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On the exponential Diophantine equation $(3m^2+1)^x + (qm^2-1)^y = (rm)^z$

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Abstract. Let m, q, r be positive integers. Then we show that the equation $(3m^2+1)^x+(qm^2-1)^y = (rm)^z$ has only the positive integer solution $(x, y, z) = (1, 1, 2)$ under some conditions. The proof is based on elementary methods and Baker's method.

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§1. Introduction

Let a, b, c be fixed relatively prime positive integers greater than one. The exponential Diophantine equation

$$a^x + b^y = c^z \tag{1.1}$$

in positive integers x, y, z has been actively studied by a number of authors. It is known that the number of solutions (x, y, z) of equation (1.1) is finite. This field has a rich history. Using elementary methods such as congruences, the quadratic reciprocity law and factorizations in number fields, many authors completely determined equation (1.1) for fixed some triples (a, b, c).

In 1956, Jeśmanowicz[J] conjectured that if $a,\ b,\ c$ are Pythagorean numbers, i.e., positive integers satisfying $a^2 + b^2 = c^2$, then equation (1.1) has only the positive integer solution $(x, y, z) = (2, 2, 2)$. (cf. [Mi3], [MYW], [T4] and [LS].) As an analogue of Jeśmanowicz' conjecture, the first author proposed that if $a,\ b,\ c,\ p,\ q,\ r$ are fixed positive integers satisfying $a^p + b^q = c^r$ with $a, b, c, p, q, r \geq 2$ and $\gcd(a, b) = 1$, then equation (1.1) has only the trivial solution $(x, y, z) = (p, q, r)$ except for a handful of triples (a, b, c). (cf. [C],[Le2],[Mi1],[Mi2], [T1], [T2] and [LSS].)

On the other direction, many of the recent works on equation (1.1) concern the case where two of a, b and c are congruent to ± 1 modulo a (relatively) large divisor of the other one. In 2012, the first author[T3] showed that if m is a positive integer such that $1 \le m \le 20$ or $m \not\equiv 3 \pmod 6$, then the equation

$$(4m^2+1)^x + (5m^2-1)^y = (3m)^z \tag{1.2}$$

has only the positive integer solution $(x, y, z) = (1, 1, 2)$. The proof is based on elementary methods and Baker's method. Suy-Li[SL] established the same in the case $m \ge 90$ and $3|m$, by means of a deep result of Bilu-Hanrot-Voutier [BHV] concerning the existence of primitive prime divisors in Lucas-numbers. Finally, Bertók[Ber] has completely solved equation (1.2) for the remaining cases $20 < m < 90$. His proof can be done by the help of exponential congruences. (cf. [BH].)

Now we propose the following:

Conjecture 1. Le m be a positive integer greater than one. Let $p, q, r > 1$ be positive integers satisfying $p + q = r^2$. Then the equation

$$(pm^2+1)^x + (qm^2-1)^y = (rm)^z$$

has only the positive integer solution $(x, y, z) = (1, 1, 2)$.

The above conjecture has been verified by several authors under some conditions on m, p, q, r. (cf. [MT], [TH1], [TH2], [T5], [FY], [P], [Mu], [KMS] and [DWY].)

In this paper, we consider the exponential Diophantine equation

$$(3m^2+1)^x + (qm^2-1)^y = (rm)^z \quad \textit{with} \quad 3 + q = r^2, \tag{1.3}$$

with m positive integer. Applying a lower bound for linear forms in two logarithms due to Laurent [La], we show that equation (1.3) has only the positive integer solution $(x, y, z) = (1, 1, 2)$ under some conditions. Our main result is the following:

Theorem 1.1. *Let m be a positive integer. Let q and r be positive integers satisfying*

$$\left(\frac{rm}{qm^2-1}\right) = -1$$

with r odd, where $\left(\frac{*}{*}\right)$ is the Jacobi symbol. Then equation (1.3) *has only the positive integer solution* $(x, y, z) = (1, 1, 2)$.

As a Corollary to Theorem 1.1, we derive the following:

Corollary 1.2. *Let m and r positive integers satisfying*

$$\text{(i)} \quad m \equiv 0 \mod 2, \quad m^2 \equiv -1 \mod r, \quad r \equiv 5 \mod 8,$$

or

$$\text{(ii)} \quad m \equiv 1 \mod 2, \quad m^2 \equiv 1 \mod r, \quad rm \equiv 3 \mod 4.$$

Then equation (1.3) *has only the positive integer solution* $(x, y, z) = (1, 1, 2)$.

§2. Preliminaries

Proposition 2.1 (Bennett[Ben])**.** *Le a and b be integers with $a, b \geq 2$. Then the equation*

$$a^x - b^y = 4$$

has at most one solution in positive integers x and y.

Proposition 2.2 (Cohn[Co], Le[Le1])**.** *All quadruples (S, T, m, n) of positive integers satisfying*

$$S^2 + 2^m = T^n, \quad \gcd(S, T) = 1, \; n \geq 3$$

are given by $(S, T, m, n) = (5, 3, 1, 3), (7, 3, 5, 4), (11, 5, 2, 3)$.

In order to obtain an upper bound for a solution of Pillai's equation, we need a result on lower bounds for linear forms in the logarithms of two algebraic numbers. We will introduce here some notations. Let α_1 and α_2 be real algebraic numbers with $|\alpha_1| \geq 1$ and $|\alpha_2| \geq 1$. We consider the linear form

$$\Lambda = b_2 \log \alpha_2 - b_1 \log \alpha_1,$$

where b_1 and b_2 are positive integers. As usual, the *logarithmic height* of an algebraic number α of degree n is defined as

$$h(\alpha) = \frac{1}{n} \left(\log |a_0| + \sum_{j=1}^{n} \log \max \left\{ 1, |\alpha^{(j)}| \right\} \right),$$

where a_0 is the leading coefficient of the minimal polynomial of α (over $\mathbb{Z}$) and $(\alpha^{(j)})_{1 \leq j \leq n}$ are the conjugates of α. Let A_1 and A_2 be real numbers greater than 1 with

$$\log A_i \geq \max \left\{ h(\alpha_i), \frac{|\log \alpha_i|}{D}, \frac{1}{D} \right\},$$

for $i \in \{1, 2\}$, where D is the degree of the number field $\mathbb{Q}(\alpha_1,\ \alpha_2)$ over $\mathbb{Q}$. Define

$$b' = \frac{b_1}{D \log A_2} + \frac{b_2}{D \log A_1}.$$

We choose to use a result due to Laurent [[La], Corollary 2] with $m = 10$ and $C_2 = 25.2$.

Proposition 2.3 (Laurent[La]). *Let Λ be given as above, with $\alpha_1 > 1$ and $\alpha_2 > 1$. Suppose that α_1 and α_2 are multiplicatively independent. Then*

$$\log |\Lambda| \geq -25.2\, D^4 \left(\max \left\{ \log b' + 0.38, \frac{10}{D} \right\} \right)^2 \log A_1 \log A_2.$$

§3. Proof of Theorem 1.1

3.1. The case $m = 1$

We first show that when $m = 1$, equation (1.3) has only the positive integer solution $(x, y, z) = (1, 1, 2)$.

Lemma 3.1. *Let r be an odd integer with $r \geq 3$. The the equation*

$$4^x + (r^2 - 4)^y = r^z \tag{3.1}$$

has only the positive integer solution $(x, y, z) = (1, 1, 2)$.

Proof. If $x = 1$, then it follows from Proposition 2.1 that (3.1) has only the positive integer solution $(y, z) = (1, 2)$. Thus we may suppose that $x > 1$.

If y is even, then it follows from Proposition 2.2 that (3.1) has no positive integer solutions. Hence y is odd. Taking (3.1) modulo 8 implies that $5 \equiv 5^y \equiv r^z \pmod 8$, so $r \equiv 5 \pmod 8$ and z is odd. From (3.1), we have

$$1 = \left(\frac{r}{r-2} \right)^z = \left(\frac{r-2}{r} \right) = \left(\frac{-2}{r} \right) = -1,$$

which is impossible. Therefore we have the desired result. □

3.2. The case $m \geq 2$

Let (x, y, z) be a solution of (1.3). By Lemma 3.1, we may suppose that $m \geq 2$. We first examine parities of x, y, z. Using our assumption, we show the following:

Lemma 3.2. *Let* (x, y, z) *be a solution of* (1.3). *Then*
(i) y *is odd and* z *is even.*
(ii) *If* m *is even, then* x *is odd.*

Proof. (i) Taking (1.3) modulo $m^2(\geq 4)$ implies that $1+(-1)^y \equiv 0 \mod m^2$, since $z > 1$. Hence y is odd.

From $3 + q = r^2$, it follows that $\left(\dfrac{3m^2+1}{pm^2-1}\right) = 1$. Indeed,

$$\left(\frac{3m^2+1}{qm^2-1}\right) = \left(\frac{3m^2+qm^2}{qm^2-1}\right) = \left(\frac{r^2m^2}{qm^2-1}\right) = 1.$$

By our assumption $\left(\dfrac{rm}{qm^2-1}\right) = -1$, we see that z is even from (1.3).

(ii) We first show that $\left(\dfrac{3m^2+1}{r}\right) = -1$. Put $m = 2^\alpha m_1$ with $\alpha \geq 1$ and m_1 odd. Note that $qm^2 - 1 \equiv -1 \pmod 8$, since q and m are even. Then

$$\left(\frac{m}{qm^2-1}\right) = \left(\frac{2}{qm^2-1}\right)^\alpha \left(\frac{m_1}{qm^2-1}\right) = 1 \cdot 1 = 1.$$

If $r \equiv 1 \pmod 4$, then

$$\left(\frac{r}{qm^2-1}\right) = \left(\frac{qm^2-1}{r}\right) = \left(\frac{-3m^2-1}{r}\right) = \left(\frac{3m^2+1}{r}\right).$$

If $r \equiv 3 \pmod 4$, then

$$\left(\frac{r}{qm^2-1}\right) = -\left(\frac{qm^2-1}{r}\right) = -\left(\frac{-3m^2-1}{r}\right) = \left(\frac{3m^2+1}{r}\right).$$

By our assumption $\left(\dfrac{rm}{qm^2-1}\right) = -1$, we have

$$-1 = \left(\frac{rm}{qm^2-1}\right) = \left(\frac{r}{qm^2-1}\right)\left(\frac{m}{qm^2-1}\right) = \left(\frac{3m^2+1}{r}\right),$$

as desired.

Taking (1.3) modulo r, together with our assumption $3 + q = r^2$, implies that

$$(3m^2+1)^x \equiv -(qm^2-1)^y \equiv -(-3m^2-1)^y \equiv (-1)^{y+1}(3m^2+1)^y \mod r.$$

Then

$$(-1)^x = \left(\frac{-1}{r}\right)^{y+1} (-1)^y = -1,$$

since y is odd. Hence x is odd. □

Using a congruence method, we can easily show that if m is even, then equation (1.3) has only the positive integer solution $(x, y, z) = (1, 1, 2)$.

Lemma 3.3. *If m is even, then equation* (1.3) *has only the positive integer solution* $(x, y, z) = (1, 1, 2)$.

Proof. If $z \leq 2$, then $(x, y, z) = (1, 1, 2)$ from (1.3). Hence we may suppose that $z \geq 3$. Taking (1.3) modulo m^3 implies that

$$1 + 3m^2x - 1 + qm^2y \equiv 0 \mod m^3,$$

so

$$3x + qy \equiv 0 \mod m,$$

which is impossible, since x is odd, q is even and m is even. We therefore obtain our assertion. □

In what follows, we may suppose that m is odd.

Lemma 3.4. *If m is odd, then $x = 1$.*

Proof. Now suppose that $x \geq 2$. We show that this will lead to a contradiction.

In view of $3 + q = r^2$ with r odd and m is odd, we see that

$$3m^2 + 1 \equiv 4 \mod 8, \quad qm^2 - 1 \equiv 5 \mod 8.$$

Then, taking (1.3) modulo 8, together with the fact that z is even, implies that

$$5^y \equiv (rm)^z \equiv 1 \mod 8.$$

Hence y is even, which contradicts Lemma 3.2. We therefore conclude that $x = 1$. □

3.3. Pillai's equation $c^z - b^y = a$

From Lemma 3.4, it follows that $x = 1$ in (1.3), provided that m is odd. If $z \leq 2$, then we obtain $x = 1$ and $z = 2$ from (1.3). From now on, we may suppose that $z \geq 4$, since z is even. Hence our theorem is reduced to solving Pillai's equation

$$c^z - b^y = a \tag{3.2}$$

with $z \geq 4$, where $a = 3m^2 + 1$, $b = qm^2 - 1$ and $c = rm$.

We now want to obtain a lower bound for y.

Lemma 3.5. $y \geq \dfrac{m^2-3}{q}$.

Proof. Taking (3.2) modulo m^4 implies that

$$1 + 3m^2 + qym^2 - 1 \equiv 0 \mod m^4,$$

so $3 + qy \equiv 0 \mod m^2$. Hence we obtain our assertion. □

We next want to obtain an upper bound for y.

Lemma 3.6. $y < 2521 \log c$.

Proof. From (3.2), we now consider the following linear form in two logarithms:

$$\Lambda = z \log c - y \log b \quad (> 0).$$

Using the inequality $\log(1+t) < t$ for $t > 0$, we have

$$0 < \Lambda = \log\left(\frac{c^z}{b^y}\right) = \log\left(1 + \frac{a}{b^y}\right) < \frac{a}{b^y}. \tag{3.3}$$

Hence we obtain

$$\log \Lambda < \log a - y \log b. \tag{3.4}$$

On the other hand, we use Proposition 2.3 to obtain a lower bound for Λ. It follows from Proposition 2.3 that

$$\log \Lambda \;\geq\; -25.2\ \left(\max\left\{\log b' + 0.38, 10\right\}\right)^2\ (\log b)\ (\log c), \tag{3.5}$$

where $b' = \dfrac{y}{\log c} + \dfrac{z}{\log b}$.

We note that $b^{y+1} > c^z$. Indeed,

$$b^{y+1} - c^z = (b-1)c^z - ab \geq (qm^2-2)(3+q)^2m^4 - (3m^2+1)(qm^2-1) > 0.$$

Hence $b' < \dfrac{2y+1}{\log c}$.

Put $M = \dfrac{y}{\log c}$. Combining (3.4) and (3.5) leads to

$$y \log b < \log a + 25.2\ \left(\max\left\{\log\left(2M + \frac{1}{\log c}\right) + 0.38,\ 10\right\}\right)^2\ (\log b)\ (\log c),$$

so

$$M < 1 + 25.2\ \left(\max\left\{\log\left(2M+1\right) + 0.38,\ 10\right\}\right)^2.$$

We therefore obtain $M < 2521$. This completes the proof of Lemma 3.6. □

We are now in a position to prove Theorem 1.1. Recall that $a = 3m^2 + 1$, $b = qm^2 - 1$ and $c = rm$ with $3 + q = r^2$. Since $a + b = c^2$ and z is even, equation (3.2) can be written as

$$(c^2)^Z - b^y = c^2 - b$$

with $z = 2Z$. Then $y \geq Z$. If $y = Z$, then we obtain $y = Z = 1$. Thus we may suppose that $y > Z$.

Since $c^{2Z} > b^y$, it follows from Lemma 3.6 that

$$1 \leq y - Z < y - \frac{\log b}{\log c^2} y = \frac{\log\left(c^2/b\right)}{2\log c} y < \frac{2521}{2} \log(c^2/b).$$

By definitions of b and c, we see that

$$\frac{c^2}{b} = \frac{r^2m^2}{(r^2-3)m^2 - 1} = \frac{1}{1 - \frac{3m^2+1}{r^2m^2}}.$$

Therefore $\alpha := 1 - (e^{2/2521})^{-1} < \dfrac{3m^2+1}{r^2m^2}$. Since $m \geq 2$, this yields

$$r^2 < \frac{1}{\alpha}\left(3 + \frac{1}{m^2}\right) \leq \frac{1}{\alpha}\left(3 + \frac{1}{4}\right) = 4098.251.$$

Consequently we obtain $r \leq 64$.

It follows from Lemmas 3.5, 3.6, together with $r \leq 64$, that

$$m^2 - 1 < 2521(r^2 - 3)\log(rm) \leq 10318453 \log(64m).$$

Hence we obtain $m \leq 11818$.

From (3.3), we have the inequality

$$\left|\frac{\log b}{\log c} - \frac{z}{y}\right| < \frac{a}{yb^y \log c},$$

which implies that $\left|\dfrac{\log b}{\log c} - \dfrac{z}{y}\right| < \dfrac{1}{2y^2}$, since $y \geq 3$. Thus $\dfrac{z}{y}$ is a convergent in the simple continued fraction expansion to $\dfrac{\log b}{\log c}$.

On the other hand, if $\dfrac{p_j}{q_j}$ is the j-th such convergent, then

$$\left|\frac{\log b}{\log c} - \frac{p_j}{q_j}\right| > \frac{1}{(a_{j+1}+2)q_j^2},$$

where a_{j+1} is the $(j+1)$-st partial quotient to $\dfrac{\log b}{\log c}$ (see e.g. Khinchin [K]). Put $\dfrac{z}{y} = \dfrac{p_j}{q_j}$. Note that $q_j \le y$. It follows, then, that

$$a_{j+1} > \frac{b^y \log c}{ay} - 2 \ge \frac{b^{q_j} \log c}{aq_j} - 2. \tag{3.6}$$

Finally, we checked by Magma [BC] that for each $r \le 64$, inequality (3.6) does not hold for any j with $q_j < 2521 \log(rm)$ in the range $2 \le m \le 11818$. This completes the proof of Theorem 1.1. □

§4. Proof of Corollary 1.2

Suppose that our assumptions of Corollary 1.2 are all satisfied. We may suppose that $m \ge 2$ from Lemma 3.1. By Theorem 1, it suffices to verify that $\left(\dfrac{rm}{qm^2-1}\right) = -1$ holds.

(i) In view of the proof of Lemma 3.2, we have

$$\left(\frac{rm}{qm^2-1}\right) = \left(\frac{3m^2+1}{r}\right) = \left(\frac{3(-1)+1}{r}\right) = \left(\frac{-2}{r}\right) = -1.$$

(ii) In view of $qm^2 - 1 \equiv 1 \mod 4$, we have

$$\begin{aligned}
\left(\frac{rm}{qm^2-1}\right) &= \left(\frac{r}{qm^2-1}\right)\left(\frac{m}{qm^2-1}\right) \\
&= \left(\frac{qm^2-1}{r}\right)\left(\frac{qm^2-1}{m}\right) \\
&= \left(\frac{q-1}{r}\right)\left(\frac{-1}{m}\right) \\
&= \left(\frac{-3-1}{r}\right)\left(\frac{-1}{m}\right) \\
&= \left(\frac{-1}{rm}\right) \\
&= -1.
\end{aligned}$$

This completes the proof of Corollary 1.2. □

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