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Radical transversal screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds

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Abstract. In this paper, we introduce the notion of radical transversal screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds giving characterization theorem with some non-trivial examples of such submanifolds. Integrability conditions of distributions D_1, D_2 and $RadTM$ on radical transversal screen pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold have been obtained. Further, we obtain necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic. We also study mixed geodesic radical transversal screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds.

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§1. Introduction

In 1990, B.Y. Chen defined slant immersions in complex geometry as a natural generalization of both holomorphic and totally real immersions ([3]). Further, A. Lotta introduced the concept of slant immersions of a Riemannian manifold into an almost contact metric manifold ([9]). A. Carriazo defined and studied bi-slant submanifolds of almost Hermitian and almost contact metric manifolds and gave the notion of pseudo-slant submanifolds ([2]). The theory of lightlike submanifolds of a semi-Riemannian manifold was introduced by Duggal and Bejancu ([6]). A submanifold M of a semi-Riemannian manifold $\overline{M}$ is said to be lightlike submanifold if the induced metric g on M is degenerate, i.e. there exists a non-zero $X \in \Gamma(TM)$ such that $g(X,Y) = 0$, $\forall Y \in \Gamma(TM)$.
The theory of radical transversal, transversal, semi-transversal lightlike submanifolds has been studied in ([14]). In this article, we introduce the notion

of radical transversal screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds. This new class of lightlike submanifolds of an indefinite Kaehler manifold includes radical transversal and transversal lightlike submanifolds as its sub-cases. The paper is arranged as follows. There are some basic results in section 2 . In section 3, we introduce radical transversal screen pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold giving some examples. Section 4 is devoted to the study of foliations determined by distributions on radical transversal screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds.

§2. Preliminaries

A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold ([6]) if the metric g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{2.1}$$

Now consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Riemannian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.2}$$

$$T\overline{M}|_M = TM \oplus tr(TM), \tag{2.3}$$

$$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^{\perp}). \tag{2.4}$$

Following are four cases of a lightlike submanifold $\big(M, g, S(TM), S(TM^{\perp})\big)$:
Case.1 r-lightlike if $r < min\,(m, n)$,
Case.2 co-isotropic if $r = n < m$, $S\left(TM^{\perp}\right) = \{0\}$,
Case.3 isotropic if $r = m < n$, $S\left(TM\right) = \{0\}$,
Case.4 totally lightlike if $r = m = n$, $S(TM) = S(TM^{\perp}) = \{0\}$.
The Gauss and Weingarten formulae are given as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y),\ \forall\, X, Y \in \Gamma(TM), \tag{2.5}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \ \forall V \in \Gamma(tr(TM)), \tag{2.6}$$

where $\{\nabla_X Y, A_V X\}$ and $\left\{h(X,Y), \nabla^t_X V\right\}$ belong to $\Gamma(TM)$ and $\Gamma(tr(TM))$ respectively. ∇ and ∇^t are linear connections on M and on the vector bundle $tr(TM)$ respectively. The second fundamental form h is a symmetric $F(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. From (2.5) and (2.6), for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$, we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l\left(X,Y\right) + h^s\left(X,Y\right), \tag{2.7}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s\left(X,N\right), \tag{2.8}$$

$$\overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l\left(X,W\right), \tag{2.9}$$

where $h^l(X,Y) = L\left(h(X,Y)\right)$, $h^s(X,Y) = S\left(h(X,Y)\right)$, $D^l(X,W) = L(\nabla^t_X W)$, $D^s(X,N) = S(\nabla^t_X N)$. L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$ respectively. ∇^l and ∇^s are linear connections on $ltr(TM)$ and $S(TM^{\perp})$ called the lightlike connection and screen transversal connection on M respectively.
Now by using (2.5), (2.7)-(2.9) and metric connection $\overline{\nabla}$, we obtain

$$\overline{g}(h^s(X,Y),W) + \overline{g}(Y, D^l(X,W)) = g(A_W X, Y), \tag{2.10}$$

$$\overline{g}(D^s(X,N),W) = \overline{g}(N, A_W X). \tag{2.11}$$

Denote the projection of TM on $S(TM)$ by $\overline{P}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, for any $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$, we have

$$\nabla_X \overline{P}Y = \nabla^*_X \overline{P}Y + h^*(X, \overline{P}Y), \tag{2.12}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi, \tag{2.13}$$

where $\left\{\nabla^*_X \overline{P}Y, A^*_\xi X\right\}$ and $\left\{h^*(X,\overline{P}Y), \nabla^{*t}_X \xi\right\}$ belong to $\Gamma(S(TM))$ and Γ $(Rad(TM))$ respectively. By using above equations, we obtain

$$\overline{g}(h^l(X,\overline{P}Y),\xi) = g(A^*_\xi X, \overline{P}Y), \tag{2.14}$$

$$\overline{g}(h^*(X,\overline{P}Y),N) = g(A_N X, \overline{P}Y), \tag{2.15}$$

$$\overline{g}(h^l(X,\xi),\xi)=0, \quad A^*_\xi\xi=0. \tag{2.16}$$

It is important to note that in general ∇ is not a metric connection. Since $\overline{\nabla}$ is metric connection, by using (2.7), for any $X, Y, Z \in \Gamma(TM)$, we get

$$(\nabla_X g)(Y,Z)=\overline{g}(h^l(X,Y),Z)+\overline{g}(h^l(X,Z),Y). \tag{2.17}$$

An indefinite almost Hermitian manifold $(\overline{M},\overline{g},\overline{J})$ is a 2m-dimensional semi-Riemannian manifold $\overline{M}$ with semi-Riemannian metric $\overline{g}$ of constant index q, $0 < q < 2m$ and a (1, 1) tensor field $\overline{J}$ on $\overline{M}$ such that following conditions are satisfied:

$$\overline{J}^2X=-X, \tag{2.18}$$

$$\overline{g}(\overline{J}X,\overline{J}Y)=\overline{g}(X,Y), \tag{2.19}$$

for all $X, Y \in \Gamma(T\overline{M})$.
An indefinite almost Hermitian manifold $(\overline{M}, \overline{g}, \overline{J})$ is called an indefinite Kaehler manifold if $\overline{J}$ is parallel with respect to $\overline{\nabla}$, i.e.,

$$(\overline{\nabla}_X\overline{J})Y=0, \tag{2.20}$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\overline{\nabla}$ is Levi-Civita connection with respect to $\overline{g}$.
For any vector field X tangent to M, we put

$$\overline{J}X=PX+FX, \tag{2.21}$$

where PX and FX are tangential and transversal parts of $\overline{J}X$ respectively.

§3. Radical Transversal Screen Pseudo-Slant Lightlike Submanifolds

In this section, we introduce the notion of radical transversal screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds. At first, we state the following Lemma for later use:

Lemma 3.1. *Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$, of index $2q$ such that $2q < dim(M)$. Then the screen distribution $S(TM)$ on lightlike submanifold M is Riemannian.*

The proof of above Lemma follows as in Lemma 3.1 of [12], so we omit it.
Definition 3.1. Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$ such that $2q < dim(M)$. Then we say that M is

a radical transversal screen pseudo-slant lightlike submanifold of $\overline{M}$ if the following conditions are satisfied:
(i) $\overline{J}RadTM = ltr(TM)$,
(ii) there exists non-degenerate orthogonal distributions D_1 and D_2 on M such that $S(TM) = D_1 \oplus_{orth} D_2$,
(iii) the distribution D_1 is anti-invariant, i.e. $\overline{J}D_1 \subset S(TM^{\perp})$,
(iv) the distribution D_2 is slant with angle $\theta \in [0, \pi/2)$, i.e. there exists $\theta \in [0, \pi/2)$ such that $|PX| = |\overline{J}X| \cos\theta$, for any $X \in \Gamma(D_2)$.
This constant angle θ is called the slant angle of distribution D_2. A radical transversal screen pseudo-slant lightlike submanifold is said to be proper if $D_1 \neq \{0\}$, $D_2 \neq \{0\}$ and $\theta \neq 0$.
From the above definition, we have the following decomposition

$$TM = RadTM \oplus_{orth} D_1 \oplus_{orth} D_2. \tag{3.1}$$

Let $(\mathbb{R}^{2m}_{2q}, \overline{g}, \overline{J})$ denote the manifold $\mathbb{R}^{2m}_{2q}$ with its usual Kaehler structure given by

$\overline{g} = \frac{1}{4}\Big(-\sum_{i=1}^{q}(dx^i \otimes dx^i + dy^i \otimes dy^i) + \sum_{i=q+1}^{m}(dx^i \otimes dx^i + dy^i \otimes dy^i)\Big)$,

$\overline{J}(\sum_{i=1}^{m}(X_i\partial x_i + Y_i\partial y_i)) = \sum_{i=1}^{m}(Y_i\partial x_i - X_i\partial y_i)$,

where (x^i, y^i) are the cartesian coordinates on $\mathbb{R}^{2m}_{2q}$. Now, we construct some examples of radical transversal screen pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold.

Example 1. Let $(\mathbb{R}^{12}_2, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6\}$.
Suppose M is a submanifold of $\mathbb{R}^{12}_2$ given by $x^1 = y^2 = u_1$, $x^2 = y^1 = u_2$, $x^3 = u_3\cos\beta$, $y^3 = u_3\sin\beta$, $x^4 = u_4\sin\beta$, $y^4 = u_4\cos\beta$, $x^5 = u_5$, $y^5 = u_6$, $x^6 = k\cos u_6$, $y^6 = k\sin u_6$, where k is any constant.
The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$, where

$Z_1 = 2(\partial x_1 + \partial y_2)$, $\quad Z_2 = 2(\partial x_2 + \partial y_1)$,
$Z_3 = 2(\cos\beta\partial x_3 + \sin\beta\partial y_3)$,
$Z_4 = 2(\sin\beta\partial x_4 + \cos\beta\partial y_4)$,
$Z_5 = 2(\partial x_5)$,
$Z_6 = 2(\partial y_5 - k\sin u_6\partial x_6 + k\cos u_6\partial y_6)$.

Hence $RadTM = span\,\{Z_1, Z_2\}$ and $S(TM) = span\,\{Z_3, Z_4, Z_5, Z_6\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 + \partial y_2$, $N_2 = -\partial x_2 + \partial y_1$ and $S(TM^{\perp})$ is spanned by

$W_1 = 2(\sin\beta\partial x_3 - \cos\beta\partial y_3)$,
$W_2 = 2(\cos\beta\partial x_4 - \sin\beta\partial y_4)$,
$W_3 = 2(k\cos u_6\partial x_6 + k\sin u_6\partial y_6)$,
$W_4 = 2(k^2\partial y_5 + k\sin u_6\partial x_6 - k\cos u_6\partial y_6)$.

It follows that $\overline{J}Z_1 = -2N_2$, $\overline{J}Z_2 = -2N_1$, which implies that $\overline{J}RadTM = ltr(TM)$. On the other hand, we can see that $D_1 = span\{Z_3, Z_4\}$ such that $\overline{J}Z_3 = W_1$, $\overline{J}Z_4 = W_2$, which implies that D_1 is anti-invariant with respect to $\overline{J}$ and $D_2 = span\{Z_5, Z_6\}$ is a slant distribution with slant angle $\theta = \arccos(1/\sqrt{1+k^2})$. Hence M is a radical transversal screen pseudo-slant 2-lightlike submanifold of $\mathbb{R}^{12}_2$.

Example 2. Let $(\mathbb{R}^{12}_2, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6\}$.

Suppose M is a submanifold of $\mathbb{R}^{12}_2$ given by $x^1 = u_1$, $y^1 = u_2$, $x^2 = -u_1\cos\alpha + u_2\sin\alpha$, $y^2 = u_1\sin\alpha + u_2\cos\alpha$, $x^3 = y^4 = u_3$, $x^4 = y^3 = u_4$, $x^5 = u_5\cos u_6$, $y^5 = u_5\sin u_6$, $x^6 = \cos u_5$, $y^6 = \sin u_5$.

The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$, where

$Z_1 = 2(\partial x_1 - \cos\alpha\partial x_2 + \sin\alpha\partial y_2)$,

$Z_2 = 2(\partial y_1 + \sin\alpha\partial x_2 + \cos\alpha\partial y_2)$,

$Z_3 = 2(\partial x_3 + \partial y_4)$, $\quad Z_4 = 2(\partial x_4 + \partial y_3)$,

$Z_5 = 2(\cos u_6\partial x_5 + \sin u_6\partial y_5 - \sin u_5\partial x_6 + \cos u_5\partial y_6)$,

$Z_6 = 2(-u_5\sin u_6\partial x_5 + u_5\cos u_6\partial y_5)$.

Hence $RadTM = span\{Z_1, Z_2\}$ and $S(TM) = span\{Z_3, Z_4, Z_5, Z_6\}$.

Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 - \cos\alpha\partial x_2 + \sin\alpha\partial y_2$, $N_2 = -\partial y_1 + \sin\alpha\partial x_2 + \cos\alpha\partial y_2$ and $S(TM^\perp)$ is spanned by

$W_1 = 2(\partial x_3 - \partial y_4)$, $\quad W_2 = 2(\partial x_4 - \partial y_3)$,

$W_3 = 2(\cos u_6\partial x_5 + \sin u_6\partial y_5 + \sin u_5\partial x_6 - \cos u_5\partial y_6)$,

$W_4 = 2(u_5\cos u_5\partial x_6 + u_5\sin u_5\partial y_6)$.

It follows that $\overline{J}Z_1 = 2N_2$, $\overline{J}Z_2 = -2N_1$, which implies that $\overline{J}RadTM = ltr(TM)$. On the other hand, we can see that $D_1 = span\{Z_3, Z_4\}$ such that $\overline{J}Z_3 = W_2$, $\overline{J}Z_4 = W_1$, which implies that D_1 is anti-invariant with respect to $\overline{J}$ and $D_2 = span\{Z_5, Z_6\}$ is a slant distribution with slant angle $\pi/4$. Hence M is a radical transversal screen pseudo-slant 2-lightlike submanifold of $\mathbb{R}^{12}_2$.

Now, We denote the projections on $RadTM$, D_1 and D_2 in TM by P_1, P_2 and P_3 respectively. Similarly, we denote the projections of $tr(TM)$ on $ltr(TM)$, $\overline{J}(D_1)$ and D' by Q_1, Q_2 and Q_3 respectively, where D' is non-degenerate orthogonal complementary subbundle of $\overline{J}(D_1)$ in $S(TM^\perp)$. Then, for any $X \in \Gamma(TM)$, we get

$$X = P_1X + P_2X + P_3X. \tag{3.2}$$

Now applying $\overline{J}$ to (3.2), we have

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + \overline{J}P_3X, \tag{3.3}$$

which gives

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + fP_3X + FP_3X, \tag{3.4}$$

where fP_3X(resp. FP_3X) denotes the tangential (resp. transversal) component of $\overline{J}P_3X$. Thus we get $\overline{J}P_1X \in \Gamma(ltr(TM))$, $\overline{J}P_2X \in \Gamma(\overline{J}(D_1)) \subset \Gamma(S(TM^{\perp}))$, $fP_3X \in \Gamma(D_2)$ and $FP_3X \in \Gamma(D')$. Also, for any $W \in \Gamma(tr(TM))$, we have

$$W = Q_1W + Q_2W + Q_3W. \tag{3.5}$$

Applying $\overline{J}$ to (3.5), we obtain

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + \overline{J}Q_3W, \tag{3.6}$$

which gives

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + BQ_3W + CQ_3W, \tag{3.7}$$

where BQ_3W (resp. CQ_3W) denotes the tangential (resp. transversal) component of $\overline{J}Q_3W$. Thus we get $\overline{J}Q_1W \in \Gamma(RadTM)$, $\overline{J}Q_2W \in \Gamma(D_1)$, $BQ_3W \in \Gamma(D_2)$ and $CQ_3W \in \Gamma(D')$.
Now, by using (2.20), (3.4), (3.7) and (2.7)-(2.9) and identifying the components on $RadTM$, D_1, D_2, $ltr(TM)$, $\overline{J}(D_1)$ and D', we obtain

$$\begin{aligned} P_1(A_{\overline{J}P_2Y}X) + P_1(A_{\overline{J}P_1Y}X) + P_1(A_{FP_3Y}X) = {} & P_1(\nabla_X fP_3Y) \\ & - \overline{J}h^l(X,Y), \end{aligned} \tag{3.8}$$

$$\begin{aligned} P_2(A_{\overline{J}P_2Y}X) + P_2(A_{\overline{J}P_1Y}X) + P_2(A_{FP_3Y}X) = {} & P_2(\nabla_X fP_3Y) \\ & - \overline{J}Q_2h^s(X,Y), \end{aligned} \tag{3.9}$$

$$\begin{aligned} P_3(A_{\overline{J}P_2Y}X) + P_3(A_{\overline{J}P_1Y}X) + P_3(A_{FP_3Y}X) = {} & P_3(\nabla_X fP_3Y) \\ & - BQ_3h^s(X,Y) - fP_3\nabla_XY, \end{aligned} \tag{3.10}$$

$$\nabla^l_X\overline{J}P_1Y + D^l(X,\overline{J}P_2Y) + h^l(X,fP_3Y) + D^l(X,FP_3Y) = \overline{J}P_1\nabla_XY, \tag{3.11}$$

$$\begin{aligned} Q_2\nabla^s_X\overline{J}P_2Y + Q_2\nabla^s_XFP_3Y = {} & \overline{J}P_2\nabla_XY - Q_2D^s(X,\overline{J}P_1Y) \\ & - Q_2h^s(X,fP_3Y), \end{aligned} \tag{3.12}$$

$$\begin{aligned} Q_3\nabla^s_X\overline{J}P_2Y + Q_3\nabla^s_XFP_3Y - FP_3\nabla_XY = {} & CQ_3h^s(X,Y) \\ & - Q_3h^s(X,fP_3Y) - Q_3D^s(X,\overline{J}P_1Y). \end{aligned} \tag{3.13}$$

Theorem 3.2. *Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then M is a radical transversal screen pseudo-slant lightlike submanifold of $\overline{M}$ if and only if*

(i) $\overline{J}ltr(TM)$ is a distribution on M such that $\overline{J}ltr(TM) = RadTM$,

(ii) distribution D_1 is anti-invariant with respect to $\overline{J}$, i.e. $\overline{J}D_1 \subset S(TM^{\perp})$,

(iii) there exists a constant $\lambda \in (0,1]$ such that $P^2X = -\lambda X$, for all $X \in \Gamma(D_2)$, where D_1 and D_2 are non-degenerate orthogonal distributions on M such that $S(TM) = D_1 \oplus_{orth} D_2$ and $\lambda = \cos^2\theta$, θ is slant angle of D_2.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then distribution D_1 is anti-invariant with respect to $\overline{J}$ and $\overline{J}RadTM = ltr(TM)$. Thus for any $X \in \Gamma(RadTM)$, $\overline{J}X \in ltr(TM)$. Hence $\overline{J}(\overline{J}X) \in \overline{J}(ltr(TM))$, which implies $-X \in \overline{J}(ltr(TM))$, for all $X \in \Gamma(RadTM)$, which proves (i) and (ii).
Now for any $X \in \Gamma(D_2)$, we have $|PX| = |\overline{J}X| \cos\theta$, which implies

$$\cos\theta = \frac{|PX|}{|\overline{J}X|}. \tag{3.14}$$

In view of (3.14), we get $\cos^2\theta = \frac{|PX|^2}{|\overline{J}X|^2} = \frac{g(PX,PX)}{g(\overline{J}X,\overline{J}X)} = \frac{g(X,P^2X)}{g(X,\overline{J}^2X)}$, which gives

$$g(X, P^2X) = \cos^2\theta\, g(X, \overline{J}^2X). \tag{3.15}$$

Since M is radical transversal screen pseudo-slant lightlike submanifold, $\cos^2\theta = \lambda(constant) \in (0,1]$ and therefore from (3.15), we get $g(X, P^2X) = \lambda g(X, \overline{J}^2X) = g(X, \lambda\overline{J}^2X)$, which implies

$$g(X, (P^2 - \lambda\overline{J}^2)X) = 0. \tag{3.16}$$

Now for any $X \in \Gamma(D_2)$, we have $\overline{J}^2(X) = P^2X + FPX + BFX + CFX$. Taking the tangential component, we get $P^2X = -X - BFX \in \Gamma(D_2)$, for any $X \in \Gamma(D_2)$. Thus $(P^2 - \lambda\overline{J}^2)X \in \Gamma(D_2)$. Since the induced metric $g = g|_{D_2 \times D_2}$ is non-degenerate(positive definite), by the facts above, we have $(P^2 - \lambda\overline{J}^2)X = 0$, which implies

$$P^2X = \lambda\overline{J}^2X = -\lambda X, \forall X \in \Gamma(D_2). \tag{3.17}$$

This proves (iii).
Conversely suppose that conditions (i), (ii) and (iii) are satisfied. From (i), we have $\overline{J}N \in RadTM$, for all $N \in \Gamma(ltr(TM))$. Hence $\overline{J}(\overline{J}N) \in \overline{J}(RadTM)$, which implies $-N \in \overline{J}(RadTM)$, for all $N \in \Gamma(ltr(TM))$. Thus $\overline{J}RadTM = ltr(TM)$. From (iii), we have $P^2X = \lambda\overline{J}^2X$, for all $X \in \Gamma(D_2)$, where $\lambda(constant) \in (0,1]$.
Now $\cos\theta = \frac{g(\overline{J}X,PX)}{|\overline{J}X||PX|} = -\frac{g(X,\overline{J}PX)}{|\overline{J}X||PX|} = -\frac{g(X,P^2X)}{|\overline{J}X||PX|} = -\lambda\frac{g(X,\overline{J}^2X)}{|\overline{J}X||PX|} = \lambda\frac{g(\overline{J}X,\overline{J}X)}{|\overline{J}X||PX|}$.
From above equation, we get

$$\cos\theta = \lambda\frac{|\overline{J}X|}{|PX|}. \tag{3.18}$$

Therefore (3.14) and (3.18) give $\cos^2\theta = \lambda(constant)$.
Hence M is a radical transversal screen pseudo-slant lightlike submanifold.

Theorem 3.3. *Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then M is a radical transversal screen pseudo-slant lightlike submanifold of $\overline{M}$ if and only if*

(i) $\overline{J}ltr(TM)$ is a distribution on M such that $\overline{J}ltr(TM) = RadTM$,

(ii) distribution D_1 is anti-invariant with respect to $\overline{J}$, i.e. $\overline{J}D_1 \subset S(TM^{\perp})$,

(iii) there exists a constant $\mu \in [0,1)$ such that $BFX = -\mu X$, for all $X \in \Gamma(D_2)$, where D_1 and D_2 are non-degenerate orthogonal distributions on M such that $S(TM) = D_1 \oplus_{orth} D_2$ and $\mu = \sin^2\theta$, θ is slant angle of D_2.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then distribution D_1 is anti-invariant with respect to $\overline{J}$ and $\overline{J}RadTM = ltr(TM)$. Thus for any $X \in \Gamma(RadTM)$ $\overline{J}X \in ltr(TM)$. Hence $\overline{J}(\overline{J}X) \in \overline{J}(ltr(TM))$, which implies $-X \in \overline{J}(ltr(TM))$, for all $X \in \Gamma(RadTM)$, which proves (i) and (ii).
Now, for any vector field $X \in \Gamma(D_2)$, we have

$$\overline{J}X = PX + FX, \tag{3.19}$$

where PX and FX are tangential and transversal parts of $\overline{J}X$ respectively. Applying $\overline{J}$ to (3.19) and taking tangential component, we get

$$-X = P^2X + BFX. \tag{3.20}$$

Since M is a radical transversal screen pseudo-slant lightlike submanifold, $P^2X = -\cos^2\theta X$, for all $X \in \Gamma(D_2)$, where $\cos^2\theta = \lambda(constant) \in (0,1]$ and therefore from (3.20), for any $X \in \Gamma(D_2)$, we get

$$BFX = -\sin^2\theta\, X, \tag{3.21}$$

where $\sin^2\theta = 1 - \lambda = \mu(constant) \in [0,1)$.
This proves (iii).
Conversely suppose that conditions (i), (ii) and (iii) are satisfied. From (i), we have $\overline{J}N \in RadTM$, for all $N \in \Gamma(ltr(TM))$. Hence $\overline{J}(\overline{J}N) \in \overline{J}(RadTM)$, which implies $-N \in \overline{J}(RadTM)$, for all $N \in \Gamma(ltr(TM))$. Thus $\overline{J}RadTM = ltr(TM)$. From (3.20), for any $X \in \Gamma(D_2)$, we get

$$-X = P^2X - \mu X, \tag{3.22}$$

which implies

$$P^2X = -\cos^2\theta\, X, \tag{3.23}$$

where $\cos^2\theta = 1 - \mu = \lambda(constant) \in (0,1]$.
Now the proof follows from theorem (3.2).

Corollary 3.1. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ with slant angle θ, then for any $X, Y \in \Gamma(D_2)$, we have*

$$g(PX, PY) = \cos^2\theta\, g(X, Y), \tag{3.24}$$

$$g(FX, FY) = \sin^2\theta\, g(X, Y). \tag{3.25}$$

The proof of above Corollary follows by using similar steps as in proof of Corollary 3.2 of [12].

Theorem 3.4. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ is integrable if and only if*
(i) $Q_2D^s(Y, \overline{J}P_1X) = Q_2D^s(X, \overline{J}P_1Y)$,
(ii) $Q_3D^s(Y, \overline{J}P_1X) = Q_3D^s(X, \overline{J}P_1Y)$,
(iii) $P_3A_{\overline{J}P_1X}Y = P_3A_{\overline{J}P_1Y}X$, for all $X, Y \in \Gamma(RadTM)$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. From (3.12), for any $X, Y \in \Gamma(RadTM)$, we have

$$Q_2D^s(X, \overline{J}P_1Y) = \overline{J}P_2\nabla_XY. \tag{3.26}$$

On interchanging X and Y in (3.26), we get

$$Q_2D^s(Y, \overline{J}P_1X) = \overline{J}P_2\nabla_YX. \tag{3.27}$$

From (3.26) and (3.27), we obtain

$$Q_2D^s(X, \overline{J}P_1Y) - Q_2D^s(Y, \overline{J}P_1X) = \overline{J}P_2[X, Y]. \tag{3.28}$$

From (3.13), for any $X, Y \in \Gamma(RadTM)$, we have

$$Q_3D^s(X, \overline{J}P_1Y) = CQ_3h^s(X, Y) + FP_3\nabla_XY. \tag{3.29}$$

Interchanging X and Y in (3.29), we get

$$Q_3D^s(Y, \overline{J}P_1X) = CQ_3h^s(Y, X) + FP_3\nabla_YX. \tag{3.30}$$

In view of (3.29) and (3.30), we obtain

$$Q_3D^s(X, \overline{J}P_1Y) - Q_3D^s(Y, \overline{J}P_1X) = FP_3[X, Y]. \tag{3.31}$$

From (3.10), for any $X, Y \in \Gamma(RadTM)$, we have

$$P_3 A_{\overline{J}P_1Y}X + BQ_3h^s(X,Y) = -fP_3\nabla_X Y. \tag{3.32}$$

On interchanging X and Y in (3.32), we get

$$P_3 A_{\overline{J}P_1X}Y + BQ_3h^s(Y,X) = -fP_3\nabla_Y X. \tag{3.33}$$

From (3.32) and (3.33), we obtain

$$P_3 A_{\overline{J}P_1X}Y - P_3 A_{\overline{J}P_1Y}X = fP_3[X,Y]. \tag{3.34}$$

Now the proof follows from (3.28), (3.31) and 3.34.

Theorem 3.5. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 is integrable if and only if*
(i) $Q_3(\nabla^s_Y \overline{J}P_2X) = Q_3(\nabla^s_X \overline{J}P_2Y)$ *and* $P_3A_{\overline{J}P_2X}Y = P_3A_{\overline{J}P_2Y}X$,
(ii) $D^l(X, \overline{J}P_2Y) = D^l(Y, \overline{J}P_2X)$, *for all* $X, Y \in \Gamma(D_1)$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. From (3.11), for any $X, Y \in \Gamma(D_1)$, we have

$$D^l(X, \overline{J}P_2Y) = \overline{J}P_1\nabla_X Y. \tag{3.35}$$

On interchanging X and Y in (3.35), we get

$$D^l(Y, \overline{J}P_2X) = \overline{J}P_1\nabla_Y X. \tag{3.36}$$

From (3.35) and (3.36), we obtain

$$D^l(X, \overline{J}P_2Y) - D^l(Y, \overline{J}P_2X) = \overline{J}P_1[X,Y]. \tag{3.37}$$

From (3.10), for any $X, Y \in \Gamma(D_1)$, we have

$$P_3 A_{\overline{J}P_2Y}X + BQ_3h^s(X,Y) = -fP_3\nabla_X Y. \tag{3.38}$$

On interchanging X and Y in (3.38), we get

$$P_3 A_{\overline{J}P_2X}Y + BQ_3h^s(Y,X) = -fP_3\nabla_Y X. \tag{3.39}$$

In view of (3.38) and (3.39), we obtain

$$P_3 A_{\overline{J}P_2X}Y - P_3 A_{\overline{J}P_2Y}X = fP_3[X,Y]. \tag{3.40}$$

From (3.13), for any $X, Y \in \Gamma(D_1)$, we have

$$Q_3(\nabla^s_X \overline{J} P_2 Y) - CQ_3 h^s(X,Y) = FP_3 \nabla_X Y. \tag{3.41}$$

Interchanging X and Y in (3.41), we get

$$Q_3(\nabla^s_Y \overline{J} P_2 X) - CQ_3 h^s(Y,X) = FP_3 \nabla_Y X. \tag{3.42}$$

From (3.41) and (3.42), we get

$$Q_3(\nabla^s_X \overline{J} P_2 Y) - Q_3(\nabla^s_Y \overline{J} P_2 X) = FP_3[X,Y]. \tag{3.43}$$

The proof follows from (3.37), (3.40) and (3.43).

Theorem 3.6. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 is integrable if and only if*

(i) $D^l(X, FP_3 Y) - h^l(Y, fP_3 X) = D^l(Y, FP_3 X) - h^l(X, fP_3 Y)$,

(ii) $Q_2(\nabla^s_X FP_3 Y - h^s(Y, fP_3 X)) = Q_2(\nabla^s_Y FP_3 X - h^s(X, fP_3 Y))$,

for all $X, Y \in \Gamma(D_2)$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. From (3.11), for any $X, Y \in \Gamma(D_2)$, we have

$$h^l(X, fP_3 Y) + D^l(X, FP_3 Y) = \overline{J} P_1 \nabla_X Y. \tag{3.44}$$

Interchanging X and Y in (3.44), we get

$$h^l(Y, fP_3 X) + D^l(Y, FP_3 X) = \overline{J} P_1 \nabla_Y X. \tag{3.45}$$

From (3.44) and (3.45), we obtain

$$\begin{aligned} h^l(X, fP_3 Y) - h^l(Y, fP_3 X) + D^l(X, FP_3 Y) \\ - D^l(Y, FP_3 X) = \overline{J} P_1[X,Y]. \end{aligned} \tag{3.46}$$

From (3.12), for any $X, Y \in \Gamma(D_2)$, we have

$$Q_2 \nabla^s_X FP_3 Y + Q_2 h^s(X, fP_3 Y) = \overline{J} P_2 \nabla_X Y. \tag{3.47}$$

Interchanging X and Y in (3.47), we get

$$Q_2 \nabla^s_Y FP_3 X + Q_2 h^s(Y, fP_3 X) = \overline{J} P_2 \nabla_Y X. \tag{3.48}$$

In view of (3.47) and (3.48), we obtain

$$\begin{aligned} Q_2 \nabla^s_X FP_3 Y - Q_2 \nabla^s_Y FP_3 X + Q_2 h^s(X, fP_3 Y) \\ - Q_2 h^s(Y, fP_3 X) = \overline{J} P_2[X,Y]. \end{aligned} \tag{3.49}$$

The proof follows from (3.46) and (3.49).

Theorem 3.7. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ is a metric connection if and only if $\overline{J}Q_2D^s(X,N) = 0$ and $BQ_3D^s(X,N) = fP_3A_NX$, for all $X \in \Gamma(TM)$ and $N \in \Gamma(ltr(TM))$.*

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ on M is a metric connection if and only if $RadTM$ is parallel distribution with respect to ∇ ([6]). From (2.20), for any $X \in \Gamma(TM)$ and $N \in \Gamma(ltr(TM))$, we have

$$\overline{\nabla}_X\overline{J}N = \overline{J}\,\overline{\nabla}_XN. \tag{3.50}$$

From (2.7), (2.8) and (3.50), we obtain

$$\overline{\nabla}_X\overline{J}N = -\overline{J}A_NX + \overline{J}\nabla^l_XN + \overline{J}Q_2D^s(X,N) + \overline{J}Q_3D^s(X,N). \tag{3.51}$$

On comparing tangential components of both sides of above equation, we get

$$\nabla_X\overline{J}N = -fP_3A_NX + \overline{J}\nabla^l_XN + \overline{J}Q_2D^s(X,N) + BQ_3D^s(X,N), \tag{3.52}$$

which completes the proof.

§4. Foliations Determined by Distributions

In this section, we obtain necessary and sufficient conditions for foliations determined by distributions on a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold to be totally geodesic.

Theorem 4.1. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ defines a totally geodesic foliation if and only if $P_1A_{\overline{J}P_2Z}X + P_1A_{FP_3Z}X = P_1\nabla_XfP_3Z$, for all $X \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$.*

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. It is easy to see that $RadTM$ defines a totally geodesic foliation if and only if $\nabla_XY \in \Gamma(RadTM)$, for all $X,Y \in \Gamma(RadTM)$. Since $\overline{\nabla}$ is metric connection, using (2.7) and (2.19), for any $X,Y \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$, we get

$$\overline{g}(\nabla_XY,Z) = \overline{g}((\overline{\nabla}_X\overline{J})Z - \overline{\nabla}_X\overline{J}Z, \overline{J}Y). \tag{4.1}$$

In view of (2.20), (3.4) and (4.1), we obtain

$$\overline{g}(\nabla_XY,Z) = -\overline{g}(\overline{\nabla}_X(\overline{J}P_2Z + fP_3Z + FP_3Z), \overline{J}Y). \tag{4.2}$$

From (2.7), (2.9) and (4.2), we get $\overline{g}(\nabla_X Y, Z) = \overline{g}(A_{\overline{J}P_2Z}X + A_{FP_3Z}X - \nabla_X fP_3Z, \overline{J}Y)$, which gives

$$\overline{g}(\nabla_X Y, Z) = \overline{g}(P_1 A_{\overline{J}P_2Z}X + P_1 A_{FP_3Z}X - P_1\nabla_X fP_3Z, \overline{J}Y). \tag{4.3}$$

This completes the proof.

Theorem 4.2. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 defines a totally geodesic foliation if and only if*
(i) $\overline{g}(h^s(X, fZ), \overline{J}Y) = -\overline{g}(\nabla^s_X FZ, \overline{J}Y)$,
(ii) $\overline{g}(h^s(X, \overline{J}N), \overline{J}Y) = 0$,
for all $X, Y \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$ *and* $N \in \Gamma(ltr(TM))$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in D_1$, for all $X, Y \in \Gamma(D_1)$. From (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we obtain

$$\overline{g}(\nabla_X Y, Z) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}Z). \tag{4.4}$$

Since $\overline{\nabla}$ is metric connection, using (4.4), we get

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \overline{J}Z, \overline{J}Y). \tag{4.5}$$

In view of (2.7), (2.9) and (4.5), we obtain

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(h^s(X, fZ) + \nabla^s_X FZ, \overline{J}Y). \tag{4.6}$$

Now from (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we have

$$\overline{g}(\nabla_X Y, N) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}N). \tag{4.7}$$

From (4.7), we get

$$\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N). \tag{4.8}$$

Also, from (2.7) and (4.8), we obtain

$$\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, h^s(X, \overline{J}N)). \tag{4.9}$$

Thus the proof is completed.

Definition 4.1. A radical transversal screen pseudo-slant lightlike submanifold M of an indefinite Kaehler manifold $\overline{M}$ is said to be mixed geodesic screen pseudo-slant lightlike submanifold if its second fundamental form h satisfies $h(X, Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$. Thus M is mixed geodesic radical transversal screen pseudo-slant lightlike submanifold if $h^l(X, Y) = 0$ and $h^s(X, Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$.

Corollary 4.1. *Let M be a mixed geodesic radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 defines a totally geodesic foliation if and only if*
(i) $\overline{g}(\nabla^s_X FZ, \overline{J}Y) = 0$,
(ii) $\overline{g}(h^s(X, \overline{J}N), \overline{J}Y) = 0$,
for all $X, Y \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$ *and* $N \in \Gamma(ltr(TM))$.

Proof. Since M is a mixed geodesic radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$, we have $h^s(X, Z) = 0$ for all $X \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$. Now the proof follows from theorem 4.2.

Theorem 4.3. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 defines a totally geodesic foliation if and only if*
(i) $\overline{g}(fY, A_{\overline{J}Z}X) = \overline{g}(FY, \nabla^s_X \overline{J}Z)$,
(ii) $\overline{g}(fY, A^*_{\overline{J}N}X) = \overline{g}(FY, h^s(X, \overline{J}N))$,
for all $X, Y \in \Gamma(D_2)$, $Z \in \Gamma(D_1)$ *and* $N \in \Gamma(ltr(TM))$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution D_2 defines a totally geodesic foliation if and only if $\nabla_X Y \in D_2$, for all $X, Y \in \Gamma(D_2)$. From (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_2)$ and $Z \in \Gamma(D_1)$, we obtain

$$\overline{g}(\nabla_X Y, Z) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}Z). \tag{4.10}$$

Since $\overline{\nabla}$ is metric connection, using (4.10), we get

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}Z). \tag{4.11}$$

From (2.9) and (4.11), we obtain

$$\overline{g}(\nabla_X Y, Z) = \overline{g}(fY, A_{\overline{J}Z}X) - \overline{g}(FY, \nabla^s_X \overline{J}Z). \tag{4.12}$$

Now, from (2.7), (2.19) and (2.20), for all $X, Y \in \Gamma(D_2)$ and $N \in \Gamma(ltr(TM))$, we have

$$\overline{g}(\nabla_X Y, N) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}N). \tag{4.13}$$

From (4.13), we get

$$\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N). \tag{4.14}$$

In view of (2.8), (2.14) and (4.14), we obtain

$$\overline{g}(\nabla_X Y, N) = \overline{g}(fY, A^*_{\overline{J}N}X) - \overline{g}(FY, h^s(X, \overline{J}N)), \tag{4.15}$$

which completes the proof.

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