself by a group of $9 \cdot 336 \div 36 = 84$ substitutions. The latter group is transitive in 7 letters, but not in 8 or in 9. It could only be formed as $\{(abcdefg)_{42}, (hi)\}$. But then the required group would contain the transposition (hi). *There is therefore no transitive group of this order in nine letters.*

Order 9 . 168. This group would contain 36 conjugate subgroups of order 7, each transformed into itself by a group of 42 substitutions. The latter is transitive in 7 letters, but not in 8 or in 9. It could only be found as a transitive H_{42} of 7 letters dimidiated with the H_2 of the 2 remaining letters. The required group will therefore contain substitutions with one cycle of 6 letters and one of 2 letters. *This can happen only with the compound H_{168} of 8 letters.**

The latter contains the substitution of order 7

$$\sigma = bcedghf.$$

The transitive subgroup of order 42 with 7 letters, of which this is a part, contains the substitution

$$cf \cdot eh \cdot dg.$$

Accordingly the required group of order 9 . 168 contains the substitution

$$s = ai \cdot cf \cdot eh \cdot dg.$$

The H_{168} of 8 letters contains also

$$t = ac \cdot bd \cdot eg \cdot fh,$$

and consequently the required group contains

$$st = aichgbdef = \rho.$$

The H_{168} of 8 letters contains

$$H_{56} = \{A(abcdefgh),\ bcedghf\}.$$

We proceed now to show that for every α

$$\rho^{\alpha} H_{56} = H_{56} \rho^{\beta},$$

and that consequently ρ and H_{56} generate a (transitive) group of 9 letters of order 504. Letting

$$\begin{aligned} \tau_2 &= ab \cdot cd \cdot ef \cdot gh, & \tau_5 &= ae \cdot bf \cdot cg \cdot dh, & \tau_8 &= ah \cdot bg \cdot cf \cdot de, \\ \tau_3 &= ac \cdot bd \cdot eg \cdot fh, & \tau_6 &= af \cdot be \cdot ch \cdot dg, & \sigma &= bcedghf, \\ \tau_4 &= ad \cdot bc \cdot eh \cdot fg, & \tau_7 &= ag \cdot bh \cdot ce \cdot df, & \rho &= aichgbdef, \end{aligned}$$

* Cf. Hölder, Math. Annalen, vol. 40, p. 82.

I find

$$\begin{array}{lll}
\rho\,\sigma = \sigma\tau_7\rho^6, & \rho\,\tau_2 = \sigma\tau_2\rho^5, & \rho\,\tau_5 = \sigma^2\tau_5\rho^3, \\
\rho^2\sigma = \sigma^6\rho^7, & \rho^2\tau_2 = \sigma^4\tau_8\rho^3, & \rho^2\tau_5 = \sigma^2\tau_6\rho^5, \\
\rho^3\sigma = \sigma^2\tau_4\rho^2, & \rho^3\tau_2 = \sigma^3\tau_3\rho^2, & \rho^3\tau_5 = \sigma^5\tau_2\rho, \\
\rho^4\sigma = \sigma^5\tau_3\rho, & \rho^4\tau_2 = \sigma^6\tau_7\rho^8, & \rho^4\tau_5 = \sigma^4\tau_7\rho^7, \\
\rho^5\sigma = \tau_6\rho^3, & \rho^5\tau_2 = \sigma^6\tau_6\rho, & \rho^5\tau_5 = \sigma^4\tau_4\rho^2, \\
\rho^6\sigma = \sigma^5\tau_6\rho^5, & \rho^6\tau_2 = \sigma^3\tau_7\rho^7, & \rho^6\tau_5 = \sigma^5\tau_7\rho^8, \\
\rho^7\sigma = \sigma^2\tau_3\rho^4, & \rho^7\tau_2 = \sigma^4\tau_3\rho^6, & \rho^7\tau_5 = \sigma^3\tau_2\rho^4, \\
\rho^8\sigma = \sigma^6\tau_4\rho^8, & \rho^8\tau_2 = \sigma\tau_8\rho^4, & \rho^8\tau_5 = \sigma^2\tau_6\rho^6, \\
 & \rho^{\alpha}\tau_3 = \tau_3\rho^{-\alpha}. &
\end{array}$$

Since, now, τ_2, τ_3, and τ_5 generate the group $A(abcdefgh)_8$, and this in combination with σ generates the group H_{56}, the preceding relations are sufficient to insure that $\rho^{\alpha}H_{56} = H_{56}\rho^{\alpha}$.

There is therefore a group of order 504 in 9 letters. If the group of order 9 . 168 exists, it must contain the group of order 504, and beside this, for example, the substitution

$$\omega = ceg \,.\, dfh.$$

The latter transforms σ into σ^2. It transforms $A(abcdefgh)_8$ into itself. It transforms ρ into $aiedcbfgh = \rho\tau_7$. Consequently it transforms the subgroup of order 504 into itself. Then in combination with the latter it generates the required group of order 1512.

There is one and only one type of transitive group of order 9 . 168 *with* 9 *letters. This is obtained by adding to the compound* H_{168} *of* 8 *letters* $[=\{(bcedghf)_{21},\ A(abcdefgh)\}]$ *the substitution aichgbdef.*

Order 9 . 56. This has already been treated under the preceding case.

There is one and only one type of transitive group of order 504 *in* 9 *letters, viz., the combination of the* H_{56} *of* 8 *letters with, for example, aichgbdef.*

This group is particularly noteworthy in view of the fact that it is *simple*, i.e., that it contains no self-conjugate subgroup.* Its substitutions which affect 9 letters form 28 cyclical subgroups of order 9, no two of which have any substitution, except identity, in common.

Order 9 . 48. This group is based on the H_{48} of eight letters

$$\{ABC = ade \,.\, bcf,\quad ABCD = acfhbdeg\}.$$

Its even substitutions form a self-conjugate subgroup of order 9 . 24 based on the H_{24} of eight letters

$$\{ABC = ade \,.\, bcf,\quad AC \,.\, BD = afbe \,.\, chdg\}.$$

* Below order 660 the only simple groups of compound order are one group for each of the orders 60, 168, 360, and 504. Cf. Hölder: Math. Ann., vol. 40, p. 55, and Cole: Amer. Jour. of Math., vol. 14, p. 378.

The latter contains 1 or 4 conjugate subgroups of order 27.* In the former case the subgroup of order 27 is transitive,† and those of its substitutions which leave a single letter unchanged form a subgroup of order 3. This subgroup must be self-conjugate within the H_{24} of the 8 letters affected. This being impossible, it follows that there are 4 conjugate subgroups of order 27.

The substitutions of the group of order 9.24 transform these 4 subgroups in 24, 12, 8, or 4 ways. Accordingly the 4 subgroups are all transformed into themselves by 9, 18, 27, or 54 substitutions, which form a self-conjugate subgroup ‡ of the group of order 9.24. The first two cases both imply a self-conjugate subgroup of order 9; the last two one of order 27. The required group of order 9.24 must therefore contain a self-conjugate subgroup of order 9.

This subgroup is transitive. It cannot be cyclical; for a brief consideration shows that it could not then be transformed into itself by $AC.BD = afbe.chdg$. Accordingly, it contains 4 subgroups of order 3. At least one of these $\{\sigma\}$ must then be transformed into itself by $ABC = ade.bcf$. We can take only

$$\sigma_1 = ade.bcf.\begin{cases} ghi \\ gih \end{cases}, \quad \sigma_2 = ade.bfc.\begin{cases} ghi \\ gih \end{cases}.$$

Of these σ_1 must be rejected since $\sigma_1^2.ade.bcf$ affects only 3 letters. Also $ACD = afg.beh$ transforms σ_2 into

$$\tau_2 = fdh.egc.\begin{cases} abi \\ bai \end{cases};$$

and

$$ade.bfc.ghi \times fdh.egc.abi = ah.bdgfe.$$

The last substitution obviously cannot occur in the required group. Consequently we must take

$$\sigma_2 = ade.bfc.gih, \quad \tau_2 = fdh.egc.bai.$$

These generate a group of order 9.

$$\begin{array}{lll} 1, & ade.bfc.gih, & aed.bcf.ghi, \\ & aib.ceg.fdh, & abi.cge.fhd, \\ & ahc.bdg.eif, & ach.bgd.efi, \\ & agf.beh.cdi, & afg.bhe.cid, \end{array}$$

and this group is transformed into itself by $ade.bcf$ and $afbe.chdg$ and therefore by the entire H_{24} of 8 letters. As

* Cf. Sylow: Math. Ann., vol. 5, p. 584–94.

† Cf. Netto: Theory of Substitutions, American edition, p. 82.

‡ Cf. Hölder, l. c., p. 57.

the H_9 and the H_{24} have no substitutions in common, their combination generates a group of order 216.

The latter is furthermore transformed into itself by $ABCD = acfhbdeg$, and therefore its combination with this substitution generates the required group of order 432.

There is a single type of primitive group of order 432 *of* 9 *letters. This is a doubly transitive group obtained by adding to the* $H_{48} = \{ABC = ade . bcf,\ ABCD = acfhbdeg\}$ *the substitution ade . bfc . gih.*

Order 9.24. One case is disposed of under the preceding order.

There is a type of primitive group of order 216 *of* 9 *letters. It is composed of the even substitutions of the preceding group.*

In the remaining case the H_{24} of 8 letters is

$$(abcd\ .\ efgh)_{12}(ae\ .\ bf\ .\ cg\ .\ dh).$$

The required group contains, as before, a self-conjugate non-cyclical subgroup of order 9. For one of the substitutions of this we must take

$$\sigma = abc\ .\ egf\ .\left\{\begin{matrix} dhi \\ dih \end{matrix}\right.$$

This is transformed by $abd\ .\ efh$ into

$$\tau = bdc\ .\ fgh\ .\left\{\begin{matrix} aei \\ aie \end{matrix}\right.$$

But

$$\sigma\tau = \left\{\begin{matrix} adficeh \\ adehcif \end{matrix}\right.$$

Consequently there is no second group of order 9 . 24.

Order 9 . 16. The two groups of order 16 of 8 letters available for the present purpose are combinations of the H_8

$$\begin{matrix} 1, & ac\ .\ ef\ .\ gh, & ac\ .\ bd\ .\ eg\ .\ fh, & abcd\ .\ efgh, \\ & bd\ .\ eh\ .\ fg, & & adbc\ .\ ehgf, \\ & ab\ .\ cd\ .\ fh, & & \\ & ad\ .\ cb\ .\ eg, & & \end{matrix}$$

with one of the two substitutions

$$aebfcgdh, \quad afbgchde.$$

In both cases the group of order 9.16 contains a self-conjugate subgroup of order 9.8 composed of the even substitutions. Those of the latter which leave i unchanged are respectively

$$\begin{array}{lll|ll}
1, & ac\,.\,bd\,.\,eg\,.\,fh, & abcd\,.\,efgh, & 1,\ ac\,.\,bd\,.\,eg\,.\,fh, & abcd\,.\,efgh, \\
 & ag\,.\,bf\,.\,ce\,.\,dh, & adcb\,.\,ehgf; & & adcb\,.\,ehgf, \\
 & ae\,.\,bh\,.\,df\,.\,cg, & & & ahcf\,.\,bgde, \\
 & af\,.\,be\,.\,ch\,.\,dg, & & & afch\,.\,bedg, \\
 & ah\,.\,bg\,.\,cf\,.\,de, & & & agec\,.\,bhdf, \\
 & & & & aecg\,.\,bfdh.
\end{array}$$

The group of order 9.8 is readily shown to contain a single, non-cyclical subgroup of order 9. The substitutions of this may be written

$$\begin{array}{lll}
1, & i12\,.\,345\,.\,678, & i21\,.\,354\,.\,687, \\
 & i36\,.\,147\,.\,258, & i63\,.\,174\,.\,285, \\
 & i48\,.\,237\,.\,156, & i84\,.\,273\,.\,165, \\
 & i75\,.\,264\,.\,183, & i57\,.\,246\,.\,138.
\end{array}$$

On examination it will be found that the only substitution, in 8 letters, of order 2, and not affecting i, which transforms this group into itself is

$$12\,.\,36, \quad 48\,.\,57.$$

Consequently only the second group of order 16 above is available for the present purpose.

We may then assume $a = 1$, $c = 2$, $b = 3$, $d = 6$. It is then readily found that there is only one type of group of order 9.8, viz.:

$$\begin{array}{lll}
1, & iac\,.\,bgh\,.\,dfe, & ica\,.\,bhg\,.\,def, \\
 & ibd\,.\,agf\,.\,che, & idb\,.\,afg\,.\,ceh, \\
 & ige\,.\,cbf\,.\,ahd, & ieg\,.\,cfb\,.\,adh, \\
 & ifh\,.\,cdg\,.\,aeb, & ihf\,.\,cgd\,.\,abe,
\end{array}$$

combined with the second H_8 above.

This group is transformed into itself by $ac\,.\,ef\,.\,gh$.

Hence there is a single type of transitive group of order 144 *in* 9 *letters. This is generated by the substitutions*

$$afbgchde, \quad ac\,.\,ef\,.\,gh, \quad iac\,.\,bgh\,.\,dfe.$$

Order 9.8. It seems hardly necessary to continue further the detailed explanation of the process by which the possible groups are obtained. I give therefore merely the results in the remaining cases.

There are three types of primitive groups of order 72 *in* 9 *letters. These have for their bases the groups* 8_{20}, 8_{21}, *and* 8_{25} *of* 8 *letters. These are respectively generated by*

(i) $abcd\,.\,efgh$, $ac\,.\,fg\,.\,eh$, $iac\,.\,bgh\,.\,dfe$;

(ii) $afbgchde$, $iac\,.\,bgh\,.\,dfe$;

(iii) $abcd\,.\,efgh$, $abcf\,.\,bgde$, $iac\,.\,bgh\,.\,dfe$.

The last group is composed of the even substitutions of the group of order $9 \cdot 16$.

The first group is noteworthy as an instance of a simply transitive primitive group with substitutions leaving more than one letter unchanged. Such a group does not occur for less than 9 letters, and for 9 letters only in this one case.

Order 36. *There is a single type of this order, generated by*

$$abcd \, . \, efgh, \quad iac \, . \, bgh \, . \, dfe.$$

This group is also simply transitive.

The remaining orders do not occur for primitive groups.

The primitive groups of 9 letters are therefore the following. The exponents attached to the order indicate the degree of transitivity.

Order	
$9!^{9)}$	$(abcdefghi)$ all,
$\frac{1}{2}9!^{7)}$	$(abcdefghi)$ pos.,
$1512^{3)}$	$\{A(abcdefgh),\ (bcedghf)_{21},\ (aichgbdef)\text{cyc.}\}$,
$504^{3)}$	$\{A(abcdefgh),\ (bcedghf)\text{cyc.},\ (aichgbdef)\text{cyc.}\}$,
$432^{2)}$	$\{ade \, . \, bcf,\ acfhbdeg,\ ade \, . \, bfc \, . \, gih\}$,
$216^{2)}$	$\{ade \, . \, bcf,\ afbe \, . \, chdg,\ ade \, . \, bfc \, . \, gih\}$,
$144^{2)}$	$\{afbgchde,\ ac \, . \, ef \, . \, gh,\ iac \, . \, bgh \, . \, dfe\}$,
$72^{1)}$	$\{abcd \, . \, efgh,\ ac \, . \, ef \, . \, gh,\ iac \, . \, bgh \, . \, dfe\}$,
$72^{2)}$	$\{afbgchde,\ iac \, . \, bgh \, . \, dfe\}$,
$72^{2)}$	$\{abcd \, . \, efgh,\ ahcf \, . \, bgde,\ iac \, . \, bgh \, . \, dfe\}$,
$36^{1)}$	$\{abcd \, . \, efgh,\ iac \, . \, bgh \, . \, dfe\}$.

There are accordingly in all $23 + 11 = 34$ transitive groups of 9 letters.

CLASSIFICATION OF MATHEMATICS.

Index du répertoire bibliographique des sciences mathématiques, publié par la Commission permanente du répertoire. Paris, Gauthier-Villars, 1893. 8vo. 14 and 80 pp.

This is a new edition, with but slight modifications, of the proceedings (*procès-verbal sommaire*) of the International Congress for Mathematical Bibliography held at the time of the Paris Exhibition of 1889 (see Bulletin of the New York Mathematical Society, vol. 2, p. 190).

The main part of the pamphlet (pp. 1–80) is occupied with the very elaborate classification of the mathematical sciences which is to serve as "index" to a complete bibliography of modern mathematics (1800–1889). In comparison with the first edition of 1889, the cross-references are increased in number, some new subdivisions are introduced, and the typographical work is much improved.

This classification is prefaced by a very brief account of the work of the congress of 1889. The resolutions adopted by this congress are given in full; they explain the scope and general plan of the bibliography, which is to be by subjects in logical order and not by authors. Nothing, however, is said about the execution of the plan; nor is there any information given in this new edition as to the progress that may have been made with the work.

The permanent committee charged with the compilation of the *répertoire* is at present (February 15, 1893) constituted as follows: President, Poincaré; Secretary, d'Ocagne; Honorary Members, Prince R. Bonaparte, Prince B. Boncompagni, Darboux, Haton de la Goupillière; Members—from Austria, Em. Weyr; Belgium, Catalan, Le Paige; Denmark, Gram; France, the President of the French Mathematical Society, D. André, Fouret, Charles Henry, G. Humbert, Kœnigs, Laisant, Raffy; Germany, Lampe, Valentin; Great Britain, Glaisher, J. S. Mackay; Greece, Stephanos; Holland, Bierens de Haan, Schoute; Italy, Guccia; Norway, Holst; Portugal, G. Teixeira; Russia, Ligin; Sweden, Eneström; United States, Craig.

NOTES.

A REGULAR meeting of the NEW YORK MATHEMATICAL SOCIETY was held Saturday afternoon, June 3, at half-past three o'clock. In the absence of the president, Professor J. M. Van Vleck occupied the chair, Mr. Charles P. Steinmetz read a paper entitled "On the flow of an incompressible liquid between coaxial cylindrical surfaces." Professor W. Woolsey Johnson communicated a paper entitled "Negative reciprocal equations" by Commander J. E. Craig, U. S. N.

Professor Van Vleck read an extract of a recent letter from Professor Felix Klein, in which Professor Klein stated that he expected to visit America during the present summer and to attend the mathematical congress at Chicago. T. S. F.

THE international congress on mathematics, astronomy, and astro-physics at Chicago will be held in the new Art Institute building on the lake front during the week beginning August 21, 1893. There seems to be every indication of a most successful session. The programme in mathematics will include a series of reviews of the recent development of particular branches of the science. Every one interested in