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CORRECTION TO "AN EXPANSION OF CONVEX HYPERSURFACES"

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The proof of case (iii) of Theorems 1.1 and 3.1 in [1] is incorrect. First, on p. 116, if $h_{11} < 0$ at (x_t, t), then it is not clear that the minimum of h_{11}/h_{22} at time t occurs at x_t as is assumed. Second, the assertion at the top of p. 117 that $\widetilde{F}_{ij,rs} = 0$ at (x_t, t) is incorrect.

We now give a correct proof of case (iii) of Theorem 3.1, and hence also of case (iii) of Theorem 1.1. Thus we assume $n = 2$ and let Γ, f, and H_0 be as in Theorem 3.1. We need to show that if H is a solution of the initial value problem

$$\begin{aligned} &\frac{\partial H}{\partial t} = F(\nabla^2 H + HI) - H \quad \text{on } S^2 \times [0, T), \\ &H(\cdot, 0) = H_0, \end{aligned} \tag{1}$$

then the eigenvalues of $\nabla^2 H + HI$ remain in a compact subset of Γ for as long as the solution exists. Problem (1) is then uniformly parabolic and we get higher order estimates as in Lemmas 3.9 and 3.10. The proofs of the existence of a smooth Γ-admissible solution on $S^2 \times [0, \infty)$ and of the assertions concerning asymptotic behavior proceed as before.

Lemmas 3.5 and 3.7 tell us that the eigenvalues of $[h_{ij}] = \nabla^2 H + HI$ remain in $\Gamma \cap [\overline{B_R(0)} - B_r(0)]$ for some controlled positive constants R and r for as long as the solution exists and is Γ-admissible. We shall prove that the eigenvalues of $[h_{ij}]$ in fact lie in a compact subset of $\Gamma \cap [\overline{B_R(0)} - B_r(0)]$.

Since H_0 is Γ-admissible and Γ is open, H_0 is also Γ'-admissible for some slightly narrower symmetric, open, convex cone $\Gamma' \subset \Gamma$ with vertex at the origin. The solution H of (1), which exists at least for small T, is then Γ'-admissible for T small enough. Since $n = 2$ we have

$$f(\lambda) = \tilde{f}(\lambda) + \sigma(\lambda_1 + \lambda_2) \quad \text{for } \lambda \in \Gamma', \tag{2}$$

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for some constant $\sigma \geq 0$, where $\tilde{f} \in C^\infty(\overline{\Gamma'} - \{0\})$ is a symmetric, concave, degree one homogeneous function on $\overline{\Gamma'}$ with

$$\tilde{f} \geq 0 \quad \text{on } \Gamma' \tag{3}$$

and

$$\tilde{f} \equiv 0 \quad \text{on } \partial\Gamma'. \tag{4}$$

From (2), (3), and the normalization $f(1, 1) = 1$ we see that $\sigma \leq \frac{1}{2}$.

Let F and $\widetilde{F}$ be the functions on $\mathfrak{M}(\Gamma)$ and $\mathfrak{M}(\Gamma')$ corresponding to f and $\tilde{f}$, respectively, as explained in [1, p. 104]. Then $F = \widetilde{F} + \sigma L$ on $\mathfrak{M}(\Gamma')$, where L is given by $L(h_{ij}) = h_{11} + h_{22}$. From the proof of Lemma 3.7 (see (3.36)) $F = F(\nabla^2 H + HI)$ satisfies

$$\frac{\partial F}{\partial t} = F_{ij}\nabla_{ij}F + (\mathscr{T} - 1)F, \tag{5}$$

where $\mathscr{T} = F_{11} + F_{22}$, and consequently we have

$$\frac{\partial \widetilde{F}}{\partial t} + \sigma\frac{\partial L}{\partial t} = F_{ij}\nabla_{ij}\widetilde{F} + \sigma F_{ij}\nabla_{ij}L + (\mathscr{T} - 1)\widetilde{F} + \sigma(\mathscr{T} - 1)L, \tag{6}$$

where $\widetilde{F} = \widetilde{F}(\nabla^2 H + HI)$ and $L = L(\nabla^2 H + HI)$. From the proof of Lemma 3.5 (see (3.29), (3.30)) we see that

$$\frac{\partial L}{\partial t} \leq F_{ij}\nabla_{ij}L - (\mathscr{T} + 1)L + 4(\widetilde{F} + \sigma L), \tag{7}$$

which in combination with (6) gives

$$\begin{aligned}\frac{\partial \widetilde{F}}{\partial t} &\geq F_{ij}\nabla_{ij}\widetilde{F} + (\mathscr{T} - 1)\widetilde{F} - 4\sigma\widetilde{F} + (2\sigma\mathscr{T} - 4\sigma^2)L \\ &\geq F_{ij}\nabla_{ij}\widetilde{F} - 4\sigma\widetilde{F},\end{aligned}$$

since $\widetilde{F}, L \geq 0$, $\mathscr{T} \geq 1$, and $\sigma \in [0, \frac{1}{2}]$. This leads to

$$\widetilde{F}_{\min}(t) \geq \widetilde{F}_{\min}(0)e^{-4\sigma t}, \tag{8}$$

where $\widetilde{F}_{\min}(t) = \min_{S^2} \widetilde{F}(\nabla^2 H(\cdot, t) + H(\cdot, t)I)$. If $\tilde{f} > 0$ on Γ', (8) implies that the eigenvalues of $[h_{ij}]$ remain in Γ' for as long as H exists, and we are finished in this case.

If $\tilde{f}$ is zero somewhere in Γ', then $\tilde{f} \equiv 0$ in Γ' by (4) and the concavity of $\tilde{f}$; consequently $\sigma = \frac{1}{2}$ and

$$f(\lambda) = \tfrac{1}{2}(\lambda_1 + \lambda_2) \quad \text{for } \lambda \in \Gamma'.$$

Since $n = 2$ it is clear that we can find a function $\hat{f}$ having the same properties as $\tilde{f}$ above and in addition such that $\hat{f} > 0$ in Γ' and $\hat{f}(1, 1) = 1$. Consider the initial value problem

$$\text{(9)} \qquad \begin{aligned} &\frac{\partial H}{\partial t} = F_\varepsilon(\nabla^2 H + HI) - H \quad \text{on } S^2 \times [0, \infty), \\ &H(\cdot, 0) = H_0, \end{aligned}$$

where F_ε corresponds to the function f_ε given by

$$f_\varepsilon(\lambda) = \tfrac{1}{2}(1 - \varepsilon)(\lambda_1 + \lambda_2) + \varepsilon \hat{f}(\lambda).$$

Since $\hat{f} \in C^\infty(\overline{\Gamma'} - \{0\})$, for $\varepsilon > 0$ small enough, (9) is uniformly parabolic on any Γ'-admissible solution, with parabolicity constants controlled independently of ε. By what we have proved above for the case that $\tilde{f} > 0$ on Γ', we see that (9) has a unique smooth Γ'-admissible solution H_ε for all sufficiently small $\varepsilon > 0$, and by Lemmas 3.5 and 3.7 with F replaced by F_ε, the eigenvalues of $\nabla^2 H_\varepsilon + H_\varepsilon I$ remain in $\Gamma' \cap [\overline{B_R(0)} - B_r(0)]$ for suitable controlled positive constants R and r, independent of ε. Furthermore, the higher order estimates of Lemmas 3.9 and 3.10 are valid, since (9) is uniformly parabolic for $\varepsilon > 0$ sufficiently small, so we conclude that as $\varepsilon \to 0^+$, H_ε converges in any $\tilde{C}^{k,\alpha}(S^2 \times [t, \infty))$ norm for any $t > 0$ to a solution $H \in C^\infty(S^2 \times [0, \infty))$ of

$$\text{(10)} \qquad \begin{aligned} &\frac{\partial H}{\partial t} = \frac{1}{2}\Delta H \quad \text{on } S^2 \times [0, \infty), \\ &H(\cdot, 0) = H_0. \end{aligned}$$

Furthermore, the eigenvalues of $\nabla^2 H + HI$ lie in $\overline{\Gamma'} \cap [\overline{B_R(0)} - B_r(0)]$. This completes the proof in the case that $\tilde{f} \equiv 0$.

Remark. The same argument with minor modifications is valid for $n \geq 3$, provided that, for a suitable cone $\Gamma' \subset \Gamma$, f has the form

$$f(\lambda) = \tilde{f}(\lambda) + \sigma \sum_{i=1}^n \lambda_i \quad \text{for } \lambda \in \Gamma',$$

for some constant $\sigma \geq 0$ and some function $\tilde{f}$ on Γ' having the properties described above. But in general a function f satisfying the hypotheses of Theorem 3.1 does not have such a decomposition unless $n = 2$.

References

[1] J. I. E. Urbas, *An expansion of convex hypersurfaces*, J. Differential Geometry **33** (1991) 91–125.

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