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NON-ZERO DEGREE MAPS AND SURFACE BUNDLES OVER S^1

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1. Introduction

In this paper we study non-zero degree maps $f : M \to N$ between 3-dimensional compact orientable irreducible ∂-irreducible manifolds.

An important question in topology is to decide whether there exists a map of non-zero degree between given manifolds of the same dimension. One can think of the existence of such a map as defining a partial ordering on the set of homeomorphic classes of compact connected manifolds of a given dimension. As suggested by M.Gromov, this partial order can be defined as follows: say that M dominates N, denoted by $M \geq N$, if there is a non-zero degree allowable map from M to N. From H.C.Wang's theorem and Gromov's work [18, Chap 6], it follows that each closed hyperbolic orientable n-manifolds with $n \neq 3$ dominates only finitely many closed orientable hyperbolic n-manifolds. We show that this result fails in dimension 3. This was first established by the second author.

In the case of surface bundles over S^1 we show that if a bundle over S^1 dominates an irreducible ∂-irreducible 3-manifold N, then either the first Betti number decreases or N is a bundle over S^1. Moreover using Thurston's norm on $H^1(.,R)$, we give a necessary and sufficient condition for such maps to be homotopic to a covering or a homeomorphism.

We apply those facts to study W.Thurston's conjecture which claims that any complete finite volume hyperbolic 3-manifold is finitely covered by a surface bundle over S^1. We prove that for any integer $n > 0$, there are infinitely many closed hyperbolic orientable 3-manifolds with first Betti number n such that no tower of abelian coverings over M contains a surface fiber bundle over S^1. So if Thurston's conjecture is true, the coverings involved must be much more complicated than towers of

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abelian coverings. There are few examples known of manifolds covered by surface bundles which are not bundles (cf. [7], [14]).

The content of this paper is as follows :

In Section 2, we consider non-zero degree maps from surface bundles to compact irreducible ∂-irreducible 3-manifolds. Using Thurston's norm on $H^1(.,R)$, we give a necessary and sufficient condition for such maps to be homotopic to a covering or a homeomorphism (Theorem 2.1 and Corollary 2.3). This improves Gabai's result [7] about 3-manifolds covered by a bundle over S^1 with the same first Betti number to the case of non-zero degree maps (Corollary 2.2). As an application we generalize a result of Edmonds and Levingston [5, Theorem 5.2], about finite group action on a surface bundle over S^1 with first betti number 1 to the case of arbitrary first Betti number, but assuming that the quotient manifold is irreducible (Proposition 2.4). Some sufficient conditions for degree-one map to be homotopic to a homeomorphism are also given in term of the homological monodromy. We obtain some applications to degree-one maps between fibered link complements (Proposition 2.6 and Corollary 2.7).

In Section 3 we introduce methods to construct non-zero degree maps between 3-manifolds. Based on those methods, we show that there is a closed orientable hyperbolic 3-manifold which maps onto infinitely many non-homeomorphic hyperbolic 3-manifolds, moreover all these maps are surjective on fundamental groups (Theorem 3.4). A direct corollary is that there is a map of degree-two between two hyperbolic 3-manifolds admiting infinitely many factorizations up to homotopy. We also show (Proposition 3.3) that any closed 3-manifold N is the image by a degree one map of a hyperbolic 3-manifold M, which is a surface bundle over S^1, with $\beta_1(M) = \beta_1(N) + 1$.

In Section 4, we construct the examples related to Thurston's conjecture mentioned above. We prove that for any integer $n \geq 0$, there are infinitely many closed hyperbolic orientable 3-manifolds with first Betti number n such that no tower of abelian coverings over M contains a surface bundle over S^1. Moreover we show that any immersed surface in this tower which is either embedded or homologically non-trivial is not a virtual fiber. We also give an example of non-trivial regular coverings between hyperbolic integer homology 3-spheres, answering a question of Luft and Sjerve [11, p.468]. Actually there is an infinite tower of regular covering between hyperbolic homology 3-spheres (dicussed with M.Baker).

We end this section by some basic definitions and notation that we will use in the paper. We also state some well-known facts about non-zero degree maps.

Let $f : M \to N$ be a map between orientable compact connected n-manifolds. We say that f is proper if $f^{-1}(\partial N) = \partial M$. We say that f is allowable if f is proper and the degrees of all possible restrictions $f| : F \to S$ have the same sign, where F is a component of ∂M and S is a component of ∂N. If M is closed or ∂M contains only one component, or if f is a branched covering, then f is always allowable. In dimension 2 the notion of "allowable map" here coincides with the notion of A. Edmonds [4] up to deformation of $(M, \partial M)$. Suppose f is proper. Then f induces homomorphisms $f_* : \pi_1(M) \to \pi_1(N)$, $f_\# : H_*(M, \partial M) \to H_*(N, \partial N)$, $f^\# : H^*(N, R) \to H^*(M, R)$. The degree of f, $\deg(f)$, is given by the equation $f_\#([M]) = \deg(f)[N]$, where $[M] \in H_3(M, \partial M; Z)$ and $[N] \in H_3(N, \partial N; Z)$ are the chosen fundamental classes of M and N.

Lemma 1.0. *Suppose $f : M \to N$ is a proper non-zero degree map between compact irreducible orientable 3-manifolds. Then the following holds:*

i) *$f_*(\pi_1(M))$ is a finite index subgroup of $\pi_1(N)$.*
ii) *$f_\# : H_*(M, \partial M, R) \to H_*(N, \partial N, R)$ is surjective, in particular $\beta_1(M) \geq \beta_1(N)$.*
iii) *If $f : M \to N$ is a degree-one allowable map, then ∂M and ∂N have the same number of components.*

In this paper all the 3-manifolds considered are compact, irreducible, and orientable, and all maps are allowable and of non-zero degree.

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2. Nonzero degree maps from the surface bundles over S^1

Theorem 2.1. *Let M and N be two compact irreducible ∂-irreducible orientable 3-manifolds. Suppose M is a bundle over S^1 and denote by $\alpha \in H^1(M, R)$ the integer cohomology class corresponding to the fibration of M. Let $f : M \to N$ be an allowable non-zero degree map. If there is a rational cohomology class $\beta \in H^1(N, R)$ such that $f^\#(\beta) = \alpha$. Then the following hold:*

i) *N is also a bundle over S^1 and f is properly homotopic to a fiber preserving map.*
ii) *Moreover if $x(\alpha) = |\deg(f)|x(\beta)$, then f is properly homotopic to a fiber preserving covering, where x denotes the Thurston*

norm on $H^1(.,R)$.

In Theorem 2.1, if M and N have the same first Betti numbers $\beta_1(M) = \beta_1(N)$, then $f^\#$ is an isomorphism. In [6], D. Gabai has proved that if $f : M \to N$ is a covering between two compact orientable irreducible ∂-irreducible 3-manifolds, and M fibers over S^1 but N does not, then $\text{rank}H^1(M,R) > \text{rank}H^1(N,R)$. So we have the following improvement of Gabai's result, which replace covering by "non-zero degree map".

Corollary 2.2. *Let M and N be two compact irreducible ∂-irreducible orientable 3-manifolds. Suppose M is a surface bundle over S^1 and $\beta_1(M) = \beta_1(N)$. If there is an allowable map $f : M \to N$ of non-zero degree, then N is also a surface bundle over S^1, and f is properly homotopic to a fiber preserving map.*

Proof of Theorem 2.1. In the following we use (F,ϕ) to denote the surface bundle over S^1 with connected fiber F and monodromy $\phi : F \to F$.

Suppose that $M = (F,\phi)$. Since M is irreducible and ∂-irreducible, F is not S^2 or D^2. Let $\alpha \in H^1(M,R)$ be the integer cohomology class associated to the fibration. Then α defines a surjective homomorphism $h_\alpha : \pi_1(M) \to Z \to 0$ whose kernel $\ker h_\alpha$ is $\pi_1(F)$.

Since $\alpha \in imf^\#$, the image of $f^\#$, and $f^\#$ is injective, there is a unique rational cohomology class $\beta \in H^1(N,R)$ such that $f^\#(\beta) = \alpha$. Let $\beta' = d\beta$ be a primitive integer class in $H^1(N,R)$, where d is an integer. Then $f^\#(\beta') = d\alpha$. Since β' is a primitive integer cohomology class, it defines a surjective homomorphism $h_{\beta'} : \pi_1(N) \to Z \to 0$.

We have the following commutative diagram:

$$(*)\qquad \begin{array}{ccccccccc}
0 & \longrightarrow & ker h_\alpha & \longrightarrow & \pi_1(M) & \xrightarrow{h_\alpha} & Z & \longrightarrow & 0 \\
 & & {\scriptstyle \bar{f}_*}\big\downarrow & & {\scriptstyle f_*}\big\downarrow & & {\scriptstyle d\times}\big\downarrow & & \\
0 & \longrightarrow & ker h_{\beta'} & \longrightarrow & \pi_1(N) & \xrightarrow{h_{\beta'}} & Z & \longrightarrow & 0
\end{array}$$

This comes from the fact that $f^\#(\beta') = d\alpha$ and $h_\alpha(\gamma) =< [\gamma], \alpha >$, where $\gamma \in \pi_1(M)$ and $< .,. >$ is the Kroneker product, or equivalently, $h_\alpha(\gamma)$ is the algebraic intersection number of the loop γ and the fiber F. Thus using the same notation for N, one obtains that

$$h_{\beta'}(f_*(\gamma)) =< [f \circ \gamma], \beta' >=< [\gamma], f^\#(\beta') >=< [\gamma], d\alpha >= dh_\alpha(\gamma).$$

Since the induced homomorphism $Z \to Z$ is injective, it follows that

$$ker h_{\beta'} \cap f_*(\pi_1(M)) = \bar{f}_*(ker h_\alpha);$$

therefore the injective homomorphism $\ker h_{\beta'} \to \pi_1(N)$ induces an injection of the quotient set $kerh_{\beta'}/\bar{f}(\ker h_\alpha)$ into the finite set $\pi_1(N)/f_*(\pi_1(M))$. So $\bar{f}_*(kerh_\alpha)$ is of finite index in $kerh_{\beta'}$. Since $kerh_\alpha = \pi_1(F)$ is finitely generated, $kerh_{\beta'}$ is also finitely generated.

By Stalling's fibration theorem [17], $kerh_{\beta'} = \pi_1(S)$, where S is a connected surface properly embedded in N , and the homomorphism $h_{\beta'}$ corresponds to a fibration of N over S^1 with fiber S ; therefore $N = (S, \psi)$.

Moreover the diagram (*) of homomorphisms between $K(\pi, 1)$ spaces can be realized by a unique, up to homotopy, fiber preserving map g. Thus f is homotopic to g. Moreover, if ∂M is not empty it is a non-empty union of tori, then ∂N is also a non-empty union of tori. Since f is allowable, we may assume that f has been first deformed so that the restriction $f|_{\partial M}$ is a covering. In this case all the deformations above can be made relatively to ∂M, and f is properly homotopic to a fiber preserving map g.

To prove part ii) of Theorem 2.1, by Waudhausen's Theorem [9] it is sufficient to prove that $f_* : \pi_1(M) \to \pi_1(N)$ is injective. Since the induced homomorphism $Z \to Z$ is injective in the diagram (*), the injectivity of f_* is equivalent to that of $\bar{f}_* : \pi_1(F) \to \pi_1(S)$.

From part i) of Theorem 2.1 we may assume that f is a fiber preserving map whose restriction on F induces the map $\bar{f} : F \to S$. Moreover $\deg f = d \deg \bar{f}$. Since F is a fiber of the fibration over S^1 associated to α and F is not S^2 or D^2, we have the Thurston norm $x(\alpha) = -\chi(F)$. In the same way $x(\beta') = -\chi(S)$ (see [19]).

By assumption $x(\alpha) = |\deg f| x(\beta)$ where $\beta = (1/d)\beta'$ in $H^1(N, R)$; therefore $x(\alpha) = |\deg \bar{f}| x(\beta')$ and $\chi(F) = |\deg \bar{f}| \chi(S)$. From the Hurewitz formula and Edmond's work [4] it follows that $\bar{f}$ is homotopic to a covering map, so that $\bar{f}_* : \pi_1(F) \to \pi_1(S)$ is injective.

As a corollary of Theorem 2.1 when M is not a torus bundle over the circle we obtain the following charaterization of non-zero degree allowable map from M to N as above which are properly homotopic to a homeomorphism.

Corollary 2.3. *Let M and N be two compact irreducible ∂-irreducible orientable 3-manifolds. Suppose M is a bundle over the S^1 with a fiber of negative Euler characteristic. Then a non-zero degree allowable map $f : M \to N$ is properly homotopic to a homeomorphism if and only if $f^\# : H^1(N, R) \to H^1(M, R)$ is an isometry with respect to the Thurston's norm.*

Proof. From part i) of Theorem 2.1, it follows that $M = (F, \phi)$, $N = (S, \psi)$ and f may be assumed to be fiber preserving. Therefore, if $\alpha \in H^1(M,R)$ and $\beta \in H^1(N,R)$ are associated rational cohomology classes such that $f^{\#}(\beta) = \alpha \neq 0$, then $x(\alpha) \geq |\deg f| x(\beta)$. Since $f^{\#}$ is an isometry with respect to Thurston's norm, it follows that $x(\alpha) = x(\beta)$ and thus $\deg f = \pm 1$, because $x(\alpha) > 0$. So we can apply part ii) of Theorem 2.1 to conclude the proof

The next result has been proved in [5, Theorem 5.2 and Corollary 5.3] under the additional condition $\beta_1(M) = 1$.

Proposition 2.4. *Suppose M is an orientable 3-manifold which is a surface bundle over S^1, and G is a orientation preserving finite group action on M. If $\beta_1(M) = \beta_1(M/G)$ and M/G is irreducible, then G can be conjugated to preserve the fiber structure of M.*

Proof. If $\beta_1(M) = \beta_1(N) = 1$, the proposition has been proved (see [5], Theorem 5.2, Remarks after Theorem 5.2, Corollary 5.3). So below we may assume that $\beta_1(M) = \beta_1(N) > 1$.

Suppose $M = (F, \phi)$. Then F is neither S^2 nor D^2, and M is irreducible ∂-irreducible. Now the quotient map $q : M \to M/G$, as a branched covering, is an allowable map of non-zero degree. Since M/G is irreducible and $\beta_1(M) = \beta_1(M/G)$, by Theorem 2.1 M/G is a surface bundle (S, φ), and $q : M \to M/G$ is homotopic to a fiber preserving map $f : (F, \phi) \to (S, \psi)$. By Theorem 2.1 of [5], we may assume that S has been deformed so that $q^{-1}(S)$ in M is 2-sided incompressible. Let $d = \deg(q) = \deg(f)$. Then

$$q_{\#}([q^{-1}(S)]) = d[S] = f_{\#}([f^{-1}(S)]).$$

Since $q_{\#}, f_{\#} : H_2(M, \partial M; R) \to H_2(M/G, \partial M/G; R)$ are the same isomorphisms, $q^{-1}(S)$ and $f^{-1}(S)$ are in the same homology class. Since $q^{-1}(S)$ is 2-sided incompressible and $f^{-1}(S)$ is a union of parallel copies of the fiber F, by the well-known argument of Stallings $q^{-1}(S)$ is isotopic to the union of parallel copies of F. The remaining argument is the same as that of Theorem 5.2 of [5].

A direct corollary of Proposition 2.4 is

Corollary 2.5. *Suppose L is a fibered link in a homology sphere Σ^3, and G is an orientation preserving finite group action on $M = \overline{\Sigma^3 - N(L)}$. If each boundary component is invariant under G, and M/G is irreducible, then G can be conjugated to preserve the fiber structure of M.*

In the remaining of this section we restrict ourselves to the case of degree-one allowable maps from surface bundles to 3-manifolds.

Let F be a connected orientable surface. Then the algebraic intersection pair is a symplectic form on $H_1(F, Q)$, and any homeomorphism $\phi : F \to F$ induces a homomorphism $\phi_{\#} : H_1(F, Q) \to H_1(F, Q)$. Moreover any proper compact subsurface $F_0 \subset F$, such that neither F_0 nor

$F - F_0 = F'$ is a disc or an annulus, corresponds to a proper symplectic subspace $V = H_1(F_0, Q)$ in $H_1(F, Q)$.

Proposition 2.6. *Let M and N be two compact irreducible ∂-irreducible orientable 3-manifolds with the same first Betti number. Suppose M is a bundle (F, ϕ) over S^1 such that the homological monodromy $\phi_\# : H_1(F, Q) \to H_1(F, Q)$ has the following property : any symplectic proper subspace $V \subset H_1(F, Q)$ with $\phi_\#(V) \subset V$ satisfies $V \cap ker(\phi_\# - Id) \neq \{0\}$. Then any degree-one allowable map $f : M \to N$ is properly homotopic to a homeomorphism. This is in particular true if the homological monodromy $\phi_\#$ is irreducible (i.e., has no proper invariant subspace) or is the identity.*

Proof. Let $M = (F, \phi)$. Then from Theorem 2.1 it follows that $N = (S, \psi)$ is a bundle over S^1. We can assume that f is fiber preserving. Moreover, since $\deg f$=1, the preimage of each fibers S of N under f is a fiber F of M and the restriction map $f| : F \to S$ is a degree-one allowable map. Therefore, by [4, Theorem 4.1], we can assume that $f| : F \to S$ is either a homeomorphism or a 2-dimensional pinch map. If $f|$ is a homeomorphism, the suspension of this homeomorphism gives a homeomorphism between M and N. If $f|$ is a pinch map, there is an essential separating simple closed curve c on F such that:

i) $F = F_0 \cup F'$, where neither F_0 nor F' is a disc or an annulus (otherwise S will be a disk or a sphere);
ii) $f(F') = x_0$ is a point in S;
iii) $f| : intF_0 \to S - x_0$ is a homeomorphism.

Since $f : M \to N$ is fiber preserving, the commutativity of the diagram below implies that the restriction maps $f| : F \to S$ and $\psi f| \phi^{-1} : F \to S$ are homotopic. In particular $f|_\# = (\psi \circ f|_1 \circ \varphi^{-1})_\#$

$$\begin{array}{ccc} H_1(F,Q) & \xrightarrow{\phi_\#} & H_1(F,Q) \\ {\scriptstyle f|_\#}\downarrow & & {\scriptstyle f|_\#}\downarrow \\ H_1(S,Q) & \xrightarrow{\psi_\#} & H_1(S,Q) \end{array}$$

Since $f| : F \to S$ is a pinch, $kerf|_\# = V$ is isomorphic to $H_1(F_0, Q)$ and is a proper sympletic subspace of $H_1(F, Q)$ such that $\phi_\#(V) \subset V$ because of the commutativity of the square diagram.

From the fact that $H_1(M, Q) = Q \oplus H_1(F, Q)/Im(\phi_\# - Id)$ and $H_1(N, Q) = Q \oplus H_1(S, Q)/Im(\psi_\# - Id)$, an easy calculation shows that $kerf_\# = V/(\phi_\# - Id)(V)$. Since $f_\# : H_1(M, Q) \to H_1(N, Q)$ is an isomorphism, $kerf_\# = 0$ and the restriction of $(\phi_\# - Id)$ has to be invertible on V. This would contradict the hypothesis on the homology monodromy.

By considering a fibered link in a homology sphere Σ^3 we obtain:

Corollary 2.7. *Let E be the exterior of a fibered link L in a homology 3-sphere Σ with an irreducible Alexander polynomial $\Delta_L(t)$ in $Z(t, t^{-1})$. Then any degree-one allowable map $f : E \to N$ is properly homotopic to a homeomorphism, where N is a compact irreducible, ∂-irreducible orientable 3-manifold.*

Proof. Since $\Delta_L(t)$, up to a power of t, is the characteristic polynomial of the monodromy of L, it follows that the monodromy is homologically irreducible over Q. To apply Proposition 2.6, it remains to show that E and N have the same first Betti number.

Let m be the number of boundary components of E. Then $\beta_1(E) = m$. Since f is of non-zero degree, we have $\beta_1(E) \geq \beta_1(N)$. Since f is of degree one and allowable, so m is also the number of components of ∂N, and we have $\beta_1(N) \geq m$. Hence $\beta_1(E) = \beta_1(N)$, the corollary follows from Proposition 2.6.

Remarks about Section 2. (1) If f is only a proper map of non-zero degree, then Theorem 2.1 is still true but the homotopy between f and a fiber preserving map in i) or a covering in ii) is no longer a proper homotopy. Moreover, the condition "∂-irreducible" can be removed for all results in this section which do not involve the Thurston's norm.

(2) With respect to Corollary 2.3, if M is a torus bundle over the circle and N is irreducible, it was known that any non-zero degree map $f : M \to N$ is homotopic to a covering; in particular, if f is degree one, then f is homotopic to a homeomorphism [22, Theorem 4], and there are no conditions on the first Betti numbers. If we put the condition on the first Betti numbers, then the result about torus bundles over S^1 can be generalized to 1-punctured torus bundles over S^1 by Theorem 2.1.

3. Partial order of 3-manifolds and construction of non-zero degree one maps

As suggested by M.Gromov, one can define a partial order on the set of homeomorphic classes of connected compact orientable 3-manifolds as follows: say that M dominates N, denoted by $M \geq N$, if there is a non-zero degree allowable map from M to N, and that M d-dominates N, denoted by $M \geq_d N$, if there is a degree-d allowable map from M to N. (For a general study of this partial ordering see the second author work [22]).

From H.C.Wang's theorem and Gromov's work, it follows that each closed hyperbolic orientable n-manifolds with $n \neq 3$ dominates only finitely many closed orientable hyperbolic n-manifolds. In this section we show that there is a closed orientable hyperbolic 3-manifold which 2-dominates infinitely many hyperbolic 3-manifolds, and also that each 3-manifold is 1-dominated by a hyperbolic surface bundle, which improves the early result of Brooks that each 3-manifold is 2-dominated by a hyperbolic surface bundle.

Definition 3.1. Suppose M is a compact 3-manifold, k is a knot in M and α is a simple closed curve on $\overline{\partial M - N(k)}$. We use $M(k,\alpha)$ to denote the manifold obtained from M by Dehn surgery on k with surgery slope α, that is, $M(k,\alpha) = \overline{M - N(k)} \cup_h D^2 \times S^1$, where the gluing map h identifies $\partial D^2 \times y$ with α for a point $y \in S^1$.

We call a knot k in M null-homotopic, if k is homotopic to a point.

The following fact is useful to construct degree-1 maps between 3-manifolds.

Proposition 3.2. *Suppose M is a closed irreducible, orientable 3-manifold, and k is a null homotopic knot in M. Then there is a degree-one map $f : M(k,\alpha) \to M$.*

Proof. Since k is null-homotopic in M, k can be obtained from a trivial knot k' by finitely many self-crossing-changes of k'. Let D' be an embedded disk in M bounded by k'. If we let D' move following the self-crossing-changes from k' to k, then each self-crossing-change of k' corresponds to an identification of a pair of arcs in D'. Thus it is easy to see that the singular disk D obtained in M with $\partial D = k$ has the homotopy type of a graph.

Let $N(D)$ be a regular neighborhood of D in M. Then $N(D)$ is an irreducible handlebody. We assume that $N(D)$ is reasonable large so that $N(k) \subset N(D)$. Since $\alpha \subset \partial N(k)$ is homotopic to a multiple of k in $N(k)$, α is homotopic to zero in $N(D)$. We are going to construct a degree-one map

$$f : M(k,\alpha) = \overline{M - N(k)} \cup D^2 \times S^1 \to \overline{M - N(k)} \cup N(k) = M$$

by three steps.

Step 1. Define $f : \overline{M - N(k)} \to \overline{M - N(k)}$ to be the identity.

Step 2. Since α is homotopic to zero in $N(D)$, we extend f in Step 1 to $\overline{M - N(k)} \cup (D^2 \times *)$ by sending $(D^2 \times *)$ to $N(D)$, where $* \in S^1$ and $\partial D^2 \times *$ is α.

Step 3. Now $\overline{D^2 \times S^1 - D^2 \times *}$ is a 3-ball D^3. Since $N(D)$ is irreducible and $f|(\partial D^3) \subset N(D)$, we can extend the map to whole $M(k,\alpha)$ by sending D^3 into $N(D)$.

Since M is a closed 3-manifold, $N(D)$ is a proper subset of M. From Step 1, we know that the degree of f is one.

Proposition 3.3. *Suppose M is a closed orientable irreducible 3-manifold. Then there is a closed orientable hyperbolic 3-manifold M_1 such that:*

i) $\beta_1(M_1) = \beta_1(M) + 1$;
ii) *M_1 is a surface bundle over S^1, with pseudo-Anosov monodromy (i.e., M_1 is a hyperbolic 3-manifold) ;*
iii) *there is a degree-one map $f : M_1 \to M$.*

Proof. By a theorem of Soma [15, Theorem 2], there is a fibered hyperbolic knot k in M such that any 3-manifold obtained by a non-trivial surgery on M along k is hyperbolic. Furthermore, from Soma's construction the knot k can be chosen to be null-homotopic in M, since it can be obtained from a null-homotopic fibered knot in M first by taking a 2-Strands cable and then by doing a finite number of Murasugi sums (cf. also [6, Theorem 7.7]).

Let M_1 be obtained from M by a longitudinal surgery along k. Then M_1 is a hyperbolic surface bundle A standard calculation shows that $\beta_1(M_1) = \beta_1(M) + 1$. By Proposition 3.2, there is a degree-one map $f : M_1 \to M$.

Theorem 3.4. *There is a closed orientable hyperbolic 3-manifold M which maps by a degree-2 map onto infinitely many non-homeomorphic orientable closed hyperbolic 3-manifolds. Moreover the induced homomorphisms are surjective on the fundamental groups.*

Proof. Suppose X is a compact irreducible, *partial*-irreducible orientable 3-manifold with ∂X a torus. We will denote by $X(\alpha)$ the closed orientable 3-manifold obtained from X by Dehn filling: $X(\alpha) = X \cup_\phi S^1 \times D^2$, where $\phi : \partial X \to \partial S^1 \times D^2$ sends the simple closed curve $\alpha \subset \partial X$ to $* \times \partial D^2$. If X is hyperbolic, $X(\alpha)$ will be hyperbolic for all but finitely many slopes α by Thurston's hyperbolic surgery Theorem [18, Chap.4.]. Then with Proposition 3.3, to prove Theorem 3.4, it is sufficient to find an orientable, connected irreducible 3-manifold M which maps by a degree-2 map onto $X(\alpha)$ for any simple closed curve α on ∂X. This follows from the following:

Proposition 3.5. *Let X be an orientable, hyperbolic 3-manifold with ∂X a torus. Let $D(X)$ be the double of X. Then $D(X)$ maps by a degree-2 map onto any manifolds $X(\alpha)$. Moreover the induced homomorphisms are surjective on the fundamental groups.*

Proof. Let α be an oriented simple closed curve on ∂X. We identify $T^2 \times I$ with $S^1 \times A$, where A is an annulus. Let $\partial A = c_+ \cup c_-$, $c_+ \subset T^2 \times 0$, $c_- \subset T^2 \times 1$. Let β be a dual curve to α on ∂X, $\alpha \cap \beta = 1$. Define $Y = X^+ \cup_{\phi_1} S^1 \times A \cup_{\phi_2} X^-$, where X^+ and X^- are two copies of X, $\phi_1 : \partial X^+ \to S^1 \times c^+$ is given by $\phi_1(\alpha) = c^+$, $\phi_1(\beta) = S^1 \times *$ and

$\phi_2 : S^1 \times c^- \to \partial X^-$ is given by $\phi_2(c^-) = -\alpha$, $\phi_2(S^1 \times *) = \beta$. Since $\phi_2 \circ \phi_1 : \partial X^+ \to \partial X^-$ is given by the matrix $\left(\begin{smallmatrix} -1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, it follows that Y is homeomorphic to the double DX of X.

Let $p_\alpha : S^1 \times A \to S^1 \times B$ be the double orientable covering, where B is the Möbius band. Let τ be the covering involution. Then τ can be extended to an orientation reversing involution τ on Y, by just exchanging X^+ and X^-. In fact $Y/\tau = X \cup_{\psi_\alpha} (S^1 \times B)$, where $\psi_\alpha : \partial X \to S^1 \times \partial B$ is given by $\psi_\alpha(\alpha) = * \times \partial B$, $\psi_\alpha(\beta) = S^1 \times *$.

Now we can construct a degree-2 map $f_\alpha : Y \to X(\alpha)$ by composition of $p_\alpha : Y \to Y/\tau = X \cup \psi_\alpha(S^1 \times B)$ and $h_\alpha : X \cup_{\psi_\alpha} S^1 \times B \to X(\alpha)$ which is defined as follows:

$h_\alpha| : X \to X$ is the identity.

$h_\alpha| : * \times B \to * \times D^2$ is a proper degree-one map.

$h_\alpha| : S^1 \times B - * \times B \to S^1 \times D^2 - * \times D^2 = D^3$, is then a proper degree-one map.

Since the $\pi_1(X(\alpha))$ is carried by X and a component of $f_\alpha^{-1}(X)$ maps onto X homeomorphically under f_α, f_α is π_1-surjective.

Remark on Theorem 3.4 and Proposition 3.5. (1) One cannot expect that there is an orientable 3-manifold M which maps with degree one onto such family of 3-manifolds as in Proposition 3.5. The obstruction lies in the torsion part of the first homology group of M. For details, see [2] or [10].

(2) If X is a complete hyperbolic manifold with finite volume, then $X(\alpha)$ is also so for almost all α. Moreover if $|M|$ denotes the simplicial volume of a 3-manifold M, then $|DX| = 2|X|$, and $|X(\alpha)|$ tends to $|X|$ as the length of α tends to infinite. It follows that DX is of minimal simplicial volume with respect to the property of having degree-two maps onto all the manifolds $X(\alpha)$.

Corollary 3.7. *There is a degree-two map between two hyperbolic 3-manifolds with infinitely many different factorizations up to homotopy.*

Proof. Let k be a null-homotopic hyperbolic knot in a closed hyperbolic manifolds N. Let $X = N - intN(k)$, the exterior of k in N. Then we have the following sequence of maps:

$$M \xrightarrow{f} D(X) \xrightarrow{p_\alpha} X \cup S^1 \times B \xrightarrow{h_\alpha} X(\alpha) \xrightarrow{g_\alpha} N.$$

The maps p_α and h_α have been constructed in the proof of Proposition 3.5, The hyperbolic 3-manifold M and the maps f are provided by Proposition 3.3. For almost all α, $X(\alpha)$ is a hyperbolic manifold and the map g_α is given by Proposition 3.2. It is not hard to verify that all these maps $g_\alpha \circ h_\alpha : X \cup S^1 \times B \to N = X \cup S^1 \times D^2$ are homotopic. So all the composition $g_\alpha \circ (h_\alpha \circ p_\alpha \circ f) : M \to N$ are homotopic to a fixed map of degree 2.

Definition 3.8. Suppose M is an orientable irreducible ∂-irreducible 3-manifold. We say M is minimal, if for any orientable irreducible ∂-irreducible 3-manifold N different from S^3 and a solid torus, $M \geq_1 N$ implies that M is homeomorphic to N.

By definition, the real projective 3-space is minimal, but to our knowledge no closed orientable hyperbolic 3-manifold is known to be minimal. In the case of non-empty boundary, according Corollary 2.7, the exterior of all fiber knots in homology spheres with irreducible Alexander polynomial are minimal manifolds. In particular the complements of the figure eight knot is a minimal 3-manifold.

4. Totally null-homotopic hyperbolic knots and coverings

The following definition appears in [1].

Definition 4.1. Suppose M is a closed orientable 3-manifold. A knot k is said to be totally null-homotopic, if k is the boundary of a singular disk D such that a regular neighborhood $N(D)$ of D is null-homotopic in M, that is to say: $i_*(\pi_1(N(D))) = \{1\}$, where $i_* : \pi_1(N(D)) \to \pi_1(M)$ is induced by the embedding of $N(D)$ in M.

If k is null-homotopic, then there is a prefered meridian-longitude system (μ, λ) such that λ is homological to zero in $\overline{M - N(k)}$. Therefore any simple closed curve on $\partial N(k)$ has a unique slope (n, m) under this system, where n, m are coprime.

Proposition 4.2. *Suppose M is a closed orientable 3-manifold. Then M contains a totally null-homotopic hyperbolic knot k.*

Proof. The proof is based on the following results of Myers [12], [13] : if M is a compact orientable 3-manifold containing no 2-sphere in ∂M, and γ is a proper arc or a simple closed curve in M, then in the proper homotopy class of γ there is a proper arc or a simple closed curve γ' such that $\overline{M - N(\gamma')}$ is simple, i.e., $\overline{M - N(\gamma')}$ is irreducible, ∂-irreducible and contains no non-boundary-parallel incompressible annulus and torus.

Let γ_1 be a null-homotopic knot in M. By Myers's result, there is a proper arc γ_2 in $\overline{M - N(\gamma_1)}$, which can be properly deformed into the boundary, such that $\overline{M - N(\gamma_1) - N(\gamma_2)}$ is simple. Let $H = N(\gamma_1) \cup N(\gamma_2)$. Then H is a handlebody of genus two, which is null-homotopic in M. Let k be a null-homotopic knot in H such that $\overline{H - N(k)}$ is simple. Now k is a knot in M. Since $\overline{M - N(K)}$ is obtained from the simple 3-manifolds $\overline{M - H}$ and $\overline{H - N(k)}$ by identifying genus-two closed surfaces in the boundaries, $\overline{M - N(k)}$ is simple. By Thurston's hyperbolization theorem [20], [21], it is equivalent to say that k is a hyperbolic knot. Since k bounds a singular disk in H and H is null-homotopic in M, k is totally null-homotopic in M.

Suppose M is a closed orientable 3-manifold, k is a knot in M and α is a simple closed curve on $\partial N(k)$. Let $p : \tilde{M} \to M$ be a finite covering. If each component of $p^{-1}(N(K))$ maps homeomorphically on $N(k)$ under p, then for any surgery manifold $M(k, \alpha)$, there is a unique induced covering $p(k, \alpha) : \tilde{M}(p^{-1}(k), p^{-1}(\alpha)) \to M(k, \alpha)$, where the surgery curve on $\partial N(\tilde{k}_i)$, $\tilde{k}_i$ a component of $p^{-1}(k)$, is a component of $p^{-1}(\alpha) \cap \partial N(\tilde{k}_i)$. In particular, if k is a null-homotopic knot in M, then this (unique) induced covering $p(k, \alpha)$ always exists. It is the pull-back of the covering $p : \tilde{M} \to M$ by the degree-one map $f : M(k; \alpha) \to M$. So there is a commutative diagramm:

$$\begin{tikzcd} M(p^{-1}(k), p^{-1}(\alpha)) \arrow[r, "\tilde{f}"] \arrow[d, "p(k,\alpha)"'] & \tilde{M} \arrow[d, "p"'] \\ M(k, \alpha) \arrow[r, "f"] & M \end{tikzcd}$$

With the notation above, we have

Lemma 4.3. *Suppose k is a totally null-homotopic knot in M. Then for any slope $\alpha \neq \lambda$, $\beta_1(\tilde{M}(p^{-1}(k), p^{-1}(\alpha)) = \beta_1(\tilde{M})$; furthermore, if $\alpha = (1, m)$, then $H_1(\tilde{M}(p^{-1}(k), p^{-1}(\alpha)), Z) = H_1(\tilde{M}, Z)$.*

Proof. Since k is totally null-homotopic, there is a singular disk D bounded by k such that $N(D)$ is null-homotopic in M. So each component of $p^{-1}(N(D))$ is homeomorphic to $N(D)$ under p; this implies that each component of $p^{-1}(k)$ bounds a singular disk, and all those singular disks are mutually disjoint.

Let $p^{-1}(k) = \{\tilde{k}_1, ..., \tilde{k}_d\}$, where d is the degree of p. We denote the component of $p^{-1}(N(D))$ associated with $\tilde{k}_i$ by $\tilde{D}_i$, and the component of $p^{-1}(\alpha)$ associated with $\tilde{k}_i$ by $\tilde{\alpha}_i$. Finally, let $\tilde{\lambda}_i = p^{-1}(\lambda) \cap \partial N(\tilde{k}_i)$. Let $\tilde{M}_1 = \tilde{M}(\tilde{k}_1, \tilde{\alpha}_1)$. Then inductively define $\tilde{M}_{i+1} = \tilde{M}_i(\tilde{k}_{i+1}, \tilde{\alpha}_{i+1})$.

Since $\tilde{\alpha}_1 \neq \tilde{\lambda}_1$ and $\tilde{\lambda}_1$ is the longitude of $\tilde{k}_1$ in $\tilde{M}$, we have $H_1(\tilde{M}_1, R) = H_1(\tilde{M}, R)$; furthermore $H_1(\tilde{M}_1, Z) = H_1(\tilde{M}, Z)$ if $\alpha = (1, m)$. Suppose $H_1(\tilde{M}_i, R) = H_1(\tilde{M}, R)$, and similarly $H_1(\tilde{M}_i, Z) = H_1(\tilde{M}, Z)$ if $\alpha = (1, m)$. Since the previous surgery are disjoint from $N(\tilde{D}_{i+1})$, $\tilde{\lambda}_{i+1}$ is still the longitude in $\tilde{M}_{i+1} - N(\tilde{k}_{i+1})$. But $\tilde{\alpha}_{i+1} \neq \tilde{\lambda}_{i+1}$, so $H_1(\tilde{M}_{i+1}, R) = H_1(\tilde{M}_i, R) = H_1(\tilde{M}, R)$. Furthermore, if $\alpha = (1, m)$, then $\alpha_{i+1} = (1, m)$ and $H_1(\tilde{M}_{i+1}, Z) = H_1(\tilde{M}_i, Z) = H_1(\tilde{M}, Z)$. Thus by induction we have

$$H_1(\tilde{M}(p^{-1}(k), p^{-1}(\alpha)), R) = H_1(\tilde{M}_d, R) = H_1(\tilde{M}, R).$$

Similarily

$$H_1(\tilde{M}(p^{-1}(k), p^{-1}(\alpha)), Z) = H_1(\tilde{M}_d, Z) = H_1(\tilde{M}, Z),$$

if $\alpha = (1, m)$.

Proposition 4.4. *Let M, N be two compact, connected irreducible, $\partial-$irreducible, orientable 3-manifolds. Suppose that M is a surface bundle over S^1. Let $f : M \to N$ be an allowable integer homology equivalence such that for any finite regular covering $p : \tilde{N} \to N$ the induced map $\tilde{f} : \tilde{M} \to \tilde{N}$ is still an integer homology equivalence, where $\tilde{M}$ is the covering of M corresponding to the subgroup $f_*^{-1}(p_*(\pi_1(\tilde{N})))$ and $\tilde{f}$ is a lift of f. Then f is homotopic to a homeomorphism.*

Proof. Suppose f is an integer homology equivalence. Then f is a degree-one map. Since $M = (F, \phi)$ is a bundle over S^1, $N = (S, \psi)$ is also so by Theorem 2.1. Moreover f can be assumed to be fiber preserving. Then the proof of Proposition 4.4 follows from the following Lemma.

Lemma 4.5. *Let $M = (F, \phi)$ and $N = (S, \psi)$ be two surface bundles over S^1. Let $M_d = (F, \phi^d)$ and $N_d = (S, \psi^d)$ be the d-fold cyclic coverings of M and N associated with the fibration over S^1. Assume that $H_1(M_d, Z)$ and $H_1(N_d, Z)$ are isomorphic for all d. Then any degree-one allowable fiber preseving map $f : M \to N$ is properly homotopic to a homeomorphism.*

Proof. Clearly the covering of M corresponding to the covering $N_d = (S, \psi^d)$ of N is $M_d = (F, \phi^d)$, and the lifts $f_d : M_d \to N_d$ are degree-one maps which are surjective in first homology groups. Since finitely generated abelian groups are hopfian, f_d induces an isomorphism $f_{d\#} : H_1(M_d, Z) \to H_1(N_d, Z)$.

Let $M_\infty = F \times R$ and $N_\infty = S \times R$ be the infinite cyclic coverings of M and N associated with the fibrations over S^1. From [3, Prop.2.4], the natural homomorphisms $i_\# : H_1(M_\infty; Z) \to \varprojlim_d H_1(M_d; Z)$ and $j_\# : H_1(N_\infty; Z) \to \varprojlim_d H_1(N_d; Z)$ are injective. By considering the lift $f_\infty : M_\infty \to N_\infty$ of f, which is a proper degree-one map, we obtain the following commutative diagram:

$$\begin{array}{ccccc}
0 & \longrightarrow & H_1(M_\infty, Z) & \xrightarrow{i_\#} & \varprojlim H_1(M_d, Z) \\
\downarrow & & {\scriptstyle (f_\infty)_\#}\downarrow & & {\scriptstyle \varprojlim (f_d)_\#}\downarrow \\
0 & \longrightarrow & H_1(N_\infty, Z) & \xrightarrow{j_\#} & \varprojlim H_1(N_d, Z)
\end{array}$$

Since $i_\#, j_\#, \varprojlim(f_d)_\#$ are injective and f_∞ is surjective, it follows that $(f_\infty)_\#$ is an isomorphism, and hence $H_1(F, Z)$ is isomorphic to $H_1(S, Z)$. Since $f : M \to N$ is fiber preserving, the restriction $f| : F \to S$ is a degree-one allowable map. By [4, Theorem 4.1], $f|$ is properly homotopic to a pinch. Since $H_1(F, Z)$ is isomorphic to $H_1(S, Z)$, $f|$ must be properly homotopic to a homeomorphism, and hence f is also so.

Theorem 4.6. *Let k be a totally null-homotopic hyperbolic knot in a hyperbolic 3-manifold M, not contained in a 3-ball. Then for any finite covering $p : \tilde{M} \to M$ and any simple closed curves $\alpha = (1, m)$ on $\partial N(k)$, $m \neq 0$, the induced covering space $\tilde{M}(p^{-1}(k), p^{-1}(\alpha))$ of $M(k, \alpha)$ is not a surface bundle over S^1.*

Proof. Since k is null-homotopic, by Proposition 3.2 there is a degree-one map $f : M(k, \alpha) \to M$, which induces an integer homology equivalence because $\alpha = (1, m)$, $m \neq 0$. By Lemma 4.3, for any finite covering $p : \tilde{M} \to M$, the induced map $\tilde{f} : \tilde{M}(p^{-1}(k), p^{-1}(\alpha)) \to \tilde{M}$ is still an integer homology equivalence, because k is totally null-homotopic. Therefore if for some finite covering $p : \tilde{M} \to M$ the induced covering $\tilde{M}(p^{-1}(k), p^{-1}(\alpha))$ is a bundle over S^1, from Proposition 4 it follows that the induced degree-one map $\tilde{f} : \tilde{M}(p^{-1}(k), p^{-1}(\alpha)) \to \tilde{M}$ is homotopic to a homeomorphism.

Following the notation in the proof of Lemma 4.3, the degree-one map $\tilde{f} : \tilde{M}(p^{-1}(k), p^{-1}(\alpha)) \to \tilde{M}$ can be written as the composition of degree-one maps $\tilde{f}_i : \tilde{M}_{i+1} \to \tilde{M}_i$, $i = 0, 1, ..., d-1$. Since $\tilde{f}$ is homotopic to a homeomorphism, each map $\tilde{f}_i$ induces isomorphism $\tilde{f}_{i*} : \pi_1(\tilde{M}_{i+1}) \to \pi_1(\tilde{M}_i)$.

If $\tilde{M}(p^{-1}(k), p^{-1}(\alpha))$ is a bundle over S^1, by Waldhausen's Theorem [23], each $\tilde{M}$ is homeomorphic to $\tilde{M}_0 = \tilde{M}$. In particular $\tilde{M}$ is a surface bundle over the circle, and $\tilde{M}(\tilde{k}_1, \alpha_1)$ is a homeomorphic to $\tilde{M}$, where $\tilde{k}_1$ is a lift of k in $\tilde{M}$, which is a totally null-homotopic knot in $\tilde{M}$ not contained in 3-ball. Then a contradiction follows from the following Lemma , which is a particular case of a more general result about Dehn surgery along null homotopic knots in irreducible 3-manifolds (cf [1]).

Lemma 4.7. *Let N be a closed irreducible surface bundle over the circle. Let k be a null homotopic knot in N, not contained in a 3-ball. Then N can never be obtained by a non-trivial Dehn surgery on k.*

Proof. Since N is a surface bundle over S^1, by [9, 14.21], $\pi_1(N)$ is hopfian. Suppose there is a non-trivial surgery manifold $N(k, \alpha)$ which is homeomorphic to N. Since k is null homotopic in N, it follows that there is a degree-one map $f : N(k, \alpha) \to N$. Then f_* is an isomorphism since the group is hopfian. In particular the core of the surgery solid torus k_α must be null homotopic in $N(k, \alpha)$, because it belongs to $ker f_*$.

Since k is not in a ball in N and N is a irreducible surface bundle over S^1, the exterior $E(k)$ of k in N is an irreducible, ∂-irreducible manifold. Since k is null homotopic in N and $H_2(N, Q) \neq 0$, it follows that

$H_2(E(k)), Q) \neq 0$. From [8, Corollary 2.7], the core of the surgery k_α is of infinite order in $\pi_1(N(k,\alpha))$ except for at most one Dehn filling along $\partial E(k)$. Since the trivial Dehn filling, yielding N, has a null homotopic core k, for any non-trivial Dehn filling, the core k_α of the surgery solid torus must be of infinite order in $\pi_1(N(k,\alpha))$. So $N(k,\alpha)$ cannot be homeomorphic to N.

Lemma 4.8. *Suppose M,k and α are as in Lemma 4.3. A finite covering $p' : \tilde{M}' \to M' = M(k,\alpha)$, corresponding to an epimorphism $\phi' : \pi_1 M(k,\alpha) \to G$ where G is the deck transformation group, is induced by a covering $p : \tilde{M} \to M$ if and only if the core of the surgery k_α in $\pi_1 M(k,\alpha)$ is in the kernel of ϕ'.*

Proof. Each component of ${p'}^{-1}(N(k_\alpha))$ maps homeomorphically on $N(k_\alpha)$ under p' if and only if k_α lies in $ker\phi'$. So the direction "only if" is clear. For the other direction, we recall that there is a degree one map $f : M(k,\alpha) \to M$. If k_α is in $Ker\phi'$, then the epimorphism $\phi' : \pi_1 M(k,\alpha) \to G$ factorizes through the epimorphism

$$f_* : \pi_1 M(k,\alpha) \to \pi_1 M$$

to induce an epimorphism $\phi : \phi_1 M \to G$. Therefore, the covering $p' : \tilde{M}' \to M(k,\alpha)$ is the pull-back by $f : M(k,\alpha) \to M$ of the covering $p : \tilde{M} \to M$ associated to the epimorphism $\phi : \pi_1 M \to G$.

Theorem 4.9. *For any integer $n \geq 0$, there are infinitely many closed hyperbolic orientable 3-manifolds with first Betti number n such that any tower of abelian covering contains no fiber bundle over the circle.*

Moreover suppose that F is an immersed surface in any 3-manifold M' belonging to those towers of abelian coverings such that F is either embedded or homologically non-trivial. Then F is not a virtual fiber (therefore F is quasi-fucshian by Bonahon and Thurston, see [16]*).*

Proof. Let k be a totally null-homotopic hyperbolic knot k in M, and let α be of slope $(1,m)$ on $\partial N(k)$. Then in the manifold $M(k,\alpha)$, the longitude λ of $\overline{M - N(k)}$ meets the meridian disk of $N(k_\alpha)$ exactly once, that is to say, k_α is null homologous in $M(k,\alpha)$. Thus for any finite abelian covering $p' : \tilde{M}' \to M' = M(k,\alpha)$ with deck transformation group G, k_α is in the kernel of $\phi' : \pi_1(M') \to G$, so p' is a covering induced by a covering $p : \tilde{M} \to M$ by Lemma 4.8. Hence the first part follows from Theorem 4.6 and its proof.

For the second part, if F is homologically non-trivial and is a virtual fiber, then there is a finite covering $p : \tilde{M} \to M'$ such that $\tilde{M} = (\tilde{F}, \phi)$ and $f_{\#2}([\tilde{F}]) = d[F]$, where d is non-zero. By Theorem 2.1 (and Poincaré duality), M' itself is fibered.

If F is a separating embedded surface and is a virtual fiber, then by standard 3-manifold argument, M' is a union of two twisted I-bundle over the surface. Therefore M' is doublely covered by a surface bundle over the circle, which is an abelian cover. This contradicts to the first part of the theorem.

The next result gives an affirmative answer to a question of Luft and Sjerve [11, p.468].

Proposition 4.10. *There is a regular covering between hyperbolic integer homology 3-spheres.*

Proof. Let $p : S^3 \to P$ be the regular covering from S^3 to the Poincaré homology 3-sphere, where the degree of p is 120. By Proposition 4.2, there is a totally null-homotopic hyperbolic knot in P. Choose a simple closed curve α in $\partial N(k)$ of slope $(1, m)$ so that $P(k, \alpha)$ is hyperbolic. Evidently, $P(k, \alpha)$ is an integer homology sphere.

Let $p(k, \alpha) : S^3(p^{-1}(k), p^{-1}(\alpha)) \to P(k, \alpha)$ be the uniquely induced regular covering. Then $H_1(S^3(p^{-1}(k), p^{-1}(\alpha)), Z) = H_1(S^3, Z)$ by Lemma 4.3. So $p(k, \alpha)$ is a regular covering between integer homology spheres.

References

[1] M.Boileau, M. Domergue & Y.Mathieu, *Sur le complément d'un noeud homotope à zéro*, Préprint, 1995.

[2] W.Browder, Surgery on simple-connected manifolds, Springer, Berlin, 1972.

[3] R.Cowsik & G.Swarup, *A remark on infinite cyclic covers*, J. Pure Appl. Algebra **11** (1977) 131-138.

[4] A. Edmonds, *Deformation of maps to branched coverings in dimension two*, Ann. of Math. **110** (1979) 113-124.

[5] A. Edmonds & C. Livingston, *Group actions on fibered three-manifolds*, Comment. Math. Helv. **58** (1983) 529-542.

[6] D. Gabai, *Detecting fibered link in S^3*, Comment. Math. Helv. **58** (1986) 519-554.

[7] ———, *On 3-manifolds finitely covered by surface bundle*, London Math. Soc. Lecture Note Ser. **112** (1988) 145-154.

[8] ———, *Foliations and the Topology of 3-manifolds.* II, J. Differential Geom. **26** (1987) 461-478.

[9] J. Hempel, *3-manifolds*, Ann. of Math. Stud., No. 86 Princeton University Press, 1976.

[10] C.Hayat-Legrand, S.Wang & H.Zieschang, *Degree one map onto lens spaces*, To appear in Pacific J. Math.

[11] E.Luft & Sjerve, *Regular covering of homology 3-spheres by homology 3-spheres*, Trans. Amer. Math. Soc. **311** (1987) 467-481.

[12] R. Myers, *Simple knots in compact orientable 3-mamifolds*, Trans. Amer. Math. Soc. **273** (1982) 75-92, **278** (1983) 271-288.

[13] ———, *Excellent 1-manifolds in compact 3-manifolds*, Topology Appl. **49** (1993) 115-127.

[14] A. Reid, *A non-Haken hyperbolic 3-manifold covered by a surface bundle*, To appear in Pacific J. Math.

[15] T. Soma, *Hyperbolic fibered links and fibre concordance*, Math. Proc. Cambridge Philos. Soc. **96** (1984) 283-294.

[16] ———, *Virtual fiber groups in 3-manifold groups*, J. London Math. Soc. (2) **43** (1991) 337-354.

[17] J. Stallings, *On fibering certain 3-manifolds, Topology of 3-manifolds and related topics*, Prentice-Hall, Englewood Cliffs, NJ, 1962.

[18] W. Thurston, *Geometry and topology on 3-manifolds*, Princeton note, 1979.

[19] ———, *A norm for the homology of 3-manifolds*, Memoirs of Amer. Math. Soc. **59** (1986) 100-130.

[20] ———, *Hyperbolic structures on 3-manifolds. I : Deformation of acylindral manifolds*, Ann. of Math. **124** (1986) 203-246.

[21] ———, *Hyperbolic structures on 3-manifolds II : Surface groupes and 3-manifolds which fiber over the circle*, Preprint, 1980.

[22] S. Wang, *The existence of maps of non-zero degree between aspherical 3-manifolds*, Math. Z. **208** (1991) 147-160.

[23] F. Waldhausen, *On irreducible 3-manifolds which are sufficiently large*, Ann. of Math. **87** (1968) 56-88.

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