# THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938 EDITED BY S. S. WILKS, 1938-1949

### THE OFFICIAL JOURNAL OF THE INSTITUTE OF MATHEMATICAL STATISTICS

#### Contents

A multi-dimensional linear growth birth and death process	727
Weak convergence of a two-sample empirical process and a new approach to Chernoff-Savage theorem	755
Most powerful tests for some non-exponential familiesEmil Spjøtvoll	772
Estimation of stochastic systems; arbitrary system process with additive white noise observations error	785
A potential theory for supermartigales E. Count Curtis	802
A system of linear differential equations for the distribution of Hotelling's generalized $T_0^2$	815
On the non-central distribution of the second elementary symmetric function of the roots of a matrixK. C. Sreedharan Pillai and Arjun K. Gupta	833
Inference problems about parameters which are subjected to changes over time Chandan Mustafi	
On the distributions of direction and collianearity factors in discriminant analysis D. G. Kabe	855
Minimization of eigenvalues of a matrix and optimality of principal components  Masashi Okamoto and Mitsuyo Kanazawa	859
Limit theorems for the multi-urn Ehrefest modelDonald L. Iglehart	
On the moment generating function of Pillai's $V^{(s)}$ criterionK. C. Sreedharan Pillai	
Moment convergence of sample extremesJames Pickands III	
On the probability distribution of a filtered random telegraph signal . David S. Newman	890
An extension of Rosen's theorem to non-identically distributed random variables L. H. Koopmans	897
Asymptotic normality in nonparametric methods	
The VNM two-sample test	
On continuity properties of infinitely divisible distribution functions. Steven Orex	
Functions of processes with Markovian statesMartin Fox and Herman Rubin	
The distribution of the maximum of a semi-Markov process Lawrence D. Stone	
Upper and lower probabilities generated by a random closed interval. A. P. Dempster On Chernoff-Savage tests for ordered alternatives in randomized blocks	957
Madan Lal Puri and Pranab Kumar Sen	967
Bayes sequential design of fractional factorial experiments for the estimation of a subgroup of pre-assigned parametersS. Zacks	973

Continued on back cover

PAGE

Vol. 39, No. 3-June 1968

## THE ANNALS OF MATHEMATICAL STATISTICS

Subscription Rates. Current issues are \$30 per volume (six issues of one calendar year) in the U.S. and Canada, \$22.50 per volume elsewhere. Single issues are \$5.75. Back numbers for all issues up to and including 1956 (Volume 27) are \$16.50 per volume, \$4.50 per issue; issues from 1957 through 1964 are \$22.00 per volume, \$5.75 per issue; from 1965 onward, \$33.00 per volume, \$5.75 per issue. Complete sets (Volumes 1 to 27) are \$275.00. If additional volumes are ordered at the same time complete sets are ordered, the price is \$16.50 per volume for Volume 28 to Volume 35, and \$22.00 per volume for Volume 36 onward.

Rates to members of the Institute of Mathematical Statistics are lower (see inside back cover).

Communications concerning subscriptions, back numbers, payment of dues, membership, changes of address, etc., should be addressed to Professor George J. Resnikoff, Treasurer, Department of Statistics, California State College, Hayward, California 94542.

Papers to be submitted for publication should be sent to the Editor, Z. W. Birnbaum, Department of Mathematics, University of Washington, Seattle, Washington 98105. Abstracts and corrected galley proofs should be sent to the Managing Editor, P. L. Meyer, Department of Mathematics, Washington State University, Pullman, Washington 99163.

Preparation of manuscripts. Each manuscript should be typewritten, double spaced with wide margins at sides, top, and bottom, and the original should be submitted with one additional copy, on paper that will take corrections. Dittoed or mimeographed papers are acceptable only if completely legible. Footnotes should be reduced to a minimum, and where possible replaced by remarks in the text, or bibliography at the end of the paper; formulae in footnotes should be avoided. References should follow the style of 33 313-314.

Figures, charts, and diagrams should be professionally drawn on plain white paper or tracing cloth in black India ink twice the size they are to be printed.

Authors are asked to keep in mind the typographical difficulties of complicated mathematical formulae. The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between such pairs as zero and the letter O, the numeral 1 and the letter l, numeral 1 used as superscript and prime ('), alpha and a kappa and k, mu and u, nu and v, eta and n, etc. Subscripts or superscripts should be clearly below or above the line. Bars above groups of letters (e.g., log x) and underlined letters (e.g., x) are difficult to print and should be avoided. Symbols are automatically italicized by the printer and should not be underlined on manuscripts. Boldface letters may be indicated by underlining with a wavy line on the manuscript; boldface subscripts and superscripts are not available. Complicated exponentials should be represented with the symbol exp. In writing square roots the fractional exponent is preferable to the radical sign. Fractions are preferably written with the

solidus; thus 
$$(a + b)/(c + d)$$
 rather than  $\frac{a+b}{c+d}$ , and  $ab/cd$  rather than  $\frac{ab}{cd}$ .

Authors will ordinarily receive only galley proofs. Fifty reprints without covers will be furnished free. Additional reprints and covers will be furnished at cost.

Mail to the Annals of Mathematical Statistics should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press.

COMPOSED AND PRINTED AT THE

WAVERLY PRESS, Inc., Baltimore, Maryland 21202 U.S.A.

Second-class postage paid at Baltimore, Maryland 21202

#### EDITORIAL STAFF

#### EDITOR 1

Z. W. BIRNBAUM

#### ASSOCIATE EDITORS

R. R. BAHADUR A. P. DEMPSTER SYLVAIN EHRENFELD THOMAS S. FERGUSON RONALD K. GETOOR FRANKLIN A. GRAYBILL W. J. HALL JAMES F. HANNAN DAVID L. HANSON SAMUEL KARLIN E. L. LEHMANN EUGENE LUKACS INGRAM OLKIN FRANK PROSCHAN RONALD PYKE JEROME SACKS FRANK L. SPITZER HENRY TEICHER

#### MANAGING EDITOR

P. L. MEYER

#### PAST EDITORS OF THE ANNALS

H. C. Carver, 1930-1938
S. S. Wilks, 1938-1949
William Kruskal, 1958-1961
T. W. Anderson, 1950-1952
J. L. Hodges Jr., 1961-1964
E. L. Lehmann, 1953-1955
D. L. Burkholder, 1964-1967

Published bimonthly by the Institute of Mathematical Statistics in February, April, June, August, October and December.

#### IMS INSTITUTIONAL MEMBERS

Aerospace Corporation, El Segundo, California

ARIZONA STATE UNIVERSITY, Tempe, Arizona

ARTHUR D. LITTLE, INC., Cambridge, Massachusetts

Ballistic Research Labs., Aberdeen Proving Ground, Md.

Bell Telephone Laboratories, Inc., Technical Library, New York, N. Y.

Boeing Airplane Company, Seattle, Washington

Case Institute of Technology, Statistical Laboratory, Cleveland, Ohio Catholic University of America, Stat. Lab., Math. Dept., Washington, D. C.

CHEVRON RESEARCH Co., Richmond, California

CIBA PHARMACEUTICAL Co., Summit, New Jersey

COLORADO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS AND STATISTICS, Fort Collins, Colorado

CORNELL UNIVERSITY, MATHEMATICS DEPARTMENT, Ithaca, New York

FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS, Tallahassee, Florida

FORD MOTOR COMPANY, Dearborn, Michigan

GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES, Warren, Michigan GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS, Washington, D. C.

W. R. GRACE AND COMPANY, RESEARCH DIVISION, Clarksville, Maryland

HUGHES AIRCRAFT COMPANY RESEARCH LABS., Malibu, California

Indiana University, The Library, Bloomington, Indiana

INTERNATIONAL BUSINESS MACHINES CORPORATION, Armonk, New York

IOWA STATE UNIVERSITY, STATISTICAL LABORATORY, Ames, Iowa

Johns Hopkins University, Department of Biostatistics, Department of Statistics, Baltimore, Maryland

Kansas State University, Department of Statistics, Manhattan, Kansas

Knolls Atomic Power Laboratory, Schenectady, New York

LOCKHEED AIRCRAFT CORPORATION, ENGINEERING LIBRARY, Burbank, California Michigan State University, Department of Statistics, East Lansing, Michigan

MINNESOTA MINING AND MANUFACTURING COMPANY, APPLIED MATHEMATICS AND STATISTICS, St. Paul, Minnesota

Monsanto Chemical Company, St. Louis, Missouri

NATIONAL CASH REGISTER COMPANY, RESEARCH DEPARTMENT, Dayton, Ohio

NATIONAL SECURITY AGENCY, Fort George G. Meade, Maryland

New Mexico State University, Department of Mathematical Sciences, Las Cruces, New Mexico

NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS, EVANSTON, Illinois Ohio State University, Statistics Laboratory, Department of Mathematics, Columbus, Ohio

OKLAHOMA STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS, Stillwater, Oklahoma

OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS, Corvallis, Oregon Pennsylvania State University, Department of Mathematics, University Park, Penna.

Princeton University, Dept. of Math., Sect. of Math. Stat., Princeton, New Jersey

PROCTER & GAMBLE COMPANY, TECHNICAL LIBRARY, Cincinnati, Ohio

Purdue University Libraries, Lafayette, Indiana

ROCKFELLER UNIVERSITY, New York, N. Y.

ROCKLAND STATE HOSPITAL, Orangeburg, New York

Sandia Corporation, Sandia Base, Albuquerque, New Mexico

SOUTHERN METHODIST UNIVERSITY, Department of Statistics, Dallas, Texas Stanford University, Girshick Memorial Library, Stanford, California System Development Corporation, Santa Monica, California

Texas A & M University, Institute of Statistics, College Station, Texas

THE RAND CORPORATION, Santa Monica, California

TRW Systems, Inc., Redondo Beach, California

Union Oil Company of California, Únion Research Center, Brea, California United States Steel Corporation Library, Monroeville, Pennsylvania University of Arizona, Tucson, Arizona

University of British Columbia, Department of Mathematics, Vancouver, B. C.

University of California, Department of Mathematics, Irvine, California University of California, Department of Mathematics, Santa Barbara, California

University of California, Department of Mathematics, Riverside, California

University of California, Statistical Laboratory, Berkeley, California

University of Georgia, Department of Statistics, Athens, Georgia

University of Illinois at Chicago Circle, Department of Mathematics, Chicago, Illinois

University of Illinois, Serials Department, Urbana, Illinois

University of Iowa, Iowa City, Iowa

University of Manitoba, Department of Statistics, Winnipeg 19, Manitoba, Canada

University of Maryland, Department of Mathematics, College Park, Maryland

University of Michigan, Department of Mathematics, Ann Arbor, Michigan University of Minnesota, Department of Statistics, Minneapolis, Minnesota

University of Missouri, Department of Statistics, Columbia, Missouri University of Missouri at Rolla, Department of Mathematics, Rolla, Missouri

University of Montreal, Department of Mathematics, Montreal, Quebec, Canada

University of New Mexico, Albuquerque, New Mexico

University of North Carolina, Department of Statistics, Chapel Hill, North Carolina

University of Rochester, Rochester, New York

University of Washington, Laboratory of Statistical Research, Seattle, Washington

University of Wisconsin, Madison, Department of Statistics, Madison, Wisconsin

University of Wisconsin, Milwaukee, Department of Mathematics, Milwaukee, Wisconsin

WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS, Detroit, Michigan Westat Research, Inc., Bethesda, Maryland

Westinghouse Electric Corporation, Research Laboratories, Pittsburgh, Penna.