

# THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF  
THE INSTITUTE OF MATHEMATICAL STATISTICS

## Articles

Log log laws for empirical measures	J. KUELBS AND R. M. DUDLEY	405
On the limiting behaviour of the mode and median of a sum of independent random variables	PETER HALL	419
Conditions for attaining $\bar{d}$ by a Markovian joining	MARTIN H. ELLIS	431
Zero-range interaction at Bose-Einstein speeds under a positive recurrent single particle law	ED WAYMIRE	441
Optimal stopping in an urn	WEN-CHEN CHEN AND NORMAN STARR	451
Laplace transforms for classes of life distributions	HENRY W. BLOCK AND THOMAS A. SAVITS	465
Martingale transform and random Abel-Dini series	LOUIS H. Y. CHEN	475
Weak convergence for the maxima of stationary Gaussian processes using random normalization	WILLIAM P. MCCORMICK	483
An extension to a strong law result of Mittal and Ylvisaker for the maxima of stationary Gaussian processes	WILLIAM P. MCCORMICK	498
A compound Poisson limit for stationary sums, and sojourns of Gaussian processes	SIMEON M. BERMAN	511
Classification of coharmonic and coinvariant functions for a Lévy process	MARTIN L. SILVERSTEIN	539
A note on domains of partial attraction	R. A. MALLER	576
Domains of partial attraction and tightness conditions	NARESH C. JAIN AND STEVEN OREY	584

## Short Communications

An independence property of Brownian motion with drift	J. G. WENDEL	600
On martingales in the limit	A. BELLOW AND A. DVORETZKY	602
Local convergence of a class of martingales in multidimensional time	RICHARD F. GUNDY	607
Tight bounds for the renewal function of a random walk	D. J. DALEY	615
A renewal model with randomly selected parameters	FREDERICK SOLOMON	622
Maxima of partial sums and a monotone regression estimator	ROBERT T. SMYTHE	630
A note on a mixing condition	RICHARD C. BRADLEY, JR.	636

---

Vol. 8, No. 3—June 1980

# THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

*The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.*

---

## OFFICERS

**President:**

George E. P. Box, Department of Statistics, University of Wisconsin, 1210 W. Dayton St., Madison, Wisconsin 53706

**President-Elect:**

Peter Bickel, Department of Statistics, University of California, Berkeley, California 94720

**Past-President:**

Samuel Karlin, Department of Mathematics, Stanford University, Stanford, California 94305

**Executive Secretary:**

Martin Fox, Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48824

**Treasurer:**

Heebok Park, Department of Statistics, California State University, Hayward, California 94542  
IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545

**Program Secretary:**

William L. Harkness, Pennsylvania State University, University Park, Pennsylvania 16801

**Editor: *Annals of Statistics***

David V. Hinkley, Department of Applied Statistics, University of Minnesota, St. Paul Campus, St. Paul, Minnesota 55108

**Editor: *Annals of Probability***

R. M. Dudley, Department of Mathematics, Room 2-245, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

**Managing Editor:**

Donald R. Truax, Department of Mathematics, University of Oregon, Eugene, Oregon 97403.

---

**Membership.** Membership dues including a subscription to one *Annals* and *The Institute of Mathematical Statistics Bulletin* are \$32 per year for all members. Special rates of \$15.00 per year are available to students. Rates in each category are one-third higher for members who wish both *Annals* as well as the *Bulletin*. Inquiries regarding membership in the institute should be sent to the Treasurer at the business office.

**Subscription Rates.** Current volumes (six issues per calendar year) of the *Annals of Probability* are \$40.00. Current volumes (six issues per calendar year) of the *Annals of Statistics* are \$45.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$8.00. Back numbers of both *Annals* and the *Annals of Mathematical Statistics* (Volumes 1 through 43) may be purchased from the Treasurer.

**The Annals of Probability**, Volume 8, Number 3, June 1980. Published bimonthly in February, April, June, August, October, and December by The Institute of Mathematical Statistics, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.

---

Mail to the *Annals of Probability* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should **not** be addressed to Waverly Press.

---

Printed at the  
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage at Hayward, California and at additional mailing offices

Copyright © 1980 by the Institute of Mathematical Statistics

## EDITORIAL STAFF

### EDITOR

R. M. DUDLEY

### ASSOCIATE EDITORS

ALEXANDRA BELLOW  
SIMEON M. BERMAN  
RABI BHATTACHARYA  
MIKLÓS CSÖRGŐ  
C. A. DOLÉANS-DADE  
VÁCLAV FABIAN

C. C. HEYDE  
NARESH C. JAIN  
MARK KAC  
HARRY KESTEN  
THOMAS G. KURTZ  
M. R. LEADBETTER

THOMAS M. LIGGETT  
WALTER PHILIPP  
STANLEY SAWYER  
DAVID O. SIEGMUND  
LAJOS TAKÁCS  
JOHN B. WALSH

### EDITORIAL ASSISTANT

CYNTHIA FRIEDMAN

### MANAGING EDITOR

D. R. TRUAX

### EDITORIAL ASSISTANTS

RALPH KRUMDIECK

ESTHER ELLEN NOVITSKI

---

### PAST EDITORS ANNALS OF MATHEMATICAL STATISTICS

H. C. CARVER, 1930-1938  
S. S. WILKS, 1938-1949  
T. W. ANDERSON, 1950-1952  
E. L. LEHMANN, 1953-1955  
T. E. HARRIS, 1955-1958

WILLIAM KRUSKAL, 1958-1961  
J. L. HODGES, JR., 1961-1964  
D. L. BURKHOLDER, 1964-1967  
Z. W. BIRNBAUM, 1967-1970  
INGRAM OLKIN, 1970-1972

### ANNALS OF PROBABILITY

RONALD PYKE, 1972-1975  
PATRICK BILLINGSLEY, 1976-1978

### ANNALS OF STATISTICS

INGRAM OLKIN, 1972-1973  
I. R. SAVAGE, 1974-1976  
RUPERT G. MILLER, JR., 1977-1979

---

## EDITORIAL POLICY

The main purpose of the *Annals of Probability* and the *Annals of Statistics* is to publish contributions to the theory of probability and statistics and to their applications. The emphasis is on importance and interest, not formal novelty and correctness. Especially appropriate are authoritative expository papers and surveys of areas in vigorous development. All papers are refereed.

---

## IMS INSTITUTIONAL MEMBERS

**AEROSPACE CORPORATION**  
El Segundo, California

**ARIZONA STATE UNIVERSITY**  
Tempe, Arizona

**BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY**  
Murray Hill, N.J.

**BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS**  
Bowling Green, Ohio

**CALIFORNIA STATE UNIVERSITY, FULLERTON, DEPARTMENT OF MATHEMATICS**  
Fullerton, California

**CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS**  
Hayward, California

**CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF MATHEMATICS**  
Cleveland, Ohio

**CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS**  
Ithaca, New York

**FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS**  
Tallahassee, Florida

**FORD MOTOR COMPANY, ENGINEERING AND RESEARCH LIBRARY**  
Dearborn, Michigan

**GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES**  
Warren, Michigan

**GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS**  
Washington, D.C.

**INDIANA UNIVERSITY, MATHEMATICS DEPT.**  
Bloomington, Indiana

**INSTITUTE FOR DEFENSE ANALYSES**  
Arlington, Virginia

**INTERNATIONAL BUSINESS MACHINES CORPORATION**  
Armonk, New York

**IOWA STATE UNIVERSITY, STATISTICAL LABORATORY**  
Ames, Iowa

**JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIostatistics, DEPARTMENT OF MATHEMATICAL SCIENCES**  
Baltimore, Maryland

**KANSAS STATE UNIVERSITY, STATISTICS DEPARTMENT**  
Manhattan, Kansas

**MARQUETTE UNIVERSITY, MATHEMATICS AND STATISTICS DEPARTMENT**  
Milwaukee, Wisconsin

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY MATHEMATICS DEPARTMENT**  
Cambridge, Massachusetts

**MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS**  
Oxford, Ohio

**MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS AND PROBABILITY**  
East Lansing, Michigan

**NATIONAL SECURITY AGENCY**  
Fort George G. Meade, Maryland

**NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**  
Las Cruces, New Mexico

**NORTHERN ILLINOIS UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**  
De Kalb, Illinois

**NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS**  
Evanston, Illinois

**OHIO STATE UNIVERSITY, DEPARTMENT OF STATISTICS**  
Columbus, Ohio

**OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS**  
Corvallis, Oregon

**PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF STATISTICS**  
University Park, Pennsylvania

**PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS**  
Princeton, New Jersey

**PURDUE UNIVERSITY LIBRARIES**  
Lafayette, Indiana

**QUEEN'S UNIVERSITY, DEPT. OF MATHEMATICS, KINGSTON**  
Ontario, Canada

**RICE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**  
Houston, Texas

**THE ROCKEFELLER UNIVERSITY**  
New York, New York

**SANDIA CORPORATION, SANDIA BASE**  
Albuquerque, New Mexico

**SIMON FRASER UNIVERSITY, MATHEMATICS DEPARTMENT**  
Burnaby, Canada

**SOUTHERN ILLINOIS UNIVERSITY, MATHEMATICAL STUDIES**  
Edwardsville, Illinois

**SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS**  
Dallas, Texas

**STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY**  
Stanford, California

**STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS**  
Amherst, New York

**TEMPLE UNIVERSITY, MATHEMATICS DEPARTMENT**  
Philadelphia, Pa.

**TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS**  
Lubbock, Texas 79409

- THE TOBACCO INSTITUTE  
Washington, D.C.
- UNION OIL COMPANY OF CALIFORNIA, UNION  
RESEARCH CENTER  
Brea, California
- UNITED STATES ARMY RESEARCH AND DEVEL-  
OPMENT CENTER  
Aberdeen Proving Ground, Maryland
- UNIVERSITY OF ALBERTA, DEPARTMENT OF  
MATHEMATICS  
Edmonton, Alberta, Canada
- UNIVERSITY OF ARIZONA, DEPARTMENT OF  
MATHEMATICS AND COMMITTEE ON STA-  
TISTICS  
Tucson, Arizona
- UNIVERSITY OF BRITISH COLUMBIA, DEPART-  
MENT OF MATHEMATICS  
Vancouver, B.C., Canada
- UNIVERSITY OF CALGARY, MATHEMATICS DE-  
PARTMENT  
Calgary 44, Alberta, Canada
- UNIVERSITY OF CALIFORNIA, BERKELEY, STA-  
TISTICAL LABORATORY  
Berkeley, California
- UNIVERSITY OF CINCINNATI, DEPARTMENT OF  
MATHEMATICAL SCIENCES  
Cincinnati, Ohio
- UNIVERSITY OF GUELPH, MATHEMATICS AND  
STATISTICS DEPARTMENT  
Guelph, Ontario, Canada
- UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE,  
DEPARTMENT OF MATHEMATICS  
Chicago, Illinois
- UNIVERSITY OF ILLINOIS, MATHEMATICS DEPT.  
Urbana, Illinois
- UNIVERSITY OF IOWA, DIVISION OF MATHE-  
MATICAL SCIENCES  
Iowa City, Iowa
- UNIVERSITY OF MANITOBA, DEPARTMENT OF  
STATISTICS  
Winnipeg, Manitoba, Canada
- UNIVERSITY OF MARYLAND, DEPARTMENT OF  
MATHEMATICS  
College Park, Maryland
- UNIVERSITY OF MASSACHUSETTS, MATHEMAT-  
ICS AND STATISTICS DEPARTMENT  
Amherst, Massachusetts
- UNIVERSITY OF MICHIGAN, DEPARTMENT OF  
STATISTICS  
Ann Arbor, Michigan
- UNIVERSITY OF MINNESOTA, SCHOOL OF STA-  
TISTICS  
Minneapolis, Minnesota
- UNIVERSITY OF MISSOURI, DEPARTMENT OF  
STATISTICS  
Columbia, Missouri
- UNIVERSITY OF MISSOURI AT ROLLA, DEPART-  
MENT OF MATHEMATICS  
Rolla, Missouri
- UNIVERSITY OF MONTREAL, DEPARTMENT OF  
MATHEMATICS  
Montreal, Quebec, Canada
- UNIVERSITY OF NEBRASKA, MATHEMATICS  
AND STATISTICS DEPARTMENT  
Lincoln, Nebraska
- UNIVERSITY OF NEW MEXICO, DEPARTMENT  
OF MATHEMATICS AND STATISTICS  
Albuquerque, New Mexico
- UNIVERSITY OF NORTH CAROLINA, DEPART-  
MENT OF STATISTICS  
Chapel Hill, North Carolina
- UNIVERSITY OF OREGON, MATHEMATICS DE-  
PARTMENT  
Eugene, Oregon
- UNIVERSITY OF OTTAWA, DEPARTMENT OF  
MATHEMATICS  
Ottawa, Ontario, Canada
- UNIVERSITY OF ROCHESTER, LIBRARY  
Rochester, New York
- UNIVERSITY OF SOUTH CAROLINA, DEPART-  
MENT OF MATHEMATICS AND COMPUTER  
SCIENCE  
Columbia, South Carolina
- UNIVERSITY OF TEXAS, DEPARTMENT OF  
MATHEMATICS  
Austin, Texas
- UNIVERSITY OF TEXAS, MATHEMATICS DEPT.  
San Antonio, Texas
- UNIVERSITY OF UTAH, DEPT. OF MATHEMATICS  
Salt Lake City, Utah
- UNIVERSITY OF VICTORIA, DEPT. OF MATHE-  
MATICS  
Victoria, British Columbia, Canada
- UNIVERSITY OF WASHINGTON, DEPARTMENT  
OF MATHEMATICS  
Seattle, Washington
- UNIVERSITY OF WATERLOO, STATISTICS DE-  
PARTMENT  
Waterloo, Ont., Canada
- UNIVERSITY OF WISCONSIN, MADISON, DE-  
PARTMENT OF STATISTICS  
Madison, Wisconsin
- UNIVERSITY OF WISCONSIN, MILWAUKEE, DE-  
PARTMENT OF MATHEMATICS  
Milwaukee, Wisconsin
- VIRGINIA COMMONWEALTH UNIVERSITY, DE-  
PARTMENT OF MATHEMATICAL SCIENCES  
Richmond, Virginia
- WAYNE STATE UNIVERSITY, DEPARTMENT OF  
MATHEMATICS  
Detroit, Michigan
- WEST CHESTER STATE COLLEGE  
West Chester, Pennsylvania
- WESTINGHOUSE ELECTRIC CORPORATION, RE-  
SEARCH LABORATORIES  
Pittsburgh, Pennsylvania

# THE ANNALS OF PROBABILITY

## INSTRUCTIONS FOR AUTHORS

**Submission of Papers.** Papers to be submitted for publication should be sent to the Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will *not* normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

**Preparation of Manuscripts.** Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

**Submission of Reference Papers.** Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

**Title and Abbreviated Title.** The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

**Summary.** Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary," followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to theories or formulas in the body of the paper—it should be self-contained.

**Footnotes.** Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

**Key Words.** Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—.  
Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to *Mathematical Reviews* Index to Volume 39, June 1970. (See, also, *The Notices of the American Mathematical Society*, June 1970; *Bulletin of the Institute of Mathematical Statistics*, September 1974; or a current index issue of *Mathematical Reviews*.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

**Identification of Symbols.** Manuscripts for publication should be clearly prepared to insure that all symbols are properly identified. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay," etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics. Acronyms should be introduced sparingly.

**Figures and Tables.** Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

**Formulas.** Fractions in the text are preferably written with the solidus or negative exponent;

thus,  $(a + b)/(c + d)$  is preferred to  $\frac{a + b}{c + d}$ , and  $(2\pi)^{-1}$  or  $1/(2\pi)$  to  $2\pi$ . Also,  $a^{b(c)}$  and  $a_{b(c)}$  are pre-

ferred to  $a^{bc}$  and  $a_{bc}$ , respectively. Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

**References.** References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. *Ann. Math. Statist.* 15 229-282.

In textual material, the format "... Doob (1944) ..." is normally preferred to "... Doob [5]..." Multiple references can be distinguished as "... Doob (1944a)..." Abbreviations for journals should be taken from a current index issue of *Mathematical Reviews*.

**Proofs.** Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.)