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SIMPLE ZEROS OF DEDEKIND ZETA FUNCTIONS

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Abstract: Using Stechkin's lemma we derive explicit regions of the half complex plane $\Re(s) \leqslant 1$ in which the Dedekind zeta function of a number field K has at most one complex zero, this zero being real if it exists. These regions are Stark-like regions, i.e. given by all $s = \beta + i\gamma$ with $\beta \geqslant 1 - c/\log d_K$ and $|\gamma| \leqslant d/\log d_K$ for some absolute positive constants c and d. These regions are larger and our proof is simpler than recently published such regions and proofs.

Keywords: Dedekind zeta function, Siegel zero.

1. Introduction

Let $d_K > 1$ and $\zeta_K(s)$ denote the absolute value of the discriminant and the Dedekind zeta function of a number field K of degree $n = r_1 + 2r_2 > 1$, with r_1 real places and r_2 complex places. It is known that $\zeta_K(s)$ has a meromorphic continuation to the complex plane, with a single pole: a simple pole at $s = 1$. It is also known that $\zeta_K(s)$ has no complex zero in the complex half-plane $\Re(s) \geqslant 1$. For $c > 0$ and $d \geqslant 0$ we let $S(c, d)$ denote the region given by all $s = \beta + i\gamma$ with $\beta \geqslant \sigma_c := 1 - c/\log d_K$ and $|\gamma| \leqslant t_d := d/\log d_K$. In 1974, H. M. Stark proved an explicit result:

Theorem 1 ([6, Lemma 3]). *A Dedekind zeta function $\zeta_K(s)$ has at most one zero in the region $S(1/4, 1/4)$; if such a zero exists, it is real and simple.*

As noted in [2, Lemma 2] Stark's Theorem 1 holds true in the region $S(2(\sqrt{2}-1)^2, 0)$. In fact, we will show that Stark's proof readily yields:

Theorem 2. *Set $c_1 := 2(\sqrt{2}-1)^2 = 0.34314\cdots$ and $d_1 := 2\frac{1-S_0-S_0^2}{2+S_0} = 0.27644\cdots$, where $S_0 = 0.45433\cdots$ is the only root of $S^4+2S^3-2S^2-4S+2$ in $[0, \sqrt{2}]$. Then Theorem 1 holds true in the regions $S(c_1, 0)$ and $S(d_1, d_1)$.*

By [3, Theorem 1.1 and Corollary 1.2], Theorem 1 holds true in the regions $S(1/2, 1/2)$, $S(1/12.74, 1)$ and $S(1/1.7, 1/4)$, for d_K large enough. By [1, Theorem 1], Theorem 1 holds true in the region $S(1/2, 1/2)$ without any restriction

2010 Mathematics Subject Classification: primary: 11R42; secondary: 11R29 on d_K. By [8, Corollary 1.2], Theorem 1 holds true in (a slightly smaller region than the) the region $S(0.0875, 1)$, for d_K large enough (notice that $1/12.74 = 0.078492\cdots < 0.0875$). We use Stechkin's Lemma and our approach introduced in [5] to improve upon these results (see [4] for another recent application of Stechkin's Lemma):

Theorem 3. *Set* $\lambda := 1 - 1/\sqrt{5}$, $c_2 := c_1/\lambda = 0.62075\cdots$ *and* $d_2 := d_1/\lambda = 0.50009\cdots$, *where* c_1 *and* d_1 *are as in Theorem 2. Then Theorem 1 holds true in the regions* $S(c_2, 0)$, $S(d_2, d_2)$.

Theorem 1 also holds true in the region $S(0.59110, 1/4)$ *(notice that* $0.59110 > 1/1.7 = 0.58823\cdots$*) and for* $d_K \geqslant 8$ *in the region* $S(0.10379, 1)$.

We would like to mention that our proof is simpler than the ones in [1], [3] or [8]. However, our regions are Stark-like regions whereas in [3] and [8] asymptotically valid regions of not larger widths (still of the type $1 - c/\log d_K \leqslant \beta \leqslant 1$) but of much larger heights ($|\gamma| \leqslant 1$ instead of $|\gamma| \leqslant d/\log d_K$) are obtained.

2. Proof of Theorem 2

Let $\sigma > 1$ be real. Let K be a number field of degree n. Then

$$0 < Z_K(\sigma) := -\frac{\zeta_K'}{\zeta_K}(\sigma) = \sum_{k \geqslant 2} \frac{\Lambda_K(k)}{k^\sigma} \qquad (\sigma > 1), \tag{1}$$

where $\Lambda_K(k) \geqslant 0$ for $k \geqslant 2$, and

$$Z_K(\sigma) + \sum_\rho \Re\left(\frac{1}{\sigma - \rho}\right) = \frac{1}{\sigma - 1} + \frac{1}{2}\log d_K + \frac{1}{\sigma} + h(r_1, r_2, \sigma) \tag{2}$$

(e.g., see [6, Proof of Lemma 3]), where ρ runs over the complex zeros of $\zeta_K(s)$ such that $0 < \Re(\rho) < 1$ (counted with their multiplicities) and where

$$h(r_1, r_2, \sigma) = \frac{r_1}{2}\left(\Psi(\sigma/2) - \log\pi\right) + r_2\left(\Psi(\sigma) - \log(2\pi)\right), \tag{3}$$

with $\Psi(s) = (\Gamma'/\Gamma)(s)$. Since $\Psi(\sigma)$ increases with $\sigma > 0$ and since $r_1 \geqslant 2$ or $r_2 \geqslant 1$, it is easily seen that $1/\sigma + h(r_1, r_2, \sigma)$ is an increasing function of $\sigma > 1$ which is negative in the range $1 < \sigma \leqslant 5$.

For the remainder of this section we assume that $1 < \sigma \leqslant 5$.

Since $Z_K(\sigma)$ is positive and since each contribution

$$\Re\left(\frac{1}{\sigma - \rho}\right) = \frac{\sigma - \Re(\rho)}{|s - \rho|^2}$$

is positive for $0 < \Re(\rho) < 1 < \sigma$, we obtain

$$\frac{2}{\sigma - \sigma_c} \leqslant \Re\left(\frac{1}{\sigma - \beta_1}\right) + \Re\left(\frac{1}{\sigma - \beta_2}\right) < \frac{1}{\sigma - 1} + \frac{1}{2}\log d_K \qquad (1 < \sigma \leqslant 5) \tag{4}$$

if $\zeta_K(s)$ has at least two distinct real zeroes $\beta_2 \neq \beta_1$ or a double real zero $\beta_2 = \beta_1$ in the range $S(c, 0)$, and

$$\frac{2(\sigma-\beta)}{(\sigma-\beta)^2+\gamma^2} = 2\Re\left(\frac{1}{\sigma-\rho}\right) < \frac{1}{\sigma-1} + \frac{1}{2}\log d_K \qquad (1 < \sigma \leqslant 5) \tag{5}$$

if $\zeta_K(s)$ has at least one non-real zero $\rho = \beta + i\gamma$ in the region $S(c, d)$, for in that case ρ and $\bar{\rho}$ are two distinct zeroes of $\zeta_K(s)$ in $S(c, d)$.

Lemmas 7 and 8 applied with $A = \log d_K$ yield the desired results (notice that $1 + 2(\sqrt{2}-1)/\log d_K \leqslant 1 + 2(\sqrt{2}-1)/\log 3 < 5$ and $1 + 2S_0/\log d_K \leqslant 1 + 2S_0/\log 3 \leqslant 5$).

3. Stechkin's trick and proof of Theorem 3

Since $Z_K(\sigma)$ is a positive valued and decreasing function of $\sigma > 1$, by (1), we have

$$Z_K(\sigma) - \kappa Z_K(\tau) \geqslant Z_K(\sigma) - Z_K(\tau) \geqslant 0 \qquad \text{for } 0 \leqslant \kappa \leqslant 1 < \sigma \leqslant \tau.$$

Using (2) twice, we deduce that for $0 \leqslant \kappa \leqslant 1 < \sigma \leqslant \tau$ we have

$$\sum_{\rho} S(\sigma, \tau, \rho) \leqslant \frac{1}{\sigma-1} + \frac{1-\kappa}{2}\log d_K + F_2(\sigma, \tau) + R(r_1, r_2, \sigma, \tau), \tag{6}$$

where

$$F_2(\sigma, \tau) = -\frac{\kappa}{\tau-1} + \frac{1}{\sigma} - \frac{\kappa}{\tau}, \tag{7}$$

$R(r_1, r_2, \sigma, \tau) := h(r_1, r_2, \sigma) - \kappa h(r_1, r_2, \tau)$ and

$$S(\sigma, \tau, \rho) := \Re\left(\frac{1}{\sigma-\rho}\right) - \kappa\Re\left(\frac{1}{\tau-\rho}\right) = S(\sigma, \tau, \bar{\rho}).$$

Now, we use Stechkin's Lemma:

Lemma 4 ([7, Lemma 2]). *Suppose that $\sigma > 1$. Set $\tau = (1+\sqrt{1+4\sigma^2})/2$ and $\kappa = 1/\sqrt{5}$. If $0 < \Re(\rho) < 1$, then*

$$\Re\left(\frac{1}{\sigma-\rho}\right) + \Re\left(\frac{1}{\sigma-(1-\rho)}\right) \geqslant \kappa\left\{\Re\left(\frac{1}{\tau-\rho}\right) + \Re\left(\frac{1}{\tau-(1-\rho)}\right)\right\}.$$

Hence, **from now on**, we take $\kappa = 1/\sqrt{5}$ and $\tau = (1+\sqrt{1+4\sigma^2})/2$. Notice that

$$F_2(\sigma, \tau) = \frac{(\sigma^2-1)/5\sigma^2}{\sigma + \sqrt{(1+4\sigma^2)/5}} > 0. \tag{8}$$

The complex zeros ρ of $\zeta_K(s)$ with $0 < \Re(\rho) < 1$ come in pairs $\{\rho, 1-\rho\}$, and their combined contributions

$$T(\sigma, \tau, \rho) := S(\sigma, \tau, \rho) + S(\sigma, \tau, 1-\rho) = T(\sigma, \tau, 1-\rho)$$

are positive, by Lemma 4. Hence, we have:

Lemma 5. *Suppose that $\sigma > 1$. Set $\tau = (1+\sqrt{1+4\sigma^2})/2$ and $\kappa = 1/\sqrt{5}$. For any finite set Z of complex zeroes of $\zeta_K(s)$ in the region $1/2 < \Re(s) < 1$, we have*

$$\sum_{\rho} S(\sigma,\tau,\rho) \geqslant \sum_{\rho\in Z} T(\sigma,\tau,\rho), \tag{9}$$

where

$$T(\sigma,\tau,\rho) = \frac{1}{\sigma-\rho} + F_3(\sigma,\tau,\rho)$$

with

$$F_3(\sigma,\tau,\rho) = -\kappa\Re\left(\frac{1}{\tau-\rho}\right) + \Re\left(\frac{1}{\sigma-1+\rho}\right) - \kappa\Re\left(\frac{1}{\tau-1+\rho}\right).$$

3.1. At least two real zeroes

Assume that $\zeta_K(s)$ has at least two distinct real zeroes $\beta_2 \neq \beta_1$ or a double real zero $\beta_2 = \beta_1$ in the range $S(c,0)\cap(1/2,1)$.

For $0 < \beta < 1 < \sigma$, the function

$$\begin{aligned}\beta \mapsto F_3(\sigma,\tau,\beta) &= -\frac{\kappa}{\tau-\beta} + \frac{1}{\sigma-1+\beta} - \frac{\kappa}{\tau-1+\beta} \\ &= \frac{1-\kappa}{\sigma-1+\beta} + \kappa\left(-\frac{1}{\tau-\beta} + \frac{\tau-\sigma}{(\sigma-1+\beta)(\tau-1+\beta)}\right)\end{aligned}$$

is clearly increasing. Hence, we have

$$F_3(\sigma,\tau,\beta) \geqslant F_3(\sigma,\tau,1) = -\frac{\kappa}{\tau-1} + \frac{1}{\sigma} - \frac{\kappa}{\tau} = F_2(\sigma,\tau),$$

by (7), and

$$T(\sigma,\tau,\beta) = \frac{1}{\sigma-\beta} + F_3(\sigma,\tau,\beta) > \frac{1}{\sigma-\beta} + F_2(\sigma,\tau) \qquad (0<\beta<1).$$

Using (9), we obtain

$$\sum_{\rho} S(\sigma,\tau,\rho) \geqslant T(\sigma,\tau,\beta_1) + T(\sigma,\tau,\beta_2) \geqslant 2\left(\frac{1}{\sigma-\sigma_c} + F_2(\sigma,\tau)\right).$$

Using (6), we then obtain

$$\frac{2}{\sigma-\sigma_c} - \frac{1}{\sigma-1} \leqslant \frac{1-\kappa}{2}\log d_K - F_2(\sigma,\tau) + R(r_1,r_2,\sigma,\tau).$$

By (8) and Lemma 6, setting $\lambda = 1-\kappa = 1-1/\sqrt{5}$, we finally obtain a neat inequality (compare with (4)):

$$\frac{2}{\sigma-\sigma_c} - \frac{1}{\sigma-1} < \frac{\lambda}{2}\log d_K \qquad (1<\sigma\leqslant 7). \tag{10}$$

Lemma 7 applied with $A = \lambda\log d_K$ yields the desired first result (notice that $1+2(\sqrt{2}-1)/(\lambda\log d_K) \leqslant 1+2(\sqrt{2}-1)/(\lambda\log 3) \leqslant 7$).

3.2. At least one non-real zero

Assume that $\zeta_K(s)$ has at least one non-real complex zero $\rho = \beta + i\gamma$, $\gamma \neq 0$, in the region $S(c,d) \cap \{s;\ \Re(s) > 1/2\}$.

Then ρ and $\bar{\rho}$ are two pairwise complex zeroes of $\zeta_K(s)$ in this region and $T(\sigma,\tau,\rho) = T(\sigma,\tau,\bar{\rho})$. Hence, using Lemma (5), we obtain

$$\sum_{\rho} S(\sigma,\tau,\rho) \geqslant 2T(\sigma,\tau,\rho) = \frac{2(\sigma-\beta)}{(\sigma-\beta)^2+\gamma^2} + 2F_4(\sigma,\tau,\beta,\gamma),$$

where

$$\begin{aligned}
F_4(\sigma,\tau,\beta,\gamma) &= F_3(\sigma,\tau,\beta+i\gamma) \\
&= -\kappa\frac{\tau-\beta}{(\tau-\beta)^2+\gamma^2} + \frac{\sigma-1+\beta}{(\sigma-1+\beta)^2+\gamma^2} - \kappa\frac{\tau-1+\beta}{(\tau-1+\beta)^2+\gamma^2} \\
&\geqslant -\frac{\kappa}{\tau-\beta} - \frac{\kappa}{\tau-1+\beta} + \frac{\sigma-1+\beta}{(\sigma-1+\beta)^2+\gamma^2} \\
&= -\frac{\kappa}{\tau-\beta} - \frac{\kappa}{\tau-1+\beta} + \frac{\sigma}{\sigma^2+\gamma^2} \\
&\quad + (1-\beta)\frac{\sigma^2-\sigma+\sigma\beta-\gamma^2}{((\sigma-1+\beta)^2+\gamma^2)(\sigma^2+\gamma^2)} \\
&\geqslant -\frac{\kappa}{\tau} - \frac{\kappa}{\tau-1} + \frac{\sigma}{\sigma^2+\gamma^2} = F_2(\sigma,\tau) - \frac{\gamma^2}{\sigma(\sigma^2+\gamma^2)} \\
&\geqslant F_2(\sigma,\tau) - \gamma^2,
\end{aligned}$$

provided that $\sigma^2-\sigma+\sigma\beta-\gamma^2 \geqslant 0$, hence provided that $1/2 < \beta < 1 < \sigma$ and $|\gamma| \leqslant 1/\sqrt{2}$. By Lemma 6 and (6), we have

$$\frac{2(\sigma-\beta)}{(\sigma-\beta)^2+\gamma^2} < \frac{1}{\sigma-1} + \frac{1-\kappa}{2}\log d_K - F_2(\sigma,\tau) + 2\gamma^2 + R(r_1,r_2,\sigma,\tau).$$

Hence, we finally obtain a neat inequality (compare with (5)):

$$\frac{2(\sigma-\beta)}{(\sigma-\beta)^2+\gamma^2} < \frac{1}{\sigma-1} + \frac{1-\kappa}{2}\log d_K \qquad (1 < \sigma \leqslant 3 \text{ and } |\gamma| \leqslant 1/2). \tag{11}$$

Lemma 8 applied with $A = \lambda \log d_K$ yields the desired second result (notice that $1 + 2S_0/(\lambda\log d_K) \leqslant 1 + 2S_0/(\lambda\log 3) \leqslant 3$ and that $|\gamma| \leqslant d_1/(\lambda\log d_k)$ implies $|\gamma| \leqslant d_1/(\lambda\log 3) \leqslant 1/2)$.

Finally, Lemma 9 applied with $G = \lambda/4$ and $A = \lambda\log d_K$ yields $S_G = 0.42409\cdots$, $B_G = 0.32675\cdots$ and $B_G/\lambda = 0.59110\cdots$ (notice that $1+2S_G/A \leqslant 1+2S_G/(\lambda\log 3) \leqslant 3$ and that $|\gamma| \leqslant 1/(4\log d_k)$ implies $|\gamma| \leqslant 1/(4\log 3) \leqslant 1/2)$ whereas Lemma 9 applied with $G = \lambda$ and $A = \lambda\log d_K$ yields $S_G = 0.58436\cdots$, $B_G = 0.05737\cdots$ and $B_G/\lambda = 0.10379\cdots$ (notice that $1+2S_G/A \leqslant 1+2S_G/(\lambda\log 3) \leqslant 3$ and that $|\gamma| \leqslant 1/\log d_k$ implies $|\gamma| \leqslant 1/\log 8 \leqslant 1/2)$.

3.3. Technical Lemmas

Lemma 6. *Set* $\kappa = 1/\sqrt{5}$ *and* $\tau = (1+\sqrt{1+4\sigma^2})/2$. *Then*

$$R(r_1, r_2, \sigma, \tau) = h(r_1, r_2, \sigma) - \kappa h(r_1, r_2, \tau) = \frac{r_1}{2} A_1(\sigma) + r_2 A_2(\sigma),$$

where $A_k(\sigma) = (\Gamma'/\Gamma)(k\sigma/2) - \kappa(\Gamma'/\Gamma)(k\tau/2) - (1-\kappa)\log(k\pi)$, *by (3).*

Then, $A_1(\sigma)$ *and* $A_2(\sigma)$ *increase with* $\sigma > 1$ *and are negative in the range* $1 < \sigma \leqslant 7$. *Hence, the expression* $R(r_1, r_2, \sigma, \tau)$ *is a decreasing function of both* r_1 *and* r_2, *for* $1 < \sigma \leqslant 7$. *Therefore, for* $n = r_1 + 2r_2 > 1$, *we have*

$$\begin{aligned} R(r_1, r_2, \sigma, \tau) &\leqslant \max(R(2,0,\sigma,\tau), R(0,1,\sigma,\tau)) \\ &= \max(A_1(\sigma), A_2(\sigma)) < 0 \qquad \text{for } 1 < \sigma \leqslant 7, \\ &= \max(A_1(\sigma), A_2(\sigma)) < -\frac{1}{2} \qquad \text{for } 1 < \sigma \leqslant 3. \end{aligned}$$

Proof. Noticing that $(\Gamma'/\Gamma)'(\sigma) = \sum_{n\geqslant 0}(n+\sigma)^{-2} > \sigma^{-2}$ for $\sigma > 0$, we have

$$A_k'(\sigma) = \frac{k}{2}\sum_{n\geqslant 0}\left\{\frac{1}{(n+k\sigma/2)^2} - \frac{2\sigma}{\sqrt{1+4\sigma^2}}\frac{\kappa}{(n+k\tau/2)^2}\right\},$$

which, in using $0 < 2\sigma/\sqrt{1+4\sigma^2} < 1$ and $0 \leqslant \kappa \leqslant 1$ and $k\tau > k\sigma > 0$, yields the desired result. ∎

Lemma 7. *Assume that* $A > 0$. *For* $\sigma = 1 + 2(\sqrt{2}-1)/A$ *the upper bound*

$$\frac{2}{\sigma - \sigma_c} - \frac{1}{\sigma - 1} < \frac{1}{2}A$$

yields $\sigma_c < 1 - 2(\sqrt{2}-1)^2/A$.

Lemma 8. *Let* S_0 *and* d_1 *be as in Theorem 2. Assume that* $A > 0$ *and* $0 < \beta < 1$. *For* $\sigma = 1 + 2S_0/A$ *the upper bound*

$$\frac{2(\sigma-\beta)}{(\sigma-\beta)^2+\gamma^2} - \frac{1}{\sigma-1} < \frac{1}{2}A \tag{12}$$

yields $\beta < 1 - d_1/A$ *or* $|\gamma| > d_1/A$.

Proof. Write $\beta = 1 - b/A$, $\gamma = g/A$ and $\sigma = 1 + 2S/A$ and set $M = \max(b, |g|)$. Then (12) implies

$$\frac{2S+b}{(2S+b)^2+M^2} < \frac{S+1}{4S}.$$

Since this left hand side is a decreasing function of $b \in (0,1)$ for $S \geqslant M/2$, we obtain

$$\frac{2S+M}{(2S+M)^2+M^2} < \frac{S+1}{4S},$$

which yields

$$M > f(S) := \frac{-S^2 + S\sqrt{2 - S^2}}{S + 1}.$$

The choice $S = S_0$ for which $f'(S_0) = 0$ and $f(S_0) = 2(1 - S_0 - S_0^2)/(2 + S_0) = d_1$ is optimal. ■

Lemma 9. *For $G \in (0, 2)$, let S_G be the only zero of $2-(S+2)\sqrt{4S^2 - G^2(S+1)^2}$ in the range $S > G/(2 - G)$. Set $B_G = \frac{2-4S_G^2-2S_G^3}{(1+S_G)(2+S_G)}$. Assume that $A > 0$ and $0 < \beta < 1$. For $\sigma = 1 + 2S_G/A$ the upper bound*

$$\frac{2(\sigma - \beta)}{(\sigma - \beta)^2 + \gamma^2} - \frac{1}{\sigma - 1} < \frac{1}{2}A \tag{13}$$

yields $\beta < 1 - B_G/A$ or $|\gamma| > G/A$.

Proof. Write $\beta = 1 - b/A$, $\gamma = g/A$ and $\sigma = 1 + 2S/A$. Assume that $|g| \leqslant G$. Then

$$\frac{2S + b}{(2S + b)^2 + G^2} < \frac{S + 1}{4S},$$

which implies

$$\left(b + \frac{2S^2}{S + 1}\right)^2 > \frac{4S^2}{(S + 1)^2} - G^2$$

and

$$b > f(S) = \frac{-2S^2 + \sqrt{4S^2 - G^2(S + 1)^2}}{S + 1} \qquad (S \geqslant G/(2 - G) > 0).$$

Since

$$f'(S) = 2S\frac{2 - (S + 2)\sqrt{4S^2 - G^2(S + 1)^2}}{(S + 1)^2\sqrt{4S^2 - G^2(S + 1)^2}}$$

and since $S \mapsto 4S^2 - G^2(S + 1)^2$ increases from 0 to $+\infty$ for S increasing from $G/(2 - G)$ to $+\infty$, the choice $S = S_G$ for which $f'(S_G) = 0$ and $f(S_G) = \frac{2-4S_G^2-2S_G^3}{(1+S_G)(2+S_G)} = B$ is optimal. ■

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