# THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF
THE INSTITUTE OF MATHEMATICAL STATISTICS

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Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

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Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

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The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use in the *Mathematical Reviews Annual Subject Index*-1980. The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

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$$(2\pi)^{-1}$$
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[5] Doob, J. L. (1944). The elementary Gaussian processes. Ann. Math. Statist. 15 229–282.

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