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Correction to "On the density of geometrically finite Kleinian groups"

by

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In this erratum, we correct an error arising in the proof of Proposition 6.4 in [BB]. To do so, we will alter the statement of the proposition. We will then need an extra estimate to apply the revised proposition. With this revised proposition and new estimate in place, all other results in [BB] hold as before.

We follow the notation from [BB].

Proposition 1. *Let $\gamma(s)$ be a smooth curve in N and let $C(t)$ be the geodesic curvature of γ at $\gamma(0)=p$ in the g_t metric. For each $\varepsilon>0$ there exists a $K>0$ depending only on ε, a and $C(0)$ such that $|C(a)-C(0)|\leqslant\varepsilon$ if $\|\eta_t(p)\|\leqslant K$ and $\|\nabla^t\eta_t(p)\|\leqslant K$ for all $t\in[0,a]$.*

Proof. We first describe the error in the proof of the original proposition, and then explain how the new assumptions provide its resolution.

In the original Proposition 6.4 we had assumed that at p we had $\|D_t\eta_t\|\leqslant K$ and $D_t^*\eta_t=0$. In our choice of local coordinates near p at time $t=0$, we had

$$D_0\eta=\sum_{i,j,k}\frac{\partial\eta_i^j}{\partial x_k}e_j\otimes\omega^k\wedge\omega^i,$$

and then the bound on $\|D_0\eta_0\|$ allowed us to bound the original terms in the sum. We then saw that

$$D_0^*\eta_0=\sum_{i,j}\frac{\partial\eta_i^j}{\partial x_i}e_j.$$

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We then erroneously claimed that $D_0^*\eta_0=0$ implied that $\partial\eta_i^j/\partial x_i=0$ for each individual term. This is the error in the proof.

Our new assumptions fix this as follows. We have

$$\nabla^0\eta_0=\sum_{i,j,k}\frac{\partial\eta_i^j}{\partial x_k}e_j\otimes\omega^k\otimes\omega^i$$

at p, and therefore we have

$$\|\nabla^0\eta_0\|^2=\sum_{i,j,k}\left|\frac{\partial\eta_i^j}{\partial x_k}\right|^2.$$

In particular, a bound on $\|\nabla^0\eta_0\|$ gives a bound on each individual $|\partial\eta_i^j/\partial x_k|$. With this bound, we can then bound the derivative of the Christoffel symbols exactly as we claimed before, and the rest of the proof remains valid. □

In [BB], to apply Proposition 6.4, we used the fact that if η is harmonic strain field (as defined there), then $*D\eta$ is also a harmonic strain field, and the pointwise norm of both can be bounded in the same way via Theorem 6.5 in [BB]. Here, we need to bound $\nabla\eta$ which is no longer a form but a $(1,2)$-tensor. Our bound will be obtained using standard results about elliptic partial differential equations (PDEs).

PROPOSITION 2. *Let η be a harmonic strain field on a ball B_R of radius R centered at p. Then, there exist a constant C_R such that*

$$\|\nabla\eta(p)\|\leqslant C_R\sqrt{\int_{B_R}\|\eta\|^2\,dV}.$$

Proof. The harmonic strain field is a $(1,1)$-tensor that satisfies the equation

$$\nabla^*\nabla\eta-2\eta=0.$$

(See [Bro, p. 824].) This is a linear elliptic system with smooth coefficients; so, by standard interior regularity results, we have

$$\|\eta\|_{H^3(B_{R/2})}\leqslant C\sqrt{\int_{B_R}\|\eta\|^2\,dV}$$

for some constant C that only depends on R. (See, for example, [McL, Theorem 6.4].) Here $H^3(B_{R/2})$ is the L^2-Sobolev space of degree 3. The Sobolev embedding theorem then gives bounds on the norm of η in the C^1-topology. (See, for example, [AF, Theorem 4.12].) This, in turn, gives our pointwise bound on $\|\nabla\eta(p)\|$. □

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