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On the tests for the equality of means in the intraclass correlation model with missing data

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Abstract. In this paper, testing for the equality of mean components and of two mean vectors in repeated measures with the intraclass correlation model are treated when the missing observations occur. We consider a new test statistic for the equality of mean components in one-sample problem. Further, we derive a new test statistic for the equality of two mean vectors. The distributions of the test statistics are given under the general case of missing observations. Finally, numerical examples by Monte Carlo simulation are conducted to illustrate power of the method proposed in this paper.

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§1. Introduction

Let $\boldsymbol{x}_1^{(i)}, \boldsymbol{x}_2^{(i)}, \ldots, \boldsymbol{x}_{n^{(i)}}^{(i)}$ $(i = 1, 2)$ be distributed as $N_p(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}^{(i)})$, where $\boldsymbol{\mu}_i = (\mu_1^{(i)}, \mu_2^{(i)}, \ldots, \mu_p^{(i)})'$. In particular, we consider to test the equality of the mean components and of two mean vectors when the variables are interchangeable with respect to variances and covariances—the intraclass correlation model, that is, when $\boldsymbol{\Sigma}^{(i)}$ is of the form

$$\boldsymbol{\Sigma}^{(i)} = \sigma_i^2[(1 - \rho_i)\boldsymbol{I}_p + \rho_i \boldsymbol{1}_p \boldsymbol{1}_p'], \quad \boldsymbol{1}_p = (1, 1, \ldots, 1)' : p \times 1.$$

When the covariance matrix has the intraclass correlation form, many authors have considered testing for the equality of mean components. For one sample case, when ρ_1 is known but σ_1^2 is not, Scheffé [8] and Miller [7] have given the simultaneous confidence intervals for all contrasts $\boldsymbol{a}'\boldsymbol{\mu}_1$ for all non-null p-dimensional vector $\boldsymbol{a}$ such that $\boldsymbol{a}'\boldsymbol{1}_p = 0$. When both σ_1^2 and ρ_1 are unknown, Bhargava and Srivastava [1] has given Scheffé and Tukey types of simultaneous confidence intervals. When the observations are the monotone type of missing, Seo and Srivastava [9] gave the exact distribution of test statistic for the equality of mean components and Scheffé and Bonferroni types of simultaneous confidence intervals. Further, when missing observations are not of monotone type, Seo and Srivastava [9] gave asymptotic simultaneous confidence intervals by usual maximum likelihood ratio method and an iterative numerical method which was discussed in Srivastava [10] and Srivastava and Carter [11]. Kanda and Fujikoshi [3] studied some basic properties of maximum likelihood estimators for a multivariate normal distribution based on monotone type of missing data. When the complete data are obtained, Hotelling's T^2-statistic is used as the usual test statistic for the null hypothesis $\boldsymbol{H}_{02} : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$ against the alternative $\boldsymbol{H}_{12}$: not $\boldsymbol{H}_{02}$ (see, Hotelling [2]). Recently, when some missing observations occur, Krishnamoorthy and Pannala [6] considered approximate methods for constructing confidence region and to test $\boldsymbol{H}_{02}$ without assumption of covariance structure. On the other hand, Koizumi and Seo [5] derived the exact distribution of test statistic for $\boldsymbol{H}_{02}$ and the simultaneous confidence intervals for all contrasts in the intraclass correlation model with monotone missing data. Koizumi and Seo's procedure is an extension to that in Seo and Srivastava [9].

In this paper, we give testing procedures when incomplete data aries. At first, we consider an exact distribution of test statistic for the null hypothesis $\boldsymbol{H}_{01} : \mu_1^{(1)} = \mu_2^{(1)} = \cdots = \mu_p^{(1)}$ against the alternative $\boldsymbol{H}_{11}$: not $\boldsymbol{H}_{01}$ under the model with uniform covariance structure. Moreover, we derive an exact test for the hypothesis $\boldsymbol{H}_{02} : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$ in the intraclass correlation model with missing data. In Section 2, we give a new exact distribution of test statistic for the equality of mean components with non-monotone type of missing data. In Section 3, we derive a new exact distribution of test for the equality of two mean vectors. Finally, we investigate powers of test statistics proposed in this paper by Monte Carlo simulation.

§2. Testing for the equality of mean components

In this section, we discuss the one-sample problem. For convenience' sake, we put $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_p) \equiv \boldsymbol{\mu}_1, \boldsymbol{\Sigma} \equiv \boldsymbol{\Sigma}^{(1)}$ and $n \equiv n^{(1)}$. We consider to test the equality of the μ_ℓ's, $\ell = 1, 2, \ldots, p$, i.e., a test statistic for the null hypothesis $\boldsymbol{H}_{01}$ in the intraclass correlation model with missing data. Data set has some missing components which are of the non-monotone type (general case). Let n_ℓ and p_j $(j = 1, 2, \ldots, n)$ be the total numbers of the observed data for ℓ-th row and j-th column, respectively. The data set is called monotone type of missing observations if n_ℓ and p_j satisfy $n = n_1 \geq n_2 \geq \cdots \geq n_p$ and $p = p_1 \geq p_2 \geq \cdots \geq p_n$, otherwise it is called a general case of missing observations. We can obtain a subvector without missing part by a transformation of a sample vector with missing components. As an example, suppose that we have the observations $\boldsymbol{x}_j = (x_{1j}, *, x_{3j}, *, x_{5j})'$ for the j-th column, where "$*$" denotes a missing component. Then, we can define as $\boldsymbol{y}_j (= (y_{1j}, y_{2j}, y_{3j})') = \boldsymbol{B}_j \boldsymbol{x}_j = (x_{1j}, x_{3j}, x_{5j})'$, where

$$\boldsymbol{B}_j = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

which is distributed as $N_3(\boldsymbol{B}_j \boldsymbol{\mu}, \boldsymbol{\Sigma}_j)$, $\boldsymbol{B}_j \boldsymbol{\mu} = (\mu_1, \mu_3, \mu_5)'$ and $\boldsymbol{\Sigma}_j = \sigma^2[(1-\rho)\boldsymbol{I}_3 + \rho \mathbf{1}_3 \mathbf{1}_3'] \equiv \boldsymbol{B}_j \boldsymbol{\Sigma} \boldsymbol{B}_j'$. Therefore, in general, letting $\boldsymbol{y}_j = (y_{1j}, y_{2j}, \dots, y_{p_j j})'$, then $\boldsymbol{y}_j$'s are independently distributed as $N_{p_j}(\boldsymbol{B}_j \boldsymbol{\mu}, \boldsymbol{\Sigma}_j)$, $j = 1, 2, \dots, n$, where $\boldsymbol{B}_j$ is a $p_j \times p$ matrix and $\boldsymbol{\Sigma}_j = \sigma^2[(1-\rho)\boldsymbol{I}_{p_j} + \rho \mathbf{1}_{p_j} \mathbf{1}_{p_j}']$.

Next, let $\boldsymbol{C}_j$ be a $p_j \times p_j$ matrix such that

$$\boldsymbol{C}_j = \boldsymbol{I}_{p_j} - \frac{\nu_j}{p_j} \mathbf{1}_{p_j} \mathbf{1}_{p_j}',$$

where $\nu_j = 1 \pm (1-\rho)^{\frac{1}{2}} \{1 + (p_j - 1)\rho\}^{-\frac{1}{2}}$ (see, Bharagava and Srivastava [1]). Then, by the transformation $\boldsymbol{w}_j (= (w_{1j}, w_{2j}, \dots, w_{p_j j})') = \boldsymbol{C}_j \boldsymbol{y}_j$, we have

$$\boldsymbol{w}_j \sim N_{p_j}(\boldsymbol{C}_j \boldsymbol{B}_j \boldsymbol{\mu}, \gamma^2 \boldsymbol{I}_{p_j}),$$

where $\gamma^2 \equiv \sigma^2(1-\rho)$.

Without loss of generality, the observed original data set $\{x_{\ell j}\}$ can be grouped into s subsets of data with same missing pattern, where the c-th group($c = 1, 2, \dots, s \le 2^p - 1$) consists of $n^{(c)}$ sample vectors such that $p^{(c)}$ observations are available in p components. We note that $p^{(c)}$ denotes the total number of components after excluding the missing part. Let $y_{\ell' j'}^{(c)}$ and $w_{\ell' j'}^{(c)}$ be a (ℓ', j') component in the c-th group, respectively. Then we define the original sample means $\overline{y}_{\ell' \cdot}^{(c)}$, $\overline{y}_{\cdot j'}^{(c)}$ and $\overline{y}_{\cdot\cdot}^{(c)}$ for the c-th group as follows:

$$\overline{y}_{\ell' \cdot}^{(c)} = \frac{1}{n^{(c)}} \sum_{j'=1}^{n^{(c)}} y_{\ell' j'}^{(c)}, \quad \overline{y}_{\cdot j'}^{(c)} = \frac{1}{p^{(c)}} \sum_{\ell'=1}^{p^{(c)}} y_{\ell' j'}^{(c)}, \quad \overline{y}_{\cdot\cdot}^{(c)} = \frac{1}{p^{(c)} n^{(c)}} \sum_{\ell'=1}^{p^{(c)}} \sum_{j'=1}^{n^{(c)}} y_{\ell' j'}^{(c)}.$$

Similarly, the transformed sample means $\overline{w}_{\ell' \cdot}^{(c)}, \overline{w}_{\cdot j'}^{(c)}$ and $\overline{w}_{\cdot\cdot}^{(c)}$ are defined by

$$\overline{w}_{\ell' \cdot}^{(c)} = \frac{1}{n^{(c)}} \sum_{j'=1}^{n^{(c)}} w_{\ell' j'}^{(c)}, \quad \overline{w}_{\cdot j'}^{(c)} = \frac{1}{p^{(c)}} \sum_{\ell'=1}^{p^{(c)}} w_{\ell' j'}^{(c)}, \quad \overline{w}_{\cdot\cdot}^{(c)} = \frac{1}{p^{(c)} n^{(c)}} \sum_{\ell'=1}^{p^{(c)}} \sum_{j'=1}^{n^{(c)}} w_{\ell' j'}^{(c)},$$

respectively. Hence, we have an unbiased estimator of γ^2 for the c-th group as

$$\begin{aligned}\widehat{\gamma}^{(c)^2} &= \frac{1}{f^{(c)}}\sum_{\ell'=1}^{p^{(c)}}\sum_{j'=1}^{n^{(c)}}\left(w_{\ell' j'}^{(c)} - \overline{w}_{\ell'\cdot}^{(c)} - \overline{w}_{\cdot j'}^{(c)} + \overline{w}_{\cdot\cdot}^{(c)}\right)^2 \\ &= \frac{1}{f^{(c)}}\sum_{\ell'=1}^{p^{(c)}}\sum_{j'=1}^{n^{(c)}}\left(y_{\ell' j'}^{(c)} - \overline{y}_{\ell'\cdot}^{(c)} - \overline{y}_{\cdot j'}^{(c)} + \overline{y}_{\cdot\cdot}^{(c)}\right)^2,\end{aligned}$$

where $f^{(c)} = (p^{(c)} - 1)(n^{(c)} - 1)$. Then $(f^{(c)}\widehat{\gamma}^{(c)^2})/\gamma^2$ has χ^2-distribution with $f^{(c)}$ degrees of freedom under the null hypothesis $\boldsymbol{H}_{01}$. Hence, we can also obtain that

$$\sum_{c=1}^{s}\frac{f^{(c)}\widehat{\gamma}^{(c)^2}}{\gamma^2} \tag{2.1}$$

has χ^2-distribution with $f_1 = \sum_{c=1}^{s} f^{(c)}$ degrees of freedom.

For each of groups, we can see $\sqrt{n^{(c)}}(\overline{w}_{\ell'\cdot}^{(c)} - \overline{w}_{\cdot\cdot}^{(c)}) = \sqrt{n^{(c)}}(\overline{y}_{\ell'\cdot}^{(c)} - \overline{y}_{\cdot\cdot}^{(c)})$. Then

$$\sum_{\ell'=1}^{p^{(c)}}\left(\frac{\sqrt{n^{(c)}}(\overline{w}_{\ell'\cdot}^{(c)} - \overline{w}_{\cdot\cdot}^{(c)})}{\gamma}\right)^2 = \sum_{\ell'=1}^{p^{(c)}}\left(\frac{\sqrt{n^{(c)}}(\overline{y}_{\ell'\cdot}^{(c)} - \overline{y}_{\cdot\cdot}^{(c)})}{\gamma}\right)^2$$

has χ^2-distribution with $p^{(c)} - 1$ degrees of freedom under the null hypothesis $\boldsymbol{H}_{01}$, and this statistic is independent of (2.1). Thus, we obtain the following theorem.

Theorem 1. *Suppose that a data set has the general missing observations at random in the intraclass correlation model. Then a test statistic for the null hypothesis* $\boldsymbol{H}_{01}$ *is given by*

$$F_1 = \frac{\sum_{c=1}^{s}\sum_{\ell'=1}^{p^{(c)}} n^{(c)}(\overline{y}_{\ell'\cdot}^{(c)} - \overline{y}_{\cdot\cdot}^{(c)})^2/p^*}{\sum_{c=1}^{s} f^{(c)}\widehat{\gamma}^{(c)^2}/f_1}, \tag{2.2}$$

where the distribution of F_1 *under the null hypothesis* F*-distribution with* $p^* = \sum_{c=1}^{s}(p^{(c)} - 1)$ *and* $f_1 = \sum_{c=1}^{s}(p^{(c)} - 1)(n^{(c)} - 1)$ *degrees of freedom.*

This theorem is different from the result due to Koizumi and Seo [4]. It may be noted that the value of F_1 is directly calculated from the original data set. Also, when $s = 1$, the statistic F_1 in (2.2) can be reduced as the test statistic given by Bhargava and Srivastava [1].

§3. Testing for the equality of two mean vectors

In this section, we consider a test for the equality of two mean vectors. We assume that $\boldsymbol{x}_j^{(i)} \sim N_p(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}^{(i)}),\ i = 1, 2,\ j = 1, 2, \ldots, n$ and $\boldsymbol{\Sigma} \equiv \boldsymbol{\Sigma}^{(1)} = \boldsymbol{\Sigma}^{(2)}$. $\{x_{\ell j}^{(i)}\}$ can be grouped into s subsets of the data which have same missing pattern, respectively. In a sample from the i-th population, data set for the c-th group is a $p^{(c)} \times n^{(c)}$ matrix and $y_{\ell' j'}^{(i,c)}$ is a (ℓ', j') component in the c-th group. Data set $\{x_{\ell j}^{(i,c)}\}$ is transformed by $\boldsymbol{B}^{(c)}$ and $\boldsymbol{C}^{(c)}$ as well as Section 2, that is, $\boldsymbol{B}^{(c)}$ and $\boldsymbol{C}^{(c)}$ are $p^{(c)} \times p$ and $p^{(c)} \times p^{(c)}$ matrices, respectively. After these transformations, we can obtain $\boldsymbol{w}_{j'}^{(i,c)} \equiv \boldsymbol{C}^{(c)} \boldsymbol{y}_{j'}^{(i,c)} \equiv \boldsymbol{C}^{(c)} \boldsymbol{B}^{(c)} \boldsymbol{x}_{j'}^{(i,c)}$ and

$$\boldsymbol{w}_{j'}^{(i,c)} \sim N_{p^{(c)}}(\boldsymbol{C}^{(c)} \boldsymbol{B}^{(c)} \boldsymbol{\mu}_i, \gamma^2 \boldsymbol{I}_{p^{(c)}}).$$

Then we define sample means for each of groups as follows:

$$\overline{y}_{\ell' \cdot}^{(i,c)} = \frac{1}{n^{(c)}} \sum_{j'=1}^{n^{(c)}} y_{\ell' j'}^{(i,c)}, \qquad \overline{w}_{\ell' \cdot}^{(i,c)} = \frac{1}{n^{(c)}} \sum_{j'=1}^{n^{(c)}} w_{\ell' j'}^{(i,c)},$$

$$\overline{y}_{\cdot j'}^{(i,c)} = \frac{1}{p^{(c)}} \sum_{\ell'=1}^{p^{(c)}} y_{\ell' j'}^{(i,c)}, \qquad \overline{w}_{\cdot j'}^{(i,c)} = \frac{1}{p^{(c)}} \sum_{\ell'=1}^{p^{(c)}} w_{\ell' j'}^{(i,c)},$$

$$\overline{y}_{\cdot\cdot}^{(i,c)} = \frac{1}{p^{(c)} n^{(c)}} \sum_{\ell'=1}^{p^{(c)}} \sum_{j'=1}^{n^{(c)}} y_{\ell' j'}^{(i,c)}, \qquad \overline{w}_{\cdot\cdot}^{(i,c)} = \frac{1}{p^{(c)} n^{(c)}} \sum_{\ell'=1}^{p^{(c)}} \sum_{j'=1}^{n^{(c)}} w_{\ell' j'}^{(i,c)}.$$

And an unbiased estimator of γ^2 for the c-th group is given by

$$\begin{aligned}
\widehat{\gamma}^{(i,c)^2} &= \frac{1}{f^{(c)}} \sum_{\ell'=1}^{p^{(c)}} \sum_{j'=1}^{n^{(c)}} \left(w_{\ell' j'}^{(i,c)} - \overline{w}_{\ell' \cdot}^{(i,c)} - \overline{w}_{\cdot j'}^{(i,c)} + \overline{w}_{\cdot\cdot}^{(i,c)} \right)^2 \\
&= \frac{1}{f^{(c)}} \sum_{\ell'=1}^{p^{(c)}} \sum_{j'=1}^{n^{(c)}} \left(y_{\ell' j'}^{(i,c)} - \overline{y}_{\ell' \cdot}^{(i,c)} - \overline{y}_{\cdot j'}^{(i,c)} + \overline{y}_{\cdot\cdot}^{(i,c)} \right)^2,
\end{aligned}$$

where $f^{(c)} = (p^{(c)} - 1)(n^{(c)} - 1)$. Hence, we noting unbiased estimator of γ^2 is given by

$$\widetilde{\gamma}^2 \equiv \sum_{i=1}^{2} \sum_{c=1}^{s} \frac{f^{(c)} \widehat{\gamma}^{(i,c)^2}}{f_2}, \quad f_2 \equiv \sum_{i=1}^{2} \sum_{c=1}^{s} f^{(c)},$$

and we have

$$\sum_{i=1}^{2} \sum_{c=1}^{s} \frac{f^{(c)} \widehat{\gamma}^{(i,c)^2}}{\gamma^2}$$

possesses χ^2-distribution with f_2 degrees of freedom.

Let $\overline{\boldsymbol{w}}^{(i,c)} \equiv (\overline{w}_{1\cdot}^{(i,c)}, \overline{w}_{2\cdot}^{(i,c)}, \ldots, \overline{w}_{p^{(c)}\cdot}^{(i,c)})'$ for each of groups. Then under the null hypothesis

$$\frac{n^{(c)}(\overline{\boldsymbol{w}}^{(1,c)} - \overline{\boldsymbol{w}}^{(2,c)})'(\overline{\boldsymbol{w}}^{(1,c)} - \overline{\boldsymbol{w}}^{(2,c)})}{2\gamma^2}$$

has χ^2-distribution with $p^{(c)}$ degrees of freedom. Hence,

$$\sum_{c=1}^{s} \frac{n^{(c)}(\overline{\boldsymbol{w}}^{(1,c)} - \overline{\boldsymbol{w}}^{(2,c)})'(\overline{\boldsymbol{w}}^{(1,c)} - \overline{\boldsymbol{w}}^{(2,c)})}{2\gamma^2} \sim \chi^2_{p^{**}},$$

where $p^{**} \equiv \sum_{c=1}^{s} p^{(c)}$. Therefore, we obtain the following theorem.

Theorem 2. *Suppose that a data set has the general missing observations at random in the intraclass correlation model. Then a test statistic for the equality of two mean vectors is given by*

$$F_2 = \frac{\sum_{c=1}^{s} n^{(c)}(\overline{\boldsymbol{w}}^{(1,c)} - \overline{\boldsymbol{w}}^{(2,c)})'(\overline{\boldsymbol{w}}^{(1,c)} - \overline{\boldsymbol{w}}^{(2,c)})}{2p^{**}\widetilde{\gamma}^2}, \tag{3.1}$$

where the distribution of F_2 under the null hypothesis $\boldsymbol{H}_{02}$ is F-distribution with $p^{**} = \sum_{c=1}^{s} p^{(c)}$ and $f_2 = \sum_{i=1}^{2}\sum_{c=1}^{s}(p^{(c)}-1)(n^{(c)}-1)$ degrees of freedom.

§4. Simulation studies

In this Section, we investigate power of statistics in (2.2) and (3.1) by Monte Carlo simulation.

The power of a test statistic in (2.2) is given by

$$\Pr(F_1 > F_{p^*,f_1,\alpha} \mid \boldsymbol{H}_{11}) = \beta_1, \tag{4.1}$$

where $F_{p^*,f_1,\alpha}$ is the upper 100α percentage point of F-distribution with p^* and f_1 degrees of freedom. Put $p = 4$, $n_1 = n_2 = 40$, $n_3 = n_4 = 20$, $\sigma^2 = 1$ and $\rho = 0.5$. Then we calculate the β_1 when the value of μ_i is changed. Results of Monte Carlo simulations for the power β_1 are given in Table 1.

The power of a test statistic in (3.1) is given by

$$\Pr(F_2 > F_{p^{**},f_2,\alpha} \mid \boldsymbol{H}_{12}) = \beta_2. \tag{4.2}$$

Since F_2 statistic in (3.1) is essentially distributed as central F-distribution under the null hypothesis, the distribution of F_2 in (3.1) under the alternative

Table 1: Power of test statistic in (2.2)

$\lvert\mu_1-\mu_2\rvert$	β_1	$\lvert\mu_1-\mu_3\rvert$	β_1
0	0.050	0	0.050
0.2	0.163	0.2	0.113
0.4	0.574	0.4	0.544
0.6	0.928	0.6	0.723
0.8	0.997	0.8	0.943
1.0	1.000	1.0	0.995

Table 2: Power of test statistic in (3.1)

$\lvert\mu_1^{(1)}-\mu_1^{(2)}\rvert$	β_2	$\lvert\mu_3^{(1)}-\mu_3^{(2)}\rvert$	β_2
0	0.050	0	0.050
0.2	0.100	0.2	0.076
0.4	0.304	0.4	0.173
0.6	0.651	0.6	0.372
0.8	0.911	0.8	0.635
1.0	0.990	1.0	0.852
1.2	1.000	1.2	0.962
1.4	1.000	1.4	0.994

hypotheses is non-central F-distribution with p^{**} and f_2 degrees of freedom and non-centrality parameter ξ^2, where ξ^2 is given by

$$\xi^2 = \sum_{i=1}^{2}\sum_{c=1}^{s}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)'\boldsymbol{B}^{(c)'}\boldsymbol{C}^{(c)'}(\gamma^2\boldsymbol{V}^{(c)})^{-1}\boldsymbol{C}^{(c)}\boldsymbol{B}^{(c)}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2),$$

$\boldsymbol{V}^{(c)^{-1}} = \mathrm{diag}(n^{(c)}, n^{(c)}, \ldots, n^{(c)})$. Therefore we can obtain the powers β_1 and β_2 by integrating probability density function of non-central F-distribution. Setting the parameters are the same the one sample problem. Results of Monte Carlo simulations for the power β_2 are given in Table 2.

We note that test statistic has a high power when the sample size is large. The more missing parts are, the smaller powers β_1 and β_2 are.

In conclusion, we have derived the exact distributions of new test statistics for $\boldsymbol{H}_{01}$ and $\boldsymbol{H}_{02}$ under the assumption of intraclass correlation model with general missing observations. We have given explicit unbiased estimators when the covariance matrix has the uniform covariance structure. By using its estimator, we have derived new exact distributions of test statistics for $\boldsymbol{H}_{01}$ and $\boldsymbol{H}_{02}$. In order to evaluate new test statistics we have investigated the powers of ones. Hence our test statistics have higher powers. We may be noted that our test statistics in (2.2) and (3.1) are useful testing for the equality of means even if data sets involves the missing observations.

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